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Gaurav Shukla

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Preface

The development of nation is directly proportional to the requirement of engineers. India being a developing country, absorbs huge number of engineers every year, and their demand in coming days cannot be overlooked. There is a lot of craze among the aspirants who like to crack GATE/IES and PSUs through many competitive exams. An aspirant would like to learn lot of things in short duration but in less volume, so keeping above point of view ,I have come up with **Handbook Mechanical Engineering** .This handbook is meant for an exhaustive and precise collection of all subjects that come under **Mechanical Engineering.** It encompasses the topics of leading exams in engineering cadre i.e.,GATE,IES and PSUs.

The key features of this book are

- Each topic is summarized in an exhaustive manner in the form of key points and notes.
- Every topic is taken up separately along with key points and notes.
- Focused material in entirety to prevent ambiguity in concepts.

I am thankful to Arihant Publication (India) Limited for giving me this opportunity to write such a book which covers almost 100% syllabus of GATE,IES and PSUs and thus enlightens the path to your success.

I would also like to thank **Er.Akash Shukla** (Project Coordinator) for giving me full support during this project. Valuable suggestions are always welcome for further improvement.

Gaurav Shukla

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1

Mechanics

Force System

When a member of forces simultaneously acting on the body, it is known as force system. A force system is a collection of forces acting at specified locations. Thus, the set of forces can be shown on any free body diagram makes-up a force system.

Truss

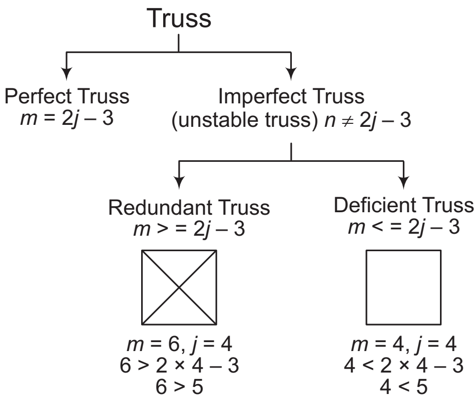
It is a rigid structure composed of number of straight members pin jointed to each other. It can sustain static or dynamic load without any relative motion to each other.

Types of Truss

1. **Plane Truss** It is defined as a truss in which members are essentially lies in a single plane.
2. **Rigid Truss** Rigid means there is no deformation take place due to internal strain in members.
3. **Simple Truss** This type of trusses built a basic triangle by adding different members are known as **simple truss**.

Classification of a Truss (based on joints)

Truss can be classified on the basis of joints (j) and members (m) in the structure. It can be easily understood with the help of following hierarchical approach.



Classification of truss

where m = number of members and j = number of joints

Key Points

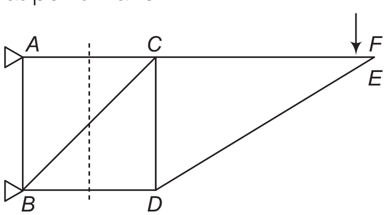
- When truss collapse under loading, then truss is known as unstable or imperfect truss.
- When truss is not collapse under the loading, then truss is known perfect truss.

Analysis of a Framed Structure (Section Method)

1. This method is used when the forces in the few members of a truss is required to found out in a truss structure.

To find out force in AC, BC and BD

- First cut a section which passes through AC, BC and BD members.
- Find out reaction at point A and B



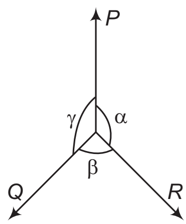
Framed structure using section method

- Find out forces F_{CA}, F_{CB} and F_{DB} in members CA, CB and DB respectively by taking moment about A and B.

2. **Analytical Method** In this method, the free body diagram of each joint is separately analysed to find magnitude of stresses in the truss members.

Lami's Theorem

If a body is in equilibrium under three concurrent forces, the each force is proportional to the sine of the angle between other two.



Three concurrent forces P, Q and R

$$\frac{P}{\sin\beta} = \frac{Q}{\sin\alpha} = \frac{R}{\sin\gamma}$$

Friction Force

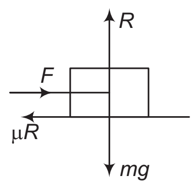
It is resistant force which acts in opposite direction at the surface in body which tend to move or its move.

Normal force $R = mg$

If $\mu\, mg > F$, the body will not move.

$\mu\, mg = F$ the body will tend to move.

$\mu\, mg < F$ the body will move.



Friction force on a body

Angle of Friction

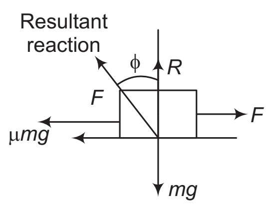
It is defined as the angle between normal reaction and resultant reaction when the body is in condition of just sliding.

$$\tan\phi = \frac{\mu\, mg}{R}$$

$$= \frac{\mu\, mg}{mg} = \mu \qquad (\because R = mg)$$

$$\phi = \tan^{-1}\mu$$

μ = coefficient of friction



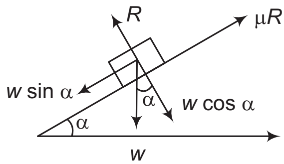
Angle of friction due to resultant reaction

Angle of Repose (α)

It is defined as angle of inclined plane with horizontal at which body is in condition of just sliding.

$$\alpha = \phi$$

Angle of friction is equal to angle of repose.



Angle of Repose (α)

Plane Motion

When all parts of the body move in a parallel planes then a rigid body said to perform plane motion.

Key Points

- The motion of rigid body is said to be translation, if every line in the body remains parallel to its original position at all times.
- In translation motion, all the particles forming a rigid body move along parallel paths.
- If all particles forming a rigid body move along parallel straight line, it is known as rectilinear translation.
- If all particles forming a rigid body does not move along a parallel straight line but they move along a curve path, then it is known as curvilinear translation.

Straight Line Motion

It defines the three equations with the relationship between velocity, acceleration, time and distance travelled by the body. In straight line motion, acceleration is constant.

$$v = u + at$$

$$s = ut + \frac{1}{2} at^2$$

$$v^2 = u^2 + 2as$$

where, u = initial velocity

v = final velocity

a = acceleration of body

t = time

s = distance travelled by body

Distance travelled in nth second

$$s_n = u + \frac{1}{2} a (2n - 1)$$

Projectile Motion

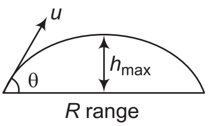
Projectile motion defines that motion in which velocity has two components, one in horizontal direction and other one in vertical direction. Horizontal component of velocity is constant during the flight of the body as no acceleration in horizontal direction.

Let the block of mass is projected at angle θ from horizontal direction

$$\text{Maximum height } h_{max} = \frac{u^2 \sin^2 \theta}{2g}$$

$$\text{Time of flight } T = \frac{2\,u \sin \theta}{g}$$

$$\text{Range } R = \frac{u^2 \sin 2\theta}{g}$$



Projectile motion

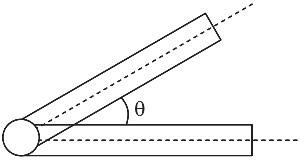
where, u = initial velocity

Key Points

- At maximum height vertical component of velocity becomes zero.
- When a rigid body move in circular paths centered on the same fixed axis, then the particle located on axis of rotation have zero velocity and zero acceleration.
- Projectile motion describe the motion of a body, when the air resistance is negligible.

Rotational Motion with Uniform Acceleration

Uniform acceleration occurs when the speed of an object changes at a constant rate. The acceleration is the same over time. So, the rotation motion with uniform acceleration can be defined as the motion of a body with the same acceleration over time. Let the rod of block rotated about a point in horizontal plane with ω angular velocity.



Rotational motion

$$\text{Angular velocity } \omega = \frac{d\theta}{dt} \text{ (change in angular displacement per unit time)}$$

$$\text{Angular acceleration } \alpha = \frac{d\omega}{dt} \Rightarrow \alpha = \frac{d^2\theta}{dt^2}$$

where θ = angle between displacement.

In case of angular velocity, the various equations with the relationships between velocity, displacement and acceleration are as follows.

$$\theta = \omega t$$
$$\alpha = 0$$
$$\omega = \omega_0 + \alpha t$$
$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$
$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

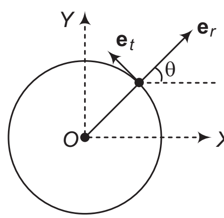
where ω_0 = initial angular velocity
ω = final angular velocity
α = angular acceleration
θ = angular displacement

Angular displacement in *n*th second

$$\theta_n = \omega_0 + \frac{1}{2}\alpha\,(2n-1)$$

Relation between Linear and Angular Quantities

There are following relations between linear and angular quantities in rotational motion.



Position of radial and tangential vectors

$$|\mathbf{e}_r| = |\mathbf{e}_t| = 1$$

$\mathbf{e}_r$ and $\mathbf{e}_t$ are radial and tangential unit vector.

