CRASH COURSE FOR
PEAK PERFORMANCE

rapid PHYSICS

- HIGH YIELD FACTS
- EASY TO GRASP
- ESSENTIAL PHYSICS FOR COMPETITIVE EXAMS

Published by : MTG Learning Media (P) Ltd.
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PREFACE

"Good things come in small packages."

Rapid Physics (**High Yield Facts** Book) is designed for people who have only enough time to glance at a book-literally. Our goal is to create an effective memory aid for those who wish to review physics.

The book covers complete syllabus in points form. The quality of the writing leaves the reader with the potential to achieve a good understanding of a given topic within a short period of learning. It gives a concise overview of the main theories and concepts of physics, for students studying physics and related courses at undergraduate level. Based on the highly successful and student friendly "at a glance" approach, the material developed in this book has been chosen to help the students grasp the essence of physics, ensuring that they can confidently use that knowledge when required.

The books has been crafted extremely well for a very specific purpose: review. A person who has been away from physics (but who understood it very well at the time) can use this book effectively for a rapid review of any basic topic. The book is so highly compressed that every page is like a food with a rich sauce that needs to be slowly savored and slowly digested for maximum benefit. You may only glance into the book, but you can think about the physics for much longer.

All the best!

Impetus by : **Prof. S.P. Arya**
& Geeta Mahajan

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CHAPTER

units and measurement

- **Units and dimensional formulae of some important physical quantities**

S. No.	Physical quantity	Dimensional formula	C.G.S. units	M.K.S. units
1.	Area	$[M^0L^2T^0]$	cm^2	m^2
2.	Volume	$[M^0L^3T^0]$	cm^3	m^3
3.	Density = mass/volume	$[ML^{-3}T^0]$	$g\ cm^{-3}$	$kg\ m^{-3}$
4.	Speed or velocity = $\frac{dx}{dt}$	$[M^0LT^{-1}]$	$cm\ s^{-1}$	$m\ s^{-1}$
5.	Acceleration = $\frac{dv}{dt}$	$[M^0LT^{-2}]$	$cm\ s^{-2}$	$m\ s^{-2}$
6.	Force = $mdv/dt = ma$	$[MLT^{-2}]$	$g\ cm\ s^{-2}$ or dyne	$kg\ m\ s^{-2}$ or N
7.	Linear Momentum ($p = mv$)	$[MLT^{-1}]$	$g\ cm\ s^{-1}$ or dyne s	$kg\ m\ s^{-1}$ or Ns
8.	Impulse = $F \cdot \Delta t$	$[MLT^{-1}]$	$g\ cm\ s^{-1}$ or dyne s	$kg\ m\ s^{-1}$ or Ns
9.	Power = work/time	$[ML^2T^{-3}]$	$erg\ s^{-1}$ or $g\ cm^2\ s^{-3}$	$J\ sec^{-1}$ or watt or $kg\ m^2\ s^{-3}$
10.	Work or Energy = force × displacement	$[ML^2T^{-2}]$	dyne cm or erg	Nm or J or $kg\ m^2\ s^{-2}$ or $g\ cm^2\ s^{-2}$
11.	Pressure or Stress = force/area	$[ML^{-1}T^{-2}]$	$dyne\ cm^{-2}$ or $g\ cm^{-1}\ s^{-2}$	$N\ m^{-2}$ or $kg\ m^{-1}\ s^{-2}$
12.	Strain = $\frac{\Delta l}{l}$ or $\frac{\Delta V}{V}$	$[M^0L^0T^0]$	No unit	No unit
13.	Specific gravity or Relative density = $\frac{\text{density of body}}{\text{density of water}}$	$[M^0L^0T^0]$	No unit	No unit