Linear velocity $v = r\omega\,\mathbf{e}_t$

Linear acceleration (Net)

$$a = -\,\omega^2 r\,\mathbf{e}_r + \frac{dv}{dt}\,\mathbf{e}_t$$

Tangential acceleration $a_t = \frac{dv}{dt}$ (rate of change of speed)

Centripetal acceleration $a_r = \omega^2 r = \frac{v^2}{r}$ $(\because\ v = r\omega)$

Net acceleration
$$a = \sqrt{a_r^2 + a_t^2}$$
$$= \sqrt{\left(\frac{v^2}{r}\right)^2 + \left(\frac{dv}{dt}\right)^2}$$

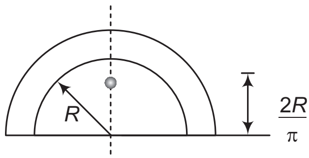
where a_r = centripetal acceleration
a_t = tangential acceleration

Centre of Mass of Continuous Body

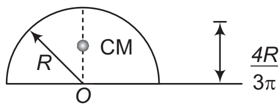
Centre of mass of continuous body can be defined as

- Centre of mass about x, $x_{CM} = \dfrac{\int x\,dm}{\int dm} = \dfrac{\int x\,dm}{M}$
- Centre of mass about y, $y_{CM} = \dfrac{\int y\,dm}{\int dm} = \dfrac{\int y\,dm}{M}$
- Centre of mass about z, $z_{CM} = \dfrac{\int z\,dm}{\int dm} = \dfrac{\int z\,dm}{M}$

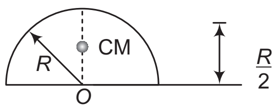
- CM of uniform rectangular, square or circular plate lies at its centre.
- CM of semicircular ring



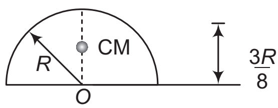
- CM of semicircular disc



- CM of hemispherical shell



- CM of solid hemisphere



Law of Conservation of Linear Momentum

The product of mass and velocity of a particle is defined as its **linear momentum** (p).

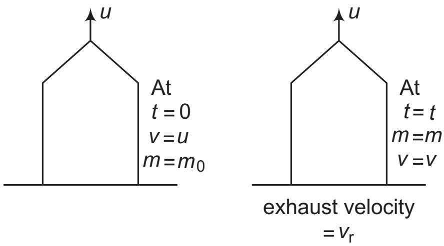
$$p = mv$$
$$p = \sqrt{2Km}$$

$$F = \frac{dp}{dt}$$

where, K = kinetic energy of the particle
F = net external force applied to body
P = momentum

Rocket Propulsion

Let m_0 be the mass of the rocket at time $t = 0$, m its mass at any time t and v its velocity at that moment. Initially, let us suppose that the velocity of the rocket is u.



Rocket propulsion

- Thrust force on the rocket $F_t = v_r\left(-\frac{dm}{dt}\right)$

  where, $-\frac{dm}{dt}$ = rate at which mass is ejecting
  v_r = relative velocity of ejecting mass (exhaust velocity)
- Weight of the rocket $w = mg$
- Net force on the rocket $F_{net} = F_t - w = v_r\left(\frac{-dm}{dt}\right) - mg$
- Net acceleration of the rocket

$$a = \frac{F}{m}$$

$$\frac{dv}{dt} = \frac{v_r}{m}\left(\frac{-dm}{dt}\right) - g$$

$$v = u - gt + v_r \ln\frac{m_0}{m}$$

where, m_0 = mass of rocket at time $t = 0$
m = mass of rocket at time t

Impulse

The product of constant force F and time t for which it acts is called the impulse (J) of the force and this is equal to the change in linear momentum which it produces.

$$\text{Impulse } J = F\,t$$

$$\Rightarrow \qquad \Delta p = p_f - p_i$$

where, F = constant force

P = linear momentum

Instantaneous Impulse *e.g.*, bat and ball contact

$$J = \int F \cdot dt \;\Rightarrow\; \Delta p = p_f - p_i$$

Key Points

- The relation between impulse and linear momentum can be understood by the following equation.

$$Ft = m\,(v - u)$$

where, F = force, t = time, m = mass, v = initial velocity, u = final velocity

- Rotation about a fixed point gives the three dimensional motion of a rigid body attached at a fixed point.

Collision

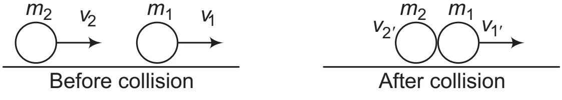
A Collision is an isolated event in which two or more moving bodies exert forces on each other for a relatively short time.

Collision between two bodies may be classified in two ways

- Head-on collision
- Oblique collision.

Head-on Collision

Let the two balls of masses m_1 and m_2 collide directly with each other with velocities v_1 and v_2 in direction as shown in figure. After collision the velocity become v_1' and v_2' along the same line.



$$v_1' = \left(\frac{m_1 - em_2}{m_1 + m_2}\right) v_1 + \left(\frac{m_2 + em_2}{m_1 + m_2}\right) v_2$$

$$v_2' = \left(\frac{m_2 - em_2}{m_1 + m_2}\right) v_2 + \left(\frac{m_1 + em_1}{m_1 + m_2}\right) v_1$$

where, m_1 = mass of body 1
m_2 = mass of body 2
v_1 = velocity of body 1
v_2 = velocity of body 2
v_1' = velocity of body 1 after collision
v_2' = velocity of body 2 after collision

where e = coefficient restitution

$$e = \frac{\text{Separation speed}}{\text{Approach speed}}$$

$$e = \frac{v_1' - v_2'}{v_2 - v_1}$$

- In case of head-on elastic collision

$$e = 1$$

- In case of head-on inelastic collision

$$0 < e < 1$$

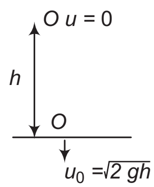
- In case of head-on perfectly inelastic collision

$$e = 0$$

If e is coefficient of restitution between ball and ground, then after nth collision with the floor, the speed of ball will remain $e^n v_0$ and it will go upto a height e^{2n} h.

$$v_n = e^n v_0 = e^n \sqrt{2gh}$$

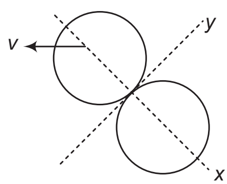
$$h_0 = e^{2n} h$$



Collision of a ball with floor

Oblique Collision

In case of oblique collision linear momentum of individual particle do change along the common normal direction. No component of impulse act along common tangent direction. So, linear momentum or linear velocity remains unchanged along tangential direction. Net momentum of both the particle remain conserved before and after collision in any direction.



Oblique collision

Moment of Inertia

Momentum of inertia can be defined as

r = distance of the body of mass, m from centre of axis.

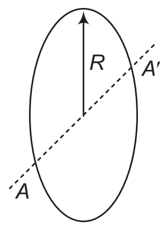
$$I = \sum_i m_i r_i^2$$

$$I = \int r^2 \, dm$$

- Very thin circular loop (ring)

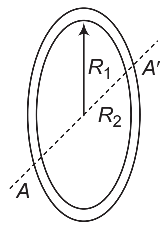
$$I = MR^2$$

where, M = mass of the body
R = radius of the ring
I = moment of inertia



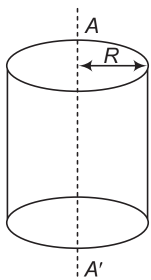
Thin circular ring

- Uniform circular loop $I = M\left(\dfrac{R_1^2 + R_2^2}{2}\right)$



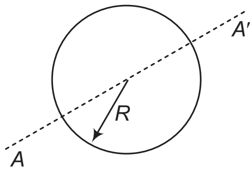
Uniform circular loop

- Uniform solid cylinder $I = \dfrac{MR^2}{2}$



Uniform solid cylinder

- Uniform solid sphere

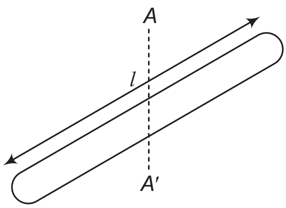


Uniform solid sphere

$$I = \frac{2}{5} MR^2$$

- Uniform thin rod

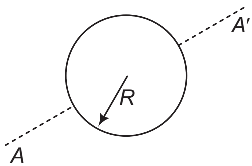
(AA') moment of inertia about the centre and perpendicular axis to the rod moment of inertia about the one corner point and perpendicular (BB') axis to the rod.



Uniform thin rod

$$I = \frac{Ml^2}{12} \quad \Rightarrow \quad I = \frac{1}{3} Ml^2$$

- Very thin spherical shell

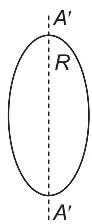


Thin sperical shell

$$I = \frac{2}{3} MR^2$$

- Thin circular sheet

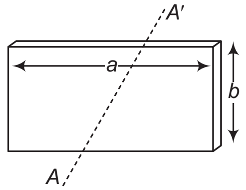
$$I = \frac{MR^2}{4}$$



Thin circular sheet

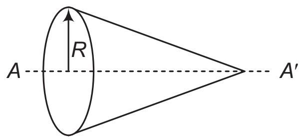
- Thin rectangular sheet

$$I = M\left(\frac{a^2 + b^2}{12}\right)$$



Thin rectangular sheet

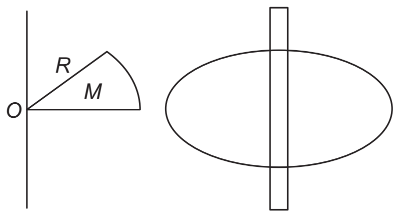
- Uniform right cone



Uniform right cone

$$I = \frac{3}{10} MR^2$$

- Uniform cone as a disc



A part of uniform cone as a disc

Suppose the given section is $\frac{1}{n}$th part of the disc, then mass of disc will be nM.

Inertia of the disc,

$$I_{disc} = \frac{1}{2}(nM) R^2$$

Inertia of the section,

$$I_{section} = \frac{1}{n} I_{disc} = \frac{1}{2} MR^2$$

Torque and Angular Acceleration of a Rigid Body

For a rigid body, net torque acting

$$\tau = I\alpha$$

where, α = angular acceleration of rigid body
I = moment of inertia about axis of rotation

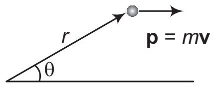
- Kinetic energy of a rigid body rotating about fixed axis

$$KE = \frac{1}{2} I\omega^2 \qquad (\omega = \text{angular velocity})$$

- Angular moment of a particle about same point

$$\mathbf{L} = \mathbf{r} \times \mathbf{p}$$

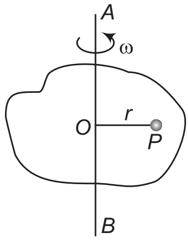
$$\mathbf{L} = m(\mathbf{r} \times \mathbf{v})$$



Angular moment

where L = angular displacement

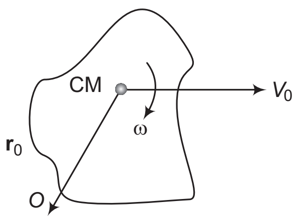
- Angular moment of a rigid body rotating about a **fixed axis**. $L = I\omega$



Angular moment of a rigid body

- Angular moment of a rigid body in combined rotation and translation

$$\mathbf{L} = \mathbf{L}_{CM} + M(\mathbf{r}_0 \times \mathbf{v}_0)$$



Combined rotation and translation in a rigid body

- Conservation of angular momentum

$$\tau = \frac{d\mathbf{L}}{dt}$$

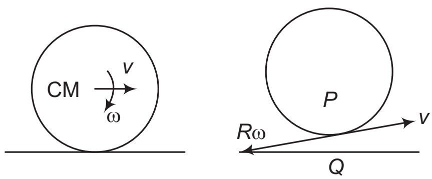
$$\frac{d\mathbf{L}}{d\mathbf{t}} = \mathbf{r} \times \mathbf{F} + \mathbf{v} \times \mathbf{p}$$

- Kinetic energy of rigid body in combined translational and rotational motion

$$K = \frac{1}{2} m v_{CM}^2 + \frac{1}{2} I_{CM} \omega^2$$

Uniform Pure Rolling

Pure rolling means no relative motion or no slipping at point of contact between two bodies.



Uniform Pure Rolling

If $v_P = v_Q \Rightarrow$ no slipping

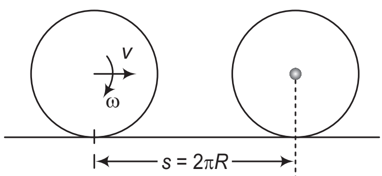
$v = R\omega$

if $v_p > v_Q \Rightarrow$ forward slipping

$v > R\omega$

if $v_P < v_Q \Rightarrow$ backward slipping

$v < R\omega$



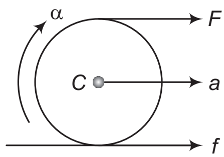
Pure Rolling

No slipping $s = 2\pi R$
Forward slipping $s > 2\pi R$
Backward slipping $s < 2\pi R$

Accelerated Pure Rolling

A pure rolling is equivalent to pure translation and pure rotation. It follows a uniform rolling and accelerated pure rolling can be defined as

$$F + f = Ma$$
$$(F - f) \cdot R = I\alpha$$



Accelerated pure rolling

F = force acting on a body, f = friction on that body

Angular Impulse

The angular impulse of a torque in a given time interval is defined as

$$\int_{t_1}^{t_2} \tau \cdot dt$$

$$\int_{t_1}^{t_2} \tau \cdot dt = L_2 - L_1$$

where, L_2 and L_1 are the angular momentum at time t_2 and t_1 respectively.

Key Points

- A force, whose line of action does not pass through centre of mass, works as force to produce translational acceleration.
- Different types of collisions are examined, whether they possess kinetic energy or not.
- The radial component of the force, which goes through the axis of rotation, has no contribution to torque.

2

Strength of Materials

Engineering Mechanics

The branch of physical science that deals with the state of rest or the state of motion is termed as **Mechanics.** Starting from the analysis of rigid bodies gravitational force and simple applied forces, the mechanics has grown to the analysis of robotics, air crafts etc. is known is **Engineering Mechanics.**

Stress

When a material in subjected to an external force, a resisting force is set up within the component. The internal resistance force per unit area acting on a material is called the stress at a point.It is a scalar quantity having unit.

$$\text{Stress} = \frac{\text{Force}}{\text{Area}}$$

Strain

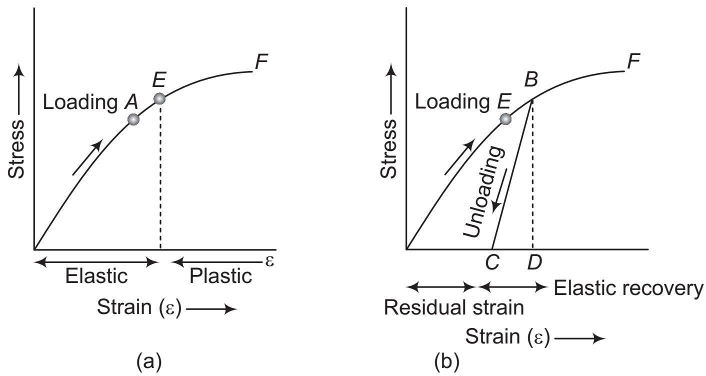
It is the deformation produced in the material due to simple stress. It usually represents the displacement between particles in the body relative to a reference length.

$$\text{Tension strain } (e_t) = \frac{\Delta L}{L}$$

$$= \frac{\text{Change in length}}{\text{Initial length}}$$

Stress-Strain Relationship

The stress-strain diagram is shown in the figure. In brittle materials there is no appreciable change in rate of strain. There is no yield point and no necking takes place.



Graph between stress-strain

In figure (a), the specimen is loaded only upto point *A*, is gradually removed the curve follows the same path *AO* and strain completely disappears. Such a behaviour is known as the **elastic behaviour.**

In figure (b), the specimen is loaded upto point *B* beyond the elastic limit *E*. When the specimen is gradually loaded the curve follows path *BC*, resulting in a residual strain *OC* or permanent strain.

Properties of Materials

Some properties of materials which judge the strength of materials are given below

Elasticity

Elasticity is the property by virtue of which a material is deformed under the load and is enabled to return to its original dimension when the load is removed.

Plasticity

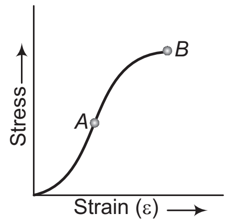
Plasticity is the converse of elasticity. A material in plastic state is permanently deformed by the application of load and it has no tendency to recover. The characteristic of the material by which it undergoes inelastic strains beyond those at elastic limit is known as **plasticity**.

Ductility

Ductility is the characteristic which permits a material to be drawn out longitudinally to a reduced section, under the action of a tensile force (large deformation).

Brittleness

Brittleness implies lack of ductility. A material is said to be brittle when it cannot be drawn out by tension to smaller section.



Stress-strain relation

Malleability

Malleability is a property of a material which permits the material to be extended in all directions without rapture. A malleable material possess a high degree of plasticity, but not necessarily great strength.

Toughness

Toughness is the property of a material which enable it to absorb energy without fracture.

Hardness

Hardness is the ability of a material to resist indentation or surface abrasion. Brinell hardness test is used to check hardness.

$$\text{Brinell Hardness Number (BHN)} = \frac{P}{\frac{\pi D}{2}\left(D - \sqrt{D^2 - d^2}\right)}$$

where, P = Standard load, D = Diameter of steel ball
d = Diameter of the indent.

Strength

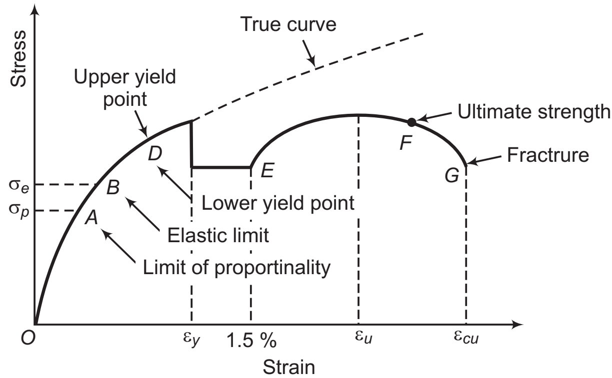
The strength of a material enables it to resist fracture under load.

Engineering Stress-Strain Curve

The stress-strain diagram is shown in figure. The curve start from origin. Showing thereby that there is no initial stress of strain in the specimen.