S. No.	Physical quantity	Dimensional formula	C.G.S. units	M.K.S. units
14.	Angular displacement	$[M^0L^0T^0]$	radian or degree	radian or degree
15.	Poisson's ratio $= \frac{\text{lateral strain}}{\text{longitudinal strain}}$	$[M^0L^0T^0]$	No unit	No unit
16.	Modulus of elasticity = stress/strain	$[ML^{-1}T^{-2}]$	dyne cm^{-2} or g cm^{-1} s^{-2}	N m^{-2} or kg $m^{-1}s^{-2}$
17.	Surface tension = force/length	$[ML^0T^{-2}]$	dynes cm^{-1} or g s^{-2}	N m^{-1} or kg s^{-2}
18.	Velocity gradient $= \frac{dv}{dx}$	$[M^0L^0T^{-1}]$	second^{-1}	second^{-1}
19.	Coefficient of viscosity $\eta = \frac{F \cdot dx}{A \cdot dv}$	$[ML^{-1}T^{-1}]$	g cm^{-1} s^{-1} or poise	kg m^{-1} s^{-1}
20.	Torque or Moment of force = Force × perpendicular distance of force from axis	$[ML^2T^{-2}]$	g cm^2 sec^{-2} or dyne cm	kg m^2 sec^{-2} or N m
21.	Gravitational constant (G) $= F \times d^2 / m_1m_2$	$[M^{-1}L^3T^{-2}]$	g^{-1} cm^3 s^{-2} or $\frac{\text{dyne cm}^2}{g^2}$	kg^{-1} m^3 s^{-2} or $\frac{\text{N m}^2}{\text{kg}^2}$
22.	Moment of Inertia $I = Mk^2$	$[ML^2T^0]$	g cm^2	kg m^2
23.	Angular velocity, ($\omega = d\theta/dt$)	$[M^0L^0T^{-1}]$	radian/s	radian/s
24.	Frequency ($f = 1/T$)	$[T^{-1}]$	cycle/s or Hz	cycle/s or Hz
25.	Angular acceleration $\alpha = d\omega/dt$	$[M^0L^0T^{-2}]$	radian/s^2	radian/s^2
26.	Angular momentum ($L = I\omega$)	$[ML^2T^{-1}]$	g cm^2 s^{-1}	kg m^2 s^{-1}
27.	Torque ($\tau = I\alpha$)	$[ML^2T^{-2}]$	g cm^2 s^{-2}	kg m^2 s^{-2}
28.	Intensity of gravitational field (= F/m)	$[M^0LT^{-2}]$	cm s^{-2}	m s^{-2}
29.	Gravitational potential = work/mass	$[M^0L^2T^{-2}]$	cm^2 s^{-2}	m^2 s^{-2}
30.	Force constant or spring constant = force/displacement	$[ML^0T^{-2}]$	g s^{-2} or dyne per cm	kg s^{-2} or N per m

S. No.	Physical quantity	Dimensional formula	C.G.S. units	M.K.S. units
31.	Heat energy	$[ML^2T^{-2}]$	calorie	kilocalorie
32.	Specific heat = $H/m\theta$	$[L^2T^{-2}\theta^{-1}]$	calorie $g^{-1}\,{}^{\circ}C^{-1}$	k cal kg^{-1} K^{-1}
33.	Mechanical equivalent of heat ($J = W/H$)	$[M^0L^0T^0]$	$\frac{\text{erg}}{\text{calorie}}$	$\frac{\text{joule}}{\text{kcal}}$
34.	Thermal conductivity $K = \frac{H}{At\,(d\theta / dx)}$	$[MLT^{-3}\theta^{-1}]$	cal cm^{-1} s^{-1} ${}^{\circ}C^{-1}$	kcal m^{-1} s^{-1} K^{-1}
35.	Universal gas constant $R = PV/nT$	$[ML^2T^{-2}\theta^{-1}]$	erg/mole°C	J/mole K
36.	Boltzmann constant $k_B = R/N$	$[ML^2T^{-2}\theta^{-1}]$	erg ${}^{\circ}C^{-1}$	J K^{-1}
37.	Planck's constant	$[ML^2T^{-1}]$	erg or g cm^2 s^{-1}	J s or kg m^2 s^{-1}
38.	Thermal Resistance	$[M^{-1}L^{-2}T^3\theta]$	$\frac{{}^{\circ}C \times \text{sec}}{\text{cal}}$	$\frac{K \times \text{sec}}{\text{kcal}}$