The stress-strain curve diagram for a ductile material like mild steel is shown in figure below

- Upto point A, Hooke's Law is obeyed and stress is proportional to strain. Point A is called **limit of proportionality.**



Stress-strain diagram for mild steel

- Point *B* is called the **elastic limit point.**
- At point *B* the cross-sectional area of the material starts decreasing and the stress decreases to a lower value to point *D*, called the **lower yield point.**
- The apparent stress decreases but the actual or true stress goes on increasing until the specimen breaks at point *C*, called the **point of fracture.**
- From point *E* ownward, the strain hardening phenomena becomes predominant and the strength of the material increases thereby requiring more stress for deformation, until point *F* is reached. Point *F* is called the **ultimate point.**

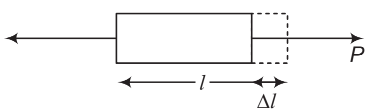
Hooke's Law (Linear elasticity)

Hooke's Law stated that within elastic limit, the linear relationship between simple stress and strain for a bar is expressed by equations.

$$\sigma \propto \varepsilon, \sigma = E\varepsilon$$

$$\frac{P}{A} = E\frac{\Delta l}{l}$$

where, E = Young's modulus of elasticity
P = Applied load across a cross-sectional area
Δl = Change in length
l = Original length.



Elastic deformation

Elongation of Bodies

Elongation of a body is defined as the transformation of a body from a reference configuration to a current configuration. A configuration is a set containing the positions of all particles of the body. *The following cases will be considered*

Bars of Varying Sections of Different Materials

It can be shown below

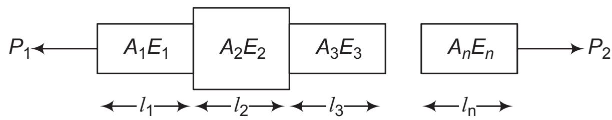


Diagram for different materials

The total deformation for such a bar is given by

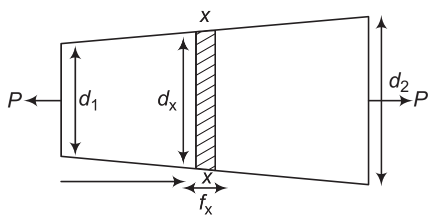
$$\Delta = \frac{P_1 l_1}{A_1 E_1} + \frac{P_2 l_2}{A_2 E_2} \ldots + \frac{P_n l_n}{A_n E_n}$$

Uniformly Tapering Circular Bar

Let us consider a uniformly tapering circular bar subjected to an axial force P. The bar of length l has diameter d_1 at one end and d_2 at other end.

$$dx = d_1 + \frac{d_2 - d_1}{l} \cdot x$$

Change in length $\Delta l = \dfrac{4Pl}{\pi E d_1 d_2}$



Tapering bars

Elongation of Bar of Uniform Section due to Self Weight

Small deformation of small cross section length dx

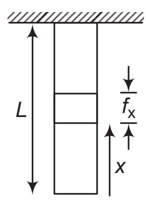
$$f_x = \frac{w_x}{A} \frac{f_x}{E}$$

where, w_x = Weight of portion below the section.

Total change in length

$$\Delta l = \frac{wL}{2AE}$$

where, w = Total weight of bar.

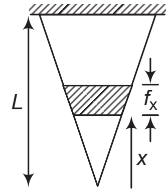


Elongation of bar

Elongation of Bar of Uniform Tapering Section

Total change in length

$$\Delta l = \frac{\rho g\ L^2}{\sigma E}$$

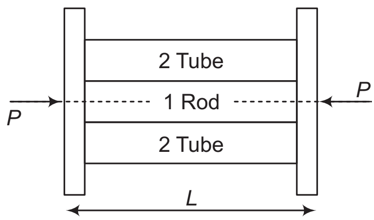


Tapering section

Compounded Bars

Consider a solid box enclosed in the hollow tube and subjected to a compression force P through rigid collars as shown in the figure.

Let P_1 and P_2 are force applied on rod and tube respectively.



Compound bars

Total force $P = P_1 + P_2$

Change in length for rod = change in length for tube

$$\Delta l_1 = \Delta l_2$$

$$\frac{P_1 L_1}{A_1 E_1} = \frac{P_2 L_2}{A_2 E_2}$$

Stress in Bolts and Nuts

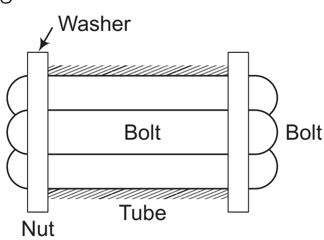
Stress in bolts and nuts are shown as in the figure.

Let stress in steel bolt is σ_s across the cross-sectional area A_s and stress in copper tube is σ_c across cross-sectional area A_c.

$$\sigma_c A_c = \sigma_s A_s$$

where, σ_c = Stress in copper

σ_c = Stress in steel



Stress in bolt and nut

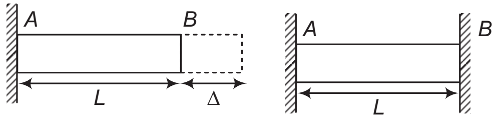
Since the final length of bolt and tube is same.

Total extension of bolt + Total compression of tube = Moment of nut

$$\frac{\sigma_s L}{E_s} + \frac{\sigma_c L}{E_c} = \text{moment of nut} \qquad \text{(Pitch of thread} \times \text{total turn)}$$

Temperature Stresses in Uniform Bars

Consider a bar of length L subjected to uniform temperature Δt increase.



Bar free to expand   Bar constrained

Increase in length of bar when bar free to expand

$$\Delta l = \varepsilon_t L = \alpha \Delta tL$$
$$\varepsilon_t = \text{thermal strain} = \alpha \ \Delta t$$

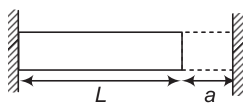
where, α = coefficient of the thermal expansion.

If the bar is constrained, net thermal stress

$$\sigma_t = \frac{\Delta l E}{L} = L \alpha \Delta t \frac{E}{L} \quad \Rightarrow \quad \sigma_t = E \alpha \Delta t$$

Suppose in one of supports yields by an amount a. The total amount of expansion checked will be $(\Delta l - a)$.

$$\sigma_t = (\Delta l - a) \frac{E}{L}$$
$$\sigma_t = (L \alpha \Delta t - a) \frac{E}{L}$$



Expansion in rod

Key Points

- The stress will be compressive when the change in temperature is positive.
- The stress will be tensile when the change in temperature is negative.

Temperature Stresses in Composite Bar

Suppose composite bar made of two materials with different coefficient of thermal expansion assume $\alpha_1 < \alpha_2$.

Tensile force in 1 = tensile force in 2

$$P_1 = P_2 = P_1$$

Final extension of 1 = final extension of 2

$$\Delta l_1 = \Delta l_2 = \Delta l$$

From geometry,

$$\Delta l_1 = \Delta l_1' + \Delta l_1''$$
$$\Delta l_2 = \Delta l_2' + \Delta l_2''$$

$\Delta l_1'$ = Free expansion of 1 due to temperature rise
$\Delta l_2'$ = Free expansion of 2 due to temperature rise
$\Delta l_1''$ = Expansion of 1 due to temperature stress (tensile)
$\Delta l_2''$ = Compression of 2 due to temperature stress (compression)

$$\Delta l_1' + \Delta l_1'' = \Delta l_2' - \Delta l_2''$$

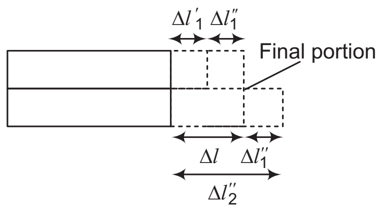
$$\Delta l_1'' + \Delta l_2'' = \Delta l_2' - \Delta l_1'$$

$$\frac{PL}{A_1 E_1} + \frac{PL}{A_2 E_2} = L\Delta t\,(\alpha_2 - \alpha_1) \quad \ldots\text{(i)}$$

or

$$P = \frac{\Delta t\,(\alpha_2 - \alpha_1)}{\dfrac{1}{A_s E_s} + \dfrac{1}{A_c E_c}}$$

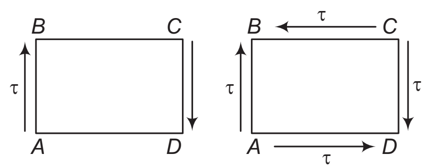
$$\Delta l = \frac{(\alpha_1 A_1 E_1 + \alpha_2 A_2 E_2)\, L\Delta t}{A_1 E_1 + A_2 E_2}$$



Expansion in composite bar

Complementry Shear Stress

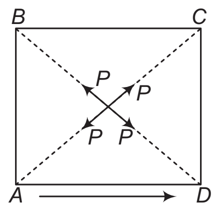
A shear stress in a given direction cannot exist without a balancing shear stress of equal intensity in a direction at right angles to it.



Complementary shear stress

A state of simple shear produces pure tensile and compressive stresses across planes inclined at 45° to those of pure shear and intensities of these direct stresses are each equal to the intensity of the pure shear stress. Linear strain of the diagonal is equal to half the shear strain (ϕ).

$$e = \frac{\phi}{2}$$



Compressive stress

Poisson's Ratio

When an axial force is applied along the longitudinal axis of a bar, the length of a bar will increase but at the same time its lateral dimension (width) will be decreased so, it is called as **Poisson' ratio.**

$$\mu = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

Value of Poisson's ratio is same in tension and compression.

Volumetric Strain

It is defined as the ratio of change in volume to the initial volume. Mathematically,

$$\text{Volumetric strain, } e_v = \frac{\text{Change in volume}}{\text{Initial volume}} = \frac{\Delta V}{V}$$

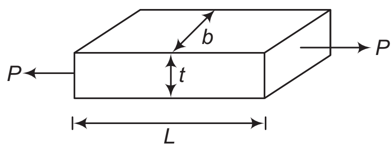
Volumetric Strain Due to Single Direct Stress

The ratio of change in volume to original volume is called **volumetric strain**.

$$e_V = e_1 + e_2 + e_3$$

$$\frac{\Delta V}{V} = \frac{fL}{L} + \frac{fb}{b} + \frac{ft}{t}$$

$$e_1 = \frac{P}{E}$$



Volumetric strain

$$e_2 = -\frac{P}{E}\mu, \; e_3 = -\mu\frac{P}{E}$$

$$\text{Volumetric strain,} \quad e_V = \frac{\Delta V}{V}, \; e_V = \frac{P}{E}(1-2\mu)$$

For the circular bar of diameter d, $V = \frac{\pi}{4}d^2L$

$$\frac{f\,V}{V} = \frac{f\,L}{L} + \frac{2fd}{d}, \; \frac{\Delta\,L}{L} = e_1 = \frac{P}{E}$$

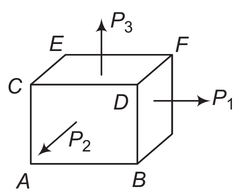
$$\frac{\Delta d}{d} = e_2 = -\mu\frac{P}{E}$$

$$e_V = \frac{P}{E}(1-2\mu), \; e = \frac{P}{E}\left(1-\frac{2}{m}\right), \qquad \left(\because \mu = \frac{1}{m}\right)$$

Volumetric Strain due to Three Mutually Perpendicular Stress System

When a body is subjected to identical pressure in three mutually perpendicular direction, then the body undergoes uniform changes in three directions without undergoing distortion of shape.

$$e_1 = \frac{P_1}{E} - \mu\frac{P_2 + P_3}{E}$$



Three stress system

$$e_2 = \frac{P_2}{E} - \mu \frac{P_3 + P_1}{E}$$

$$e_3 = \frac{P_3}{E} - \mu \frac{P_1 + P_2}{E}$$

$$e_V = e_V + e_2 + e_3$$

$$e_V = (1 - 2\mu)\left(\frac{P_1 + P_2 + P_3}{E}\right)$$

or $$e_V = \left(1 - \frac{2}{m}\right)\left(\frac{P_1 + P_2 + P_3}{E}\right) \qquad \left(\because \mu = \frac{1}{m}\right)$$

Shear Modulus or Modulus of Rigidity

Modulus of rigidity $$G = \frac{\text{shear stress}}{\text{shear strain}} = \frac{\tau}{\phi}$$

- At principal planes, shear stress is always zero.
- Planes of maximum shear stress also contains normal stress.

Relationship between *E*, *G*, *K* and μ

Modulus of rigidity $$G = \frac{E}{2(1+\mu)}$$

Bulk modulus $$K = \frac{E}{3(1-2\mu)} \quad \text{or} \quad E = \frac{9KG}{3K + G}$$

$$\mu = \frac{3K - 2G}{6K + 2G}$$

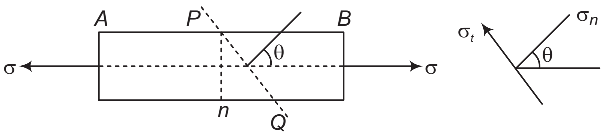
Analysis of Stress and Strain

We will derive some mathematical expressions for plains stresses and will study their graphical significance in 2 *D* and 3 *D*.

Stress on Inclined Section *PQ* due to Uniaxial Stress

Consider a rectangular cross-section and we have to calcutate the stress on an inclined section as shown in figure.

Normal stress $$\sigma_n = \sigma \cos^2 \theta$$



Stress on an inclined section

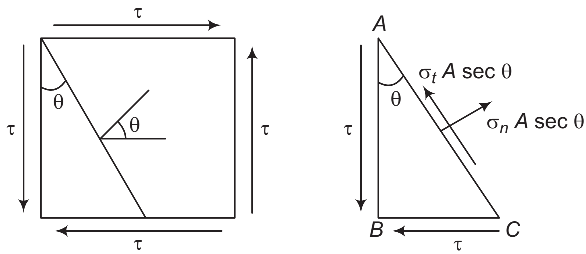
Tangential stress $\sigma_t = -\dfrac{\sigma}{2}\sin 2\theta$

Resultant stress $\sigma_r = P\cos\theta$

Stress Induced by State Simple Shear

Induced stress is divided into two components which are given as

$$\text{Normal stress, } \sigma_n = \tau \sin 2\theta$$
$$\text{Tangential stress, } \sigma_t = \tau \cos 2\theta$$



Stress simple shear

Stress Induced by Axial Stress and Simple Shear

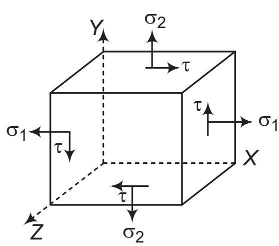
Normal stress

$$\sigma_n = \sigma_1 \cos^2\theta + \sigma_2 \sin^2\theta + \tau \sin 2\theta$$

or
$$\sigma_n = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2}$$
$$\cos 2\theta + \tau \sin 2\theta$$

Tangential stress

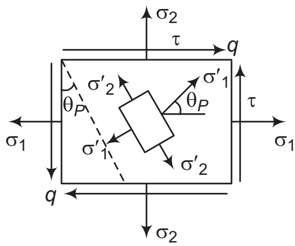
$$\sigma_t = -\left(\frac{\sigma_1 + \sigma_2}{2}\right)\sin 2\theta + \tau \cos 2\theta$$



Induced stress body diagram

Principal Stresses and Principal Planes

The plane carrying the maximum normal stress is called the **major principal plane** and normal stress is called **major principal stress**. The plane carrying the minimum normal stress is known as **minor principal stress.**



Principal stress and planes

Major principal stress (σ_1'),

$$\sigma_1' = \frac{\sigma_1 + \sigma_2}{2} + \sqrt{\frac{(\sigma_1 - \sigma_2)^2}{2} + \tau^2}$$

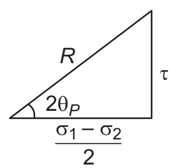
Minor principal stress (σ_2'), $\sigma_2' = \frac{\sigma_1 + \sigma_2}{2} - \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$

$$\tan 2\theta_P = \frac{2\tau}{\sigma_1 - \sigma_2}$$

$$\sigma_1' + \sigma_2' = \sigma_1 + \sigma_2$$

When $2\,\theta_P = 0$

$\Rightarrow$ $\sigma_1' = \sigma_1$ and $\sigma_2' = \sigma_2$



2 θ_P triangle

Across maximum normal stresses acting in plane shear stresses are zero.

Computation of principal Stress from Principal Strain

The three stresses normal to shear principal planes are called principal stress, while a plane at which shear strain is zero is called **principal strain.**

For two dimensional stress system, $\sigma_3 = 0$

$$\sigma_1 = \frac{E_1(\varepsilon_1 + \mu\varepsilon_2)}{1 - \mu^2}, \quad \sigma_2 = \frac{E(\mu\varepsilon_1 + \varepsilon_2)}{1 - \mu^2}$$

Maximum Shear Stress

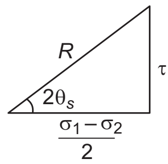
The maximum shear stress or maximum principal stress is equal of one half the difference between the largest and smallest principal stresses and acts on the plane that bisects the angle between the directions of the largest and smallest principal stress, *i.e.,* the plane of the maximum shear stress is oriented 45° from the principal stress planes.

$$\tau' = \sqrt{\frac{(\sigma_1 - \sigma_2)^2}{2} + \tau^2}$$

$$\tau' = \frac{\sigma_1' - \sigma_2'}{2}$$

$$\tan 2\theta_s = -\frac{\sigma_1 - \sigma_2}{2\tau} = -\frac{1}{\tan 2\theta_P}$$

$$2\theta_s = 2\theta_P \pm 90°, \ \theta_s = \theta_P \pm 45°$$



$2\theta_s$ triangle

Principal Strain

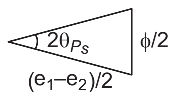
For two dimensional strain system,

$$\varepsilon_{1,2} = \frac{e_1 + e_2}{2} \pm \sqrt{\frac{e_1 - e_2}{2} + \left(\frac{\phi}{2}\right)^2}$$

where, e_1 = Strain in x-direction

e_2 = Strain in y-direction

ϕ = Shearing strain relative to OX and OY.