- **Some important abbreviations**

Symbol	Prefix	Multiplier
d	deci	10^{-1}
c	centi	10^{-2}
m	milli	10^{-3}
μ	micro	10^{-6}
n	nano	10^{-9}
p	pico	10^{-12}
f	femto	10^{-15}
a	atto	10^{-18}
da	deca	10^{1}
h	hecto	10^{2}
k	kilo	10^{3}
M	mega	10^{6}
G	giga	10^{9}
T	tera	10^{12}
P	pecta	10^{15}
E	exa	10^{18}

Significant figure rule

- So far **significant figures** are concerned, in mathematical operations like addition and subtraction, the result would be correct upto minimum number of decimal places in any of quantities involved. However, in multiplication and division, number of significant figures in the result will be limited corresponding to the minimum number of significant figures in any of the quantities involved.
  To represent the result to a correct number of significant figures, we round off as per the rules.
- There are three rules on determining how many significant figures are in a number :
  (i) Non-zero digits are always significant.
  (ii) Any zeros between two significant digits are significant.
  (iii) A final zero or trailing zeros in the decimal portion **ONLY** are significant.
- All numbers are based upon measurements (except for a very few that are defined). Since all measurements are uncertain, we must only use those numbers that are meaningful. A common ruler cannot measure something to be 22.4072643 cm long. Not all of the digits have meaning (significance) and, therefore, should not be written down. In science, only the numbers that have significance (derived from measurment) are written.
- **Rule 1:** Non-zero digits are always significant. Number like 26.38 would have four significant figures and 7.94 would have three. The problem comes with numbers like 0.00980 or 28.09.
- **Rule 2:** Any zeros between two significant digits are significant. *e.g.* 406 have 3 significant figures.
- **Rule 3:** A final zero or trailing zeros in the decimal portion only are significant.
  e.g. 0.00500 m only three number are significant.
  The zero at left of non-zero and after decimal point are not significant figures.
  Another way the number 0.00500 m can be written in scientific notation (in power of 10) is
  $0.00500 \text{ m} = 5.00 \times 10^{-3} \text{ m} = 5.00 \times 10^{-1} \text{ cm}$
  $= 5.00 \times 10^{-6} \text{ km}$
  The power 10 is not considered or irrelevant in significant figures.
- **Rule 4:** Trailing zeros in a whole number are not significant.
  e.g. 25,000 has two significant figures.
  Example
  - 0.009 g $\rightarrow$ has one significant figure and can be written as 9×10^{-3} g.
  - 6.032 Nm^{-2} $\rightarrow$ has four significant figures.
  - 0.000507 m^2 $\rightarrow$ has three significant figures.
- When some value is recorded on the basis of some actual measurement, the zeros on the right of last non-zero digit become significant. For example, m = 100 kg has three significant figures.

Rounding off

- **Rule 1:** If the digit to be dropped is less than 5, then the preceding digit is left unchanged.
  e.g. 8.22 is rounded off to 8.2.

- **Rule 2:** If the digit to be dropped is more than 5, then the preceding digit is raised by one.
  e.g. $x = 6.87$ is rounded off to 6.9.
- **Rule 3:** If the digit to be dropped is 5 followed by digit other than zero, then the preceeding digit is raised by one.
  e.g. 7.851 is rounded off to 7.9.
- **Rule 4:** If the digit to be dropped is 5 or 5 followed by zero, then preceding digit is left unchanged, if it is even.
  e.g. 5.250 rounding off to 5.2.
- **Rule 5:** If the digit to be dropped is 5 or 5 followed by zeros, then the preceding digit is raised by one, if it is odd.
  e.g. 3.750 is rounded off to 3.8.
- **Accuracy** is the extent to which a reported measurement approaches the true value of the quantity measured.

Absolute error

- This error in the measurement of a quantity is the magnitude of the difference between the true value and the measured value of the quantity.
  Let in some experiment on physical quantity is measured n times. Let the measured values be $a_1, a_2, ... a_n$. The arithmetic mean of these value is

  $$a_m = \frac{a_1 + a_2 + ... + a_n}{n} \quad \text{or} \quad a_m = \frac{1}{n}\sum_{i=1}^{n} a_i$$

  a_m is considered as true value of the quantity, if the true value of that quantity is not known.
- The absolute error in the measured values of quantity are

  $$\Delta a_1 = a_m - a_1$$

  $$\Delta a_2 = a_m - a_2$$

  . . . .

  . . . .

  . . . .

  $$\Delta a_n = a_m - a_n$$

  The absolute error may be positive in certain cases and negative in certain other cases.

Mean absolute error

It is the arithmetic mean of the magnitude of absolute errors in all the measurements of the quantity.

$$\overline{\Delta a} = \frac{|\Delta a_1| + |\Delta a_2| + ...|\Delta a_3|}{n}$$

$$\overline{\Delta a} = \frac{1}{n}\sum_{i=1}^{n} |\Delta a_i|^n$$

Hence the final result of measurement may be written as

$a = a_m \pm \overline{\Delta a}$.

Relative error or fractional error

Relative error $= \dfrac{\text{mean absolute error}}{\text{mean value}} = \dfrac{\overline{\Delta a}}{a_m}$

Percentage error $= \dfrac{\overline{\Delta a}}{a_m} \times 100\%$

Propagation or combination of errors

- **Error in sum of quantities**
  Let $x = a + b$
  Let Δa = absolute error in measurement of a
  Let Δb = absolute error in measurement of b
  The maximum absolute error in x is
  $\Delta x = \pm (\Delta a + \Delta b)$
- **Error in difference of quantities**
  Let $x = a - b$
  The maximum absolute error in x is
  $\Delta x = \pm (\Delta a + \Delta b)$
- **Error in product of quantities**
  $x = a \times b$
  Maximum possible value of (or maximum fractional error or relative error in product)
  $\dfrac{\Delta x}{x} = \pm\left(\dfrac{\Delta a}{a} + \dfrac{\Delta b}{b}\right)$
- **Error in division of quantities**
  Let $x = \dfrac{a}{b}$
  The maximum value of fractional or relative error in division of quantities is
  $\dfrac{\Delta x}{x} = \pm\left(\dfrac{\Delta a}{a} + \dfrac{\Delta b}{b}\right)$
- **Error in quantity raised to some power**
  Let $x = \dfrac{a^n}{b^m}$
  The fraction error or relative error.
  $\dfrac{\Delta x}{x} = \pm\left(n\dfrac{\Delta a}{a} + m\dfrac{\Delta b}{b}\right)$
- The initial and final temperatures of water as recorded by an observer are (40.6 ± 0.2)°C and (78.3 ± 0.3)°C. The rise in temperature with proper error limit is given by :
  Rise in temperature = $\theta = \theta_2 - \theta_1 = 78.3 - 40.6 = 37.7$°C $\quad \Delta\theta = (\Delta\theta_1 + \Delta\theta_2)$
  $= \pm\,(0.2 + 0.3) = \pm 0.5$°C
  Hence rise in temperature = (37.7 ± 0.5)°C.
- Any **order of magnitude** is an exponential change of plus-or-minus 1 in the value of a quantity or unit. The term is generally used in conjunction with power-of-10 scientific notation.

- In base 10, the most common numeration scheme worldwide, an increase of one order of magnitude is the same as multiplying a quantity by 10. An increase of two orders of magnitude is the equivalent of multiplying by 100 or 10^2. In general, an increase of n orders of magnitude is the equivalent of multiplying a quantity of 10^n. This, 2315 is one order of magnitude larger than 231.5, which in turn is one order of magnitude larger than 23.15.
- For smaller value, a decrease of one order of magnitude is the same as multiplying a quantity by 0.1. A decrease of two orders of magnitude is the equivalent of multiplying by 0.01 or 10^{-2}. In general, a decrease of n orders of magnitude is equivalent of multiplying a quantity by 10^{-n}.
- In the Standard International (SI) System of Units, most quantities can be expressed in multiple or fractional terms according to the order of magnitude. For example attaching the prefix "kilo" to a unit increases the size of the unit by three orders of magnitude, or one thousand (10^3). Attaching the prefix "micro" to a unit decreases the size of the unit by six orders of magnitude, the equivalent of multiplying it by one millionth (10^{-6}). Scientists and engineers have designated prefix multipliers from septillionths (10^{-24}) to septillions (10^{24}), a span of 48 orders of magnitude.