$2\theta_{P_S}$ triangle

$$\tan 2\theta_{P_S} = \frac{\phi/2}{\frac{e_1 - e_2}{2}}$$

$$\tan 2\theta_{P_S} = \frac{\phi}{e_1 - e_2}$$

Maximum Shear Strain

The maximum shear strain also contains normal strain which is given as

$$\frac{\phi_{max}}{2} = \sqrt{\left(\frac{e_1 - e_2}{2}\right)^2 + \left(\frac{\phi}{2}\right)^2}$$

45° Strain Rosette or Rectangular Strain Rosette

Rectangular strains Rosette are inclined 45° to each other.

$$e_a = \frac{1}{2}(\varepsilon_1 + \varepsilon_2) + \frac{1}{2}(\varepsilon_1 - \varepsilon_2)\cos 2\theta$$

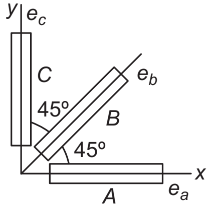
$$e_b = \frac{1}{2}(\varepsilon_1 + \varepsilon_2) - \frac{1}{2}(\varepsilon_1 - \varepsilon_2)\sin 2\theta$$

$$e_c = \frac{1}{2}(\varepsilon_1 + \varepsilon_2) - \frac{1}{2}(\varepsilon_1 - \varepsilon_2)\cos 2\theta$$

Principal strains $\varepsilon_{12} = \frac{e_a + e_b}{2} \pm \frac{1}{\sqrt{2}}$

$$= \sqrt{(e_a - e_b)^2 + (e_b - e_c)^2}$$

$$\tan 2q_{pe} = \frac{2e_b - e_a - e_c}{e_c - e_a}$$



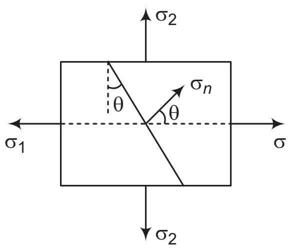
45° strain Rosette

Mohr's Circle

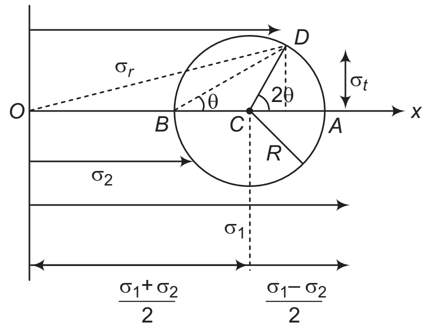
Graphically, variation of normal stress and shear stress are studied with the help of Mohr's circle. A two dimensional Mohr's circle can be constructed, if the normal streses σ_1 and σ_2.

$$\text{Normal stress } \sigma_n = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2} \cos 2\theta$$

$$\text{Shear stress } \tau = -\frac{\sigma_1 - \sigma_2}{2} \sin 2\theta$$



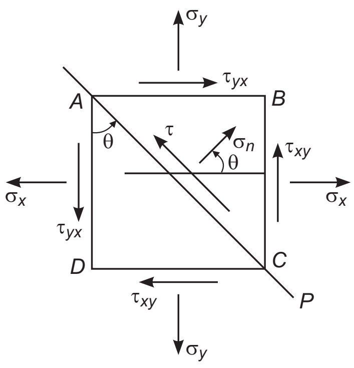
Different stress diagram



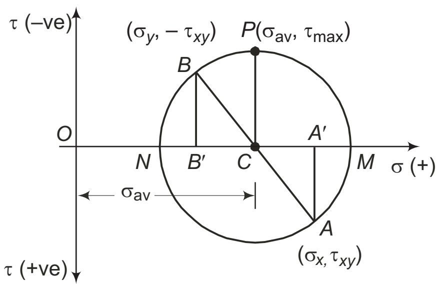
Mohr's circle for plane stress and strain

General State of Stress at an Element

The following procedure is used to draw a Mohr's circle and to find the magnitude and direction of maximum stresses from it.



Mohr's circle approach as state of stress



Mohr's circle for plane stressed and plane strain

Observations from Mohr's Circle

The followings are the observations of Mohr's circle as

- **At Point M on Circle** σ_n is maximum and shear stress is zero.
  $\therefore$ Maximum principal stress $\equiv$ coordinate of M
- **At Point N on Circle** σ_n is minimum and shear stress τ is zero.
  $\therefore$ Minimum principal stress $\equiv$ coordinate of N
- **At Point P on Circle** τ is maximum.
  $\therefore$ Maximum shear stress $\equiv$ ordinate of P (*i.e.,* radius of circle)
  Also, normal stress on plane of maximum shear stress

$$\equiv \text{abscissa of } P \left(i.e.,\ \sigma_n = \sigma_{av} = \frac{\sigma_x + \sigma_y}{2} \right)$$

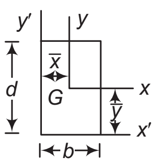
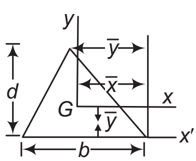
where, $\sigma_n \equiv$ Average stress

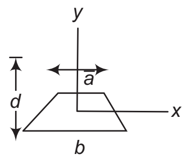
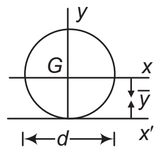
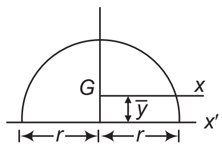
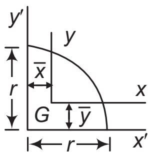
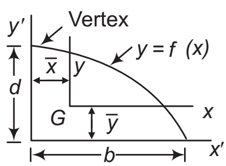
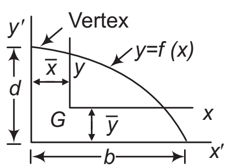
- Mohr's circle becomes zero at a point if radius of circle has the following consideration.

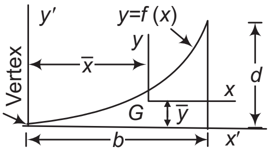
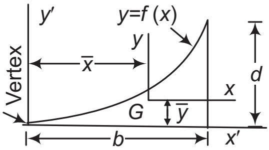
$$\text{Radius of circle} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2{}_{xy}}$$

- If $\sigma_x = \sigma_y$, then radius of Mohr's circle is zero and $\tau_{xy} = 0$

Centroid and Moment of Inertia of Different Sections

Plane Areas	Properties
	**Rectangle** $A = b \cdot d$; $\bar{x} = b/2$; $\bar{y} = d/2$ $I_x = \frac{1}{12} bd^3$; $I_y = \frac{1}{12} db^3$; $I_{xy} = 0$ $I'_x = \frac{1}{3} bd^3$; $I'_y = \frac{1}{3} db^3$; $I_{x'y'} = \frac{b^2d^2}{4}$
	**Triangle** $A = \frac{1}{2} bd$; $\bar{x} = \frac{(b + c)}{3}$; $\bar{y} = \frac{d}{3}$ $I_x = \frac{1}{36} bd^3$; $I_y = \frac{bd}{36}(b^2 - bc + c^2)$ $I_{xy} = \frac{bd^2}{72}(b - 2c)$ $I'_x = \frac{1}{12} bd^3$; $I_{y'} = \frac{bd}{12}(3b^2 - 3bc + c^2)$ $I_{x'y'} = \frac{bd^2}{24}(3b - 2c)$;

Plane Areas	Properties
 	**Trapezoid** $A = d\,\frac{(a + b)}{2}$; $\bar{y} = \frac{d}{3}\left(\frac{2a + b}{a + b}\right)$ $I_x = \frac{d^3\,(a^2 + 4ab + b^2)}{36\,(a + b)}$; $I'_x = \frac{d^3\,(3a + b)}{12}$
 	**Circle** $A = \frac{\pi}{4}\,d^2$; $\bar{y} = \frac{d}{2}$; $\bar{x} = 0$ $I_x = \frac{\pi}{64}\,d^4 = I_y$; $I_{xy} = 0$; $I'_x = \frac{5\pi}{64}\,d^4$
 	**Half circle** $A = \frac{\pi r^2}{2}$; $\bar{y} = \frac{4r}{3\pi}$ $I_x = \frac{(9\pi^2 - 64)\,r^4}{72\pi} = 0.1098r^4$; $I_y = \frac{\pi r^4}{8}$; $I_{xy} = 0$; $I'_x = \frac{\pi r^4}{8}$
 	**Quarter circle** $A = \frac{\pi r^2}{4}$; $\bar{x} = \bar{y} = \frac{4r}{3\pi}$ $I_x = I_y = \frac{(9\pi^2 - 64)\,r^4}{144\,\pi} = 0.05488\,r^4$; $I'_x = I'_y = \frac{\pi r^4}{16}$
 	**Parabolic semi-segment** $y = f(x) = h\,(1 - x^2/b^2)$; $A = \frac{2}{3}\,bd$; $\bar{x} = \frac{3}{8}\,b$; $\bar{y} = \frac{2}{5}\,d$ $I_x = \frac{16}{105}\,bd^3$; $I_y = \frac{2}{15}\,db^3$; $I_{xy} = \frac{1}{12}\,b^2d^2$
 	**Semi segment of *n*th degree** $y = f(x) = h\,(1 - x^n/b^n)$, $n > 0$, $A = bd\left(\frac{n}{n + 1}\right)$, $\bar{x} = \frac{b\,(n + 1)}{2\,(n + 2)}$; $\bar{y} = \frac{dn}{2n + 1}$ $I_x = \frac{2bd^3n^3}{(n + 1)\,(2n + 1)\,(3n + 1)}$; $I_y = \frac{db^3n}{3\,(n + 3)}$

Plane Areas	Properties
	**Parabolic spandrel** $y = f(x) = d\,(x^2/b^2)$ $A = (b \cdot d)/3\,;\ \bar{x} = 3b/4\,;\ \bar{y} = 3d/10$ $I_x = \dfrac{bd^3}{21}\,;\ I_y = \dfrac{db^3}{5}\,;\ I_{xy} = \dfrac{b^2d^2}{12}$
	**Spandrel of *n*th degree** $y = f(x) = d\,(x^n/b^n),\ n > 0$ $A = \dfrac{bd}{n+1}\,;\ \bar{x} = \dfrac{b\,(n+1)}{(n+2)}\,;\ \bar{y} = \dfrac{d\,(n+1)}{2\,(2n+1)}$ $I_x = \dfrac{bd^3}{3\,(3n+1)}\,;\ I_y = \dfrac{db^3}{(n+3)}\,;\ I_{xy} = \dfrac{b^2d^2}{4\,(n+1)}$

Shear Force and **Bending Moment Diagram**

A Shear Force Diagram (SFD) indicates how a force applied perpendicular to the axis (*i.e.,* parallel to cross-section) of a beam is transmitted along the length of that beam. A Bending Moment Diagram (BMD) will show how the applied loads to a beam create a moment variation along the length of the beam.