Trigonometric identities

- **Reciprocal identities**

$$\sin u = \frac{1}{\operatorname{cosec} u},\quad \cos u = \frac{1}{\sec u},\quad \tan u = \frac{1}{\cot u}$$

$$\operatorname{cosec} u = \frac{1}{\sin u},\quad \sec u = \frac{1}{\cos u},\quad \cot u = \frac{1}{\tan u}$$

- **Pythagorean identities**

$$\sin^2 u + \cos^2 u = 1,\qquad 1 + \tan^2 u = \sec^2 u$$

$$1 + \cot^2 u = \operatorname{cosec}^2 u$$

- **Quotient identities**

$$\tan u = \frac{\sin u}{\cos u},\quad \cot u = \frac{\cos u}{\sin u}$$

- **Co-function identities**

$$\sin\left(\frac{\pi}{2} - u\right) = \cos u,\quad \cos\left(\frac{\pi}{2} - u\right) = \sin u$$

$$\tan\left(\frac{\pi}{2} - u\right) = \cot u,\quad \operatorname{cosec}\left(\frac{\pi}{2} - u\right) = \sec u$$

$$\sec\left(\frac{\pi}{2} - u\right) = \operatorname{cosec} u,\quad \cot\left(\frac{\pi}{2} - u\right) = \tan u$$

- **Even-odd identities**

$\sin(-\theta) = -\sin\theta$, $\quad\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$, $\quad\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$

$\sec(-\theta) = \sec\theta$, $\quad\cot(-\theta) = -\cot\theta$

- **Sum-difference formulae**

$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$

$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$

$\tan(u \pm v) = \dfrac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$

- **Double angle formulae**

$\sin(2u) = 2\sin u \cos u$

$\cos(2u) = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$

$\tan(2u) = \dfrac{2\tan u}{1 - \tan^2 u}$

- **Power-reducing/half angle formulae**

$\sin^2 u = \dfrac{1 - \cos(2u)}{2}$; $\cos^2 u = \dfrac{1 + \cos(2u)}{2}$

$\tan^2 u = \dfrac{1 - \cos(2u)}{1 + \cos(2u)}$

- **Sum-to-product formulae**

$\sin u + \sin v = 2\sin\left(\dfrac{u+v}{2}\right) \cos\left(\dfrac{u-v}{2}\right)$

$\sin u - \sin v = 2\cos\left(\dfrac{u+v}{2}\right) \sin\left(\dfrac{u-v}{2}\right)$

$\cos u + \cos v = 2\cos\left(\dfrac{u+v}{2}\right) \cos\left(\dfrac{u-v}{2}\right)$

$\cos u - \cos v = -2\sin\left(\dfrac{u+v}{2}\right) \sin\left(\dfrac{u-v}{2}\right)$

- **Product-to-sum formulae**

$\sin u \sin v = \dfrac{1}{2}[\cos(u - v) - \cos(u + v)]$

$\cos u \cos v = \dfrac{1}{2}[\cos(u - v) + \cos(u + v)]$

$\sin u \cos v = \dfrac{1}{2}[\sin(u + v) + \sin(u - v)]$

$\cos u \sin v = \dfrac{1}{2}[\sin(u + v) - \sin(u - v)]$

Measurement

- The mass, length and time in the physical world have variations from atomic to astronomical range. Measurements are an integral part of all advancements in modern science and technology. Extent of uncertainty is simultaneously taken into account while reporting the results of research and exploration.

Measurement of length

- The Indian national standards of seven base units are maintained at National Physical Laboratory (NPL), New Delhi.
- The standard of length (metre) is realised by employing a stabilised helium-neon laser as a source of light.
- Very small distances such as inter-molecular distances, size of molecules, radius of atom can be measured by electron microscope. Modern science has developed tunneling microscopes to resolve microscopic distances. Rutherford developed scattering method to measure distance of closest approach which is of the order of 10^{-14} m.
- Very large distances such as the distance of a star from earth can be measured with the help of angular measurements or parallax methods. Sextant can measure height of mountains. Radar signal echo method or optical Doppler shift method provide measurements of diameters of moon and star.
- The following table displays the large variation in terms of order of magnitude of length.