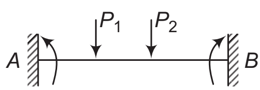
Statically Determinate Beam

A beam is said to be statically determinate if all its reaction components can be calculated by applying three conditions of static equilibrium *i.e.,*

$$\Sigma V = 0,\ \Sigma H = 0 \quad \text{and} \quad \Sigma M = 0$$

Statically Indeterminate Beam

When the number of unknown reaction components exceeds the static conditions of equilibrium, the beam is said to be statically indeterminate.



Indeterminate beam

Shear Force

Shear force has a tendency to slide the surface, it acts parallel to surface.

$$\Sigma F_{\text{vert}} = 0$$

$$V - q\,dx - (V + dV) = 0$$

$$\frac{dV}{dx} = -q \quad \Rightarrow \quad \int_A^B dV = -\int_A^B q\,dx$$

$$V_B - V_A = -\int_A^B q\,dx$$

only for distributed load not for point load.

Bending Moment

Any moment produced by forces acting on the beam must be balance by an equal opposite moment produced by internal forces acting in beam at the section. This moment is called **bending moment.**

$$\Sigma M = 0$$

$$-M - qdx\left(\frac{dx}{z}\right) - (V + dV)\,dx + M + dm = 0$$

$$\frac{dM}{dx} = V \quad \Rightarrow \quad M_B - M_A = \int V\,dx$$

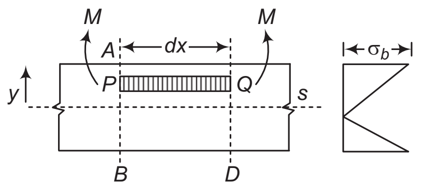
only for distributed and concentrated load not for couple.

Bending Moments and Shear Stress Distribution

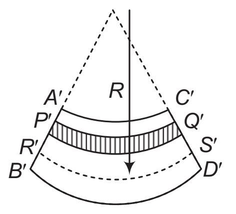
Bending stress and shear stress distribution are classified in the following groups

Bending Moment in Beam

When beam is subjected to a bending moment or bent there are induced longitudinal or bending stress in cross-section.



Bending stress in beam



Inertia about neutral axis

I = Moment of inertia about neutral axis.

$$\frac{\sigma_a}{y} = \frac{M}{I} = \frac{E}{R}$$

At the neutral axis, there is no stress of any kind. At one side of the neutral axis, there are compressive stresses, whereas on the other side there are tensile stresses.

Modulus of Section

Section modulus is a geometric property for a given cross-section used in the design of beams or flexural members.

$$z = \frac{I}{y_{max}} \quad \Rightarrow \quad \frac{M}{I} = \frac{\sigma}{y}$$

$$M = \sigma_{max} \frac{I}{y_{max}} \quad \Rightarrow \quad M = \sigma_{max} \times z$$

Rectangular section $I = \frac{bd^3}{12}$

Modulus of section $z = \frac{bd^2}{\sigma}$

Circular section $I = \frac{\pi}{64} d^4$

Modulus of section $z = \frac{\pi}{32} d^3$

Shearing Stress

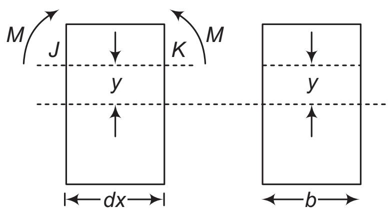
Shearing stress on a layer *JK* of beam at distance *y* from neutral axis.

$$\tau = \frac{VA\bar{y}}{Ib}$$

where,

V = Shearing force

$A\bar{y}$ = First moment of area $\tau = \frac{VQ}{Ib}$.



Shearing stress on a beam

Shear Stress in Rectangular Beam

Suppose, we have to determine the shear stress at the longitudinal layer having y distance from neutral axis.

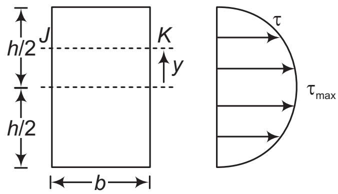
$$Q = b\left(\frac{h}{2} - y_1\right)\left(y_1 + \frac{\frac{h}{2} - y_1}{2}\right)$$

$$Q = \frac{b}{2}\left(\frac{h^2}{4} - y_1^2\right)$$

$$\tau = \frac{V}{2I}\left(\frac{h^2}{4} - y_1^2\right)$$

$$\tau_{max} = \frac{3}{2}\frac{V}{A}$$

$$\tau_{max} = 1.5\,\tau_{av}$$



Rectangular beam

Circular Beam

Centre of gravity of semi-circle lies at $\frac{4r}{3\pi}$ distance from centre or base line.

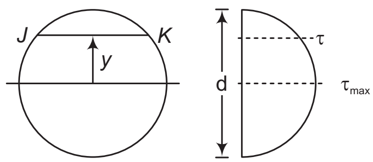
As it is symmetrical above neutral axis, hence at neutral axis shear stress will be maximum.

$$\tau = V\,\frac{(r^2 - y^2)}{3I}$$

$$Q = A\bar{y} = \frac{\pi r^2}{2}\left(\frac{4r}{3\pi}\right) = \frac{2r^3}{3}$$

$$b = 2r$$

$$\tau_{max} = \frac{V\,\frac{2r^3}{3}}{\frac{\pi r^4}{4}(2r)} = \frac{4}{3}\frac{V}{A}$$



Circular beam

For τ_{max} substituting $y = 0$

$$I = \frac{\pi d^4}{64}$$

$$\tau_{max} = \frac{4V}{3A} = \frac{4}{3}\tau_{av}$$

Shears Stress in Hollow Circular Cross-Section

In hollow circular cross-section, if we have to calculate τ at neutral axis by the formula

$$\tau_{max} = \frac{4V}{3A}\left(\frac{r_2^2 + r_1 r_2 + r_1^2}{r_2^4 - r_1^4}\right)$$

Shear Stress in Triangular Section

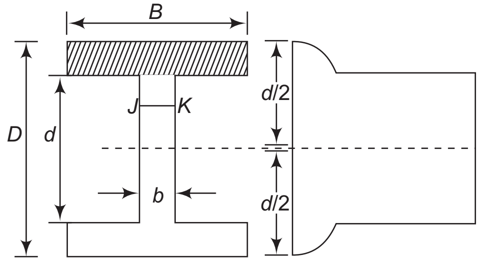
In a triangular cross-section, if we have to calculate τ at neutral axis, then in formula

$$\tau = \frac{V}{3I}(hx - x^2) \quad \Rightarrow \quad \tau_{max} = \frac{3}{2}\tau_{av}$$

Shear Stress in I-section

$$\tau = \frac{VX}{Ib}\left[\frac{B(D^2 - d^2)}{8} + \frac{b}{2}\left(\frac{d^2}{4} - y^2\right)\right]$$

$$\tau_{max} = \frac{V}{Ib}\left[\frac{B}{8}(D^2 - d^2) + \frac{bd^2}{8}\right]$$



I-Section diagram

Deflection of Beam

Deflection is defined as the vertical displacement of a point on a loaded beam. There are many methods to find out the slope and deflection at a section in a loaded beam.

Double Integration Method

This is most suitable when concentrated or udl over entire length is acting on the beam.

$$EI\frac{d^2y}{dx^2} = -M$$

Integrating one time $\quad EI\frac{dy}{dx} = -\int M$

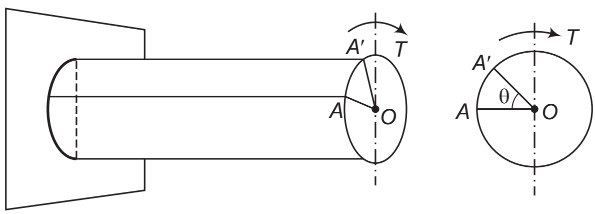
Integrating again $\quad E\,I\,y = -\iint M$

where, M = Bending moment
I = Moment of inertia of the beam section
y = Deflection of the beam
E = Modulus of elasticity of beam material.

Torsion of Shaft and Combined Stresses

If τ_r be the intensity of shear stress, on any layer at a distance r from the centre of shaft, then

$$\frac{\tau_r}{r} = \frac{T}{J} = \frac{G\theta}{l}$$



Torsion in shaft and combined stresses

Rate of twist $\left(\frac{\theta}{l}\right)$ $\quad \frac{\theta}{l} = \frac{T}{GJ}$

Total angle of twist $\quad \theta = \frac{Tl}{GJ}$

where, T = Torque, J = Polar moment of inertia
G = Modulus of rigidity, θ = Angle of twist
L = Length of shaft, GJ = Torsional rigidity

$\frac{GJ}{l} \rightarrow$ Torsional stiffness; $\frac{l}{GJ} \rightarrow$ Torsional flexibility

$\frac{EA}{l} \rightarrow$ Axial stiffness; $\frac{l}{EA} \rightarrow$ Axial flexibility

- For solid circular shaft, $J = \frac{\pi d^4}{32}$, $\tau_{max} = \frac{16T}{\pi d^3}$
- For hollow circular shaft, $J = \frac{\pi}{32}(d_o^4 - d_i^4)$
- Power transmitted by shaft, $P = \frac{2\pi NT}{60000}$ kW

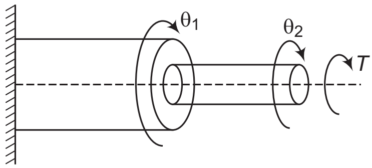
where, N = Rotation per minute.

Compound Shaft

An improved type of compound coupling for connecting in series and parallel are given below

1. **Series connection** Series connection of compound shaft as shown in figure. Due to series connection the torque on shaft 1 will be equal to shaft 2 and the total angular deformation will be equal to the sum of deformation of 1st shaft and 2nd shaft.

$$\theta = \theta_1 + \theta_2$$
$$T = T_1 = T_2$$

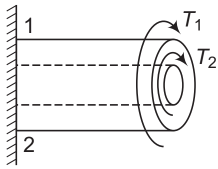


Series connection

where,
θ_1 = Angular deformation of 1st shaft
θ_2 = Angular deformation of 2nd shaft

2. **Parallel connection** Parallel connection of compound shaft as shown in figure. Due to parallel connection of compound shaft the total torque will be equal to the sum of torque of shaft 1 and torque of shaft 2 and the deflection will be same in both the shafts.

$$\theta_1 = \theta_2$$
$$T = T_1 + T_2$$



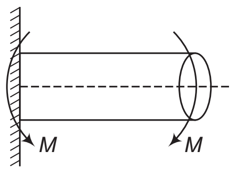
Parallel connection

Effect of Pure Bending on Shaft

The effect of pure bending on shaft can be defined by the relation

$$\sigma = \frac{32\,M}{\pi D^3}$$

where, σ = Principal stress
D = Diameter of shaft
M = Bending moment



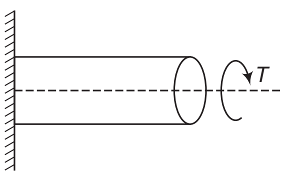
Pure bending on shaft

Effect of Pure Torsion on Shaft

It can be calculated by the formula, which are given below

$$\tau_{max} = \frac{16\,T}{\pi D^3}$$

where, τ = Torsion
D = Diameter of shaft



Pure torsion on shaft

Combined effect of bending and torsion

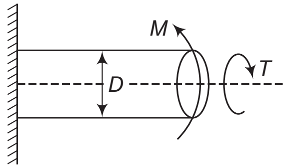
Principal stress $= \frac{16}{\pi D^3}[M \pm \sqrt{M^2 + T^2}]$

Maximum shear stress $= \frac{16}{\pi D^3}\sqrt{M^2 + T^2}$

Equivalent bending moment

$$M_{eq} = \frac{1}{2}[M + \sqrt{M^2 + T^2}]$$

Equivalent torque $T_{eq} = \sqrt{T^2 + M^2}$



Bending and torsion effect

Thin Shell

If the thickness of the wall of a shell is less than $\frac{1}{10}$th of $\frac{1}{15}$th of its diameter, it is known as a **thin shell**.

Stresses in Thin Cylindrical Shell

(a) Circumferential stress (hoop stress)

$$\sigma_c = \frac{pd}{2t} \Rightarrow \sigma_c = \frac{pd}{2t\eta}$$

where, p = Intensity of internal pressure
d = Diameter of the shell
t = Thickness of shell
η = Efficiency of joint

(b) Longitudinal stress $\sigma_l = \frac{pd}{4t} \Rightarrow \sigma_l = \frac{pd}{4t\eta}$

Change in Dimension of a Thin Cylindrical Shell due to an Internal Pressure

Change in diameter $\delta d = \varepsilon_1 d = \frac{pd}{2tE}\left(1 - \frac{1}{2m}\right) \times d$

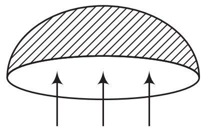
ε_1 = Circumferential strain $\varepsilon_1 = \frac{\sigma_c}{E} - \frac{\sigma_l}{mE} \Rightarrow \frac{1}{m} = \mu$

Change in length $f l = \varepsilon_2 l = \frac{pd}{2tE}\left(\frac{1}{2} - \frac{1}{m}\right) \times l$

ε_2 = Longitudinal strain $\varepsilon_2 = \frac{\sigma l}{E} - \frac{\sigma_c}{mE}$

Change in volume $\delta V = (2\varepsilon_1 + \varepsilon_2) V$

Thin Spherical Shell



1. Stresses in shell material,

$$\sigma = \frac{pd}{4t} \Rightarrow \sigma = \frac{pd}{4t\,\eta}$$

2. Change in diameter $\delta d = \varepsilon \times d$,

$$\text{where } \varepsilon = \left(\frac{\sigma}{E} - \frac{\sigma}{mE}\right)$$

$$= \frac{pd}{4t\varepsilon}\left(1 - \frac{1}{m}\right) \times d$$

Change in volume $\delta V = V \cdot 3\,\varepsilon = V \cdot \frac{pd}{4tE}\left(1 - \frac{1}{m}\right)$

Lake's Theory

Lake's theory is based on the following assumptions

Assumptions

1. Homogeneous material.
2. Plane section of cylinder, perpendicular to longitudinal axis remains under plane and pressure.

   Hoop stress at any section

$$\sigma_r = \frac{b}{r^2} + a$$

Radial pressure $p_r = \frac{b}{r^2} - a$

- **Subjected to Internal Pressure (p)**

1. At $r = r_i$, $\sigma_{r_i} = p\left(\frac{r_0^2 + r_i^2}{r_0^2 - r_i^2}\right)$

2. At $r = r_0$, $\sigma_{r_0} = \frac{2pr_i^2}{r_0^2 - r_i^2}$

- **Subjected to External Pressure (p)**

1. At $r = r_i$, $\sigma_{r_i} = \frac{-2\,pr_0^2}{r_0^2 - r_i^2}$

2. At $r = r_0$, $\sigma_{r_0} = -\,p\left(\frac{r_0^2 + r_i^2}{r_0^2 - r_i^2}\right)$

Note Radial and hoop stresses vary hyperbolically.

Columns and **Struts**

A structural member subjected to an axial compressive force is called **strut.** As per definition strut may be horizontal, inclined or even vertical. Vertical strut is called a **column.**

Euler's Column Theory

This theory has the following assumptions

- Perfectly straight column and axial load apply.
- Uniform cross-section of the column throughout its length.
- Perfectly elastic, homogeneous and isotropic material.
- Length of column is large as compared to its cross-sectional dimensions.
- The shortening of column due to direct compression is neglected.
- The failure of column occurs due to buckling alone.

Euler's Buckling (or crippling load)

The maximum load at which the column tends to have lateral displacement or tends to buckle is known as **buckling** or **crippling** load. *Load columns can be analysed with the Euler' column formulas can be given as*

$$p_E = \frac{n^2 \pi^2 EI}{l^2} \qquad (n = 1, 2, 3, \dots)$$

or

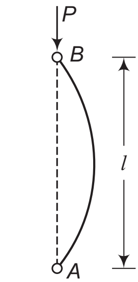
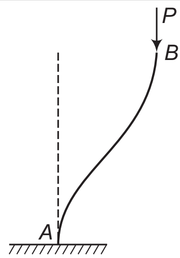
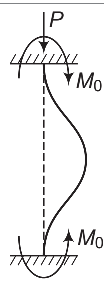
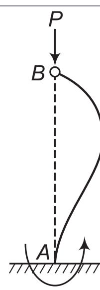
$$p_E = \frac{\pi^2 EI}{l_{\text{eff}}^2}$$

where, E = Modulus of elasticity

l = Length of column

I = Moment of inertia of column section.

Euler's Buckling for Different Structures

1.	For both end hinged $n = 1,$ $l_{\text{eff}} = l$ $p_E = \frac{\pi^2 EI}{l^2}$	 Both end hinged
2.	For one end fixed and other free $n = \frac{1}{2}$ $l_{\text{eff}} = 2l$ $p_E = \frac{\pi^2 EI}{4l^2}$	 One end fixed
3.	For both end fixed $n = 2,$ $l_{\text{eff}} = \frac{l}{2}$ $p_E = \frac{4\pi^2 EI}{EI}$	 Both end fixed
4.	For one end fixed and other hinged $n = \sqrt{2}$ $l_{\text{eff}} = \frac{l}{\sqrt{2}}$ $p_E = \frac{2\pi^2 EI}{l^2}$	 One end fixed and other hinged