1.	Diameter of proton	10^{-15} m
2.	Diameter of nucleus	10^{-14} m
3.	Diameter of hydrogen atom	10^{-10} m
4.	Height of human being	10^{0} m
5.	Length of cricket field	10^{2} m
6.	Height of Mount Everest	10^{4} m
7.	Diameter of earth	10^{7} m
8.	Diameter of sun	10^{9} m
9.	Distance of sun from earth	10^{11} m
10.	Diameter of our galaxy	10^{21} m.

Measurement of mass

- The Indian national standard of mass is a copy No. 57 of the International prototype kilogram supplied by the International Bureau of Weights and Measures (BIPM).
- Science differentiates between the inertial mass and the gravitational mass.
- Inertial mass (m) = force/acceleration. It can be measured with the help of inertia balance.
- Gravitational mass of a body determines the gravitational pull due to earth acting upon the body. It can be measured with the help of a common balance. In order to measure the weight of a body, a spring balance is used.
- The following table displays the huge variation in terms of order of magnitude of mass:

1.	Electron	10^{-30} kg
2.	Proton	10^{-27} kg
3.	Atom	10^{-25} kg
4.	Human being	10^{2} kg
5.	Hippopotamus	10^{3} kg
6.	Moon	10^{23} kg
7.	Earth	10^{25} kg
8.	Star	10^{30} kg
9.	Galaxy	10^{42} kg
10.	Universe	10^{55} kg.

Measurement of time

- The national standard of time interval, second as well as frequency, is maintained through four cesium atomic clocks mentioned at NPL.
- The standard maintained at NPL is linked to different users in a variety of ways. This process is known as dissemination. Time is disseminated through TV, radio, special telephone service for day-to-day requirements. For high level of accuracy there is a satellite-based time dissemination service which utilized the Indian Satellite INSAT.
- The following table displays the low to high range of variation of time.

1.	Time for proton to revolve in nucleus	10^{-22} s
2.	Period of atomic vibration	10^{-15} s
3.	Time taken by atom to emit light	10^{-9} s
4.	Time between heart beats	10^{0} s
5.	Time during which sun light reaches earth	10^{3} s
6.	Time during which earth rotates on its orbit (day)	10^{5} s
7.	Time during which earth revolves around sun (year)	10^{7} s
8.	Human life span	10^{9} s.

Units

- In the **System International**, length(m), mass(kg), time(s), electric current(A), thermodynamic temperature(K), luminous intensity(cd) and quantity of matter(mol) are considered as fundamental quantities. In this system, plane angle and solid angle are considered as supplementary quantities.
- The quantities that are derived using the fundamental quantities are called **derived quantities**. For example speed, volume, acceleration, force, momentum, etc.
- The units, that are used to measure the fundamental quantities are known as **fundamental units**.
- The units, that are used to measure the derived quantities are known as derived units *e.g.* metre^2, newton, cubic metre, erg, watt, etc.
- Some more quantities which are found to be fundamental in nature are added to the existing list of fundamental quantities. The system consisting of all fundamental quantities is known as "System International" and units are called SI units.
- Units should not be expressed in capital letters. They should be written as newton, joule, watt, etc.
- Units can be expressed in full or by approved symbols.
- Units must be expressed in singular form. Example: 11 metre, 26 newton, 6 joule, etc.
- **Symbols of units :** When a unit is named after a scientist, the symbol is a capital letter. For other units, the symbol is not a capital letter. Thus symbol for kelvin, the unit of temperature is K. Similarly J (joule), N (newton), W (watt), P (pascal), C (coulomb), A (ampere), F (faraday), H (henry), V (volt), Wb (weber), T (tesla), Gs (gilbert), Ω (ohm), S (siemen) represent other units. Correct symbol for gram is g, for kilogram is kg, for metre is m.
- **Single solidus :** In a symbol, more than one solidus should not be used. Acceleration should not be expressed as m/s/s. It should be written as ms^{-2} or m/s^2. The index form ms^{-2} is, however, preferred.

- **No full stop after symbol**. Volume should be written as cc and not as c.c.
- **Derived units and equivalences**

Derived unit	Quantity	Equivalence
quintal	Mass	1 q = 100 kg
metric ton	Mass	1 t = 1000 kg
atomic mass unit	Mass	1 amu = 1.66×10^{-27} kg
fermi	Length	1 f = 10^{-15} m
X-ray unit	Length	1 Xu = 10^{-13} m
angstrom unit	Length	1 Å = 10^{-10} m
micron	Length	1 µm = 10^{-6} m
astronomical unit	Length	1 AU = 1.5×10^{11} m
light year	Length	1 ly = 9.46×10^{15} m
parallactic second	Length	1 pc = 3.26 ly
parsec	Length	1 pc = 3×10^{16} m
mile	Length	1 mile = 1.61 km
nautical mile	Length	1 n mile = 1852 m
shake	Time	1 shake = 10^{-8} second
acre	Area	4047 sq. m
hectare	Area	10^4 sq. m = 2.47 acre
barn	Area	10^{-28} m^2
newton (N)	Force	10^5 dyne
pascal (P)	Pressure	1 Nm^{-2}
bar	Pressure	1 atmosphere or 10^5 pascal
torr	Pressure	133.3 pascal
gal	Acceleration	10^{-2} ms^{-2}
joule (J)	Energy	1 Nm = 10^7 erg = 0.24 cal
eV	Energy	1.6×10^{-19} J = 1.6×10^{-12} erg
horse power	Power	746 watt = 0.746 kW

Dimensions

- The **dimensions** of a physical quantity are the powers to which the fundamental units are raised to obtain the unit of that quantity.
- Angle and ratio do not have dimensions.
- The dimensional equations are used
  (a) to verify whether an equation connecting physical quantities is correct or not.
  (b) to find the relation between systems of units of the same physical quantity.
  (c) to derive equations connecting physical quantities.
- The dimensional methods are applicable to equations connecting physical quantities alone.
- The dimensional methods are applicable to equations involving the three fundamental quantities only.
- The proportionality constants are to be found experimentally.
- How to obtain a dimensional formula?
  (a) From the definition or from the defining equation:

F = Force = ma

$$= \text{mass} \times \frac{\text{Velocity change}}{\text{Time}} = \text{M}^1\text{L}^1\text{T}^{-2}.$$

(b) From a knowledge of the unit:

Planck's constant = joule - sec = $\text{M}^1\text{L}^2\text{T}^{-2}\text{T}^1 = \text{M}^1\text{L}^2\text{T}^{-1}$.

(c) From a formula in which the required quantity appears:

$$Q = KA\frac{(T_2 - T_1)}{L}\cdot t \;\; ; \;\; K = \frac{QL}{A(T_2 - T_1)t}$$

$$K = \frac{\text{M}^1\ \text{L}^2\ \text{T}^{-2}.\text{L}}{\text{L}^2\ \text{K}^{+1}\ \text{T}^1} = \text{M}^1\text{L}^1\ \text{T}^{-3}\text{K}^{-1}.$$

- **Important uses of dimensions illustrated**

  (a) Conversion from one system to the other:

  Ex: Force = $\dfrac{\text{kg-m-sec}^{-2}}{\text{g-cm-sec}^{-2}} = 1000 \times 100 = 10^5$

  1 N = 10^5 dyne.

  (b) Investigating how the unit of a derived quantity change when the fundamental units L, M, T are altered.

  Ex: If L, M and T are doubled, what happens to the unit of power.

  $$P = \text{Power} = \frac{\text{M}^1\text{L}^2}{\text{T}^3}$$

  $$P_1 = \frac{\text{M}_1^1\ \text{L}_1^2}{\text{T}_1^3} = \frac{(2\text{M})^1(2\text{L})^2}{(2\text{T})^3} = \frac{2 \times 4\text{M}^1\ \text{L}^2}{8\ \text{T}^3} = P.$$

- If L represents inductance, C represents capacitance, R represents resistance and q represents the charge, then

  (i) L/R, RC, $\sqrt{LC}$ have the dimensions of time

  (ii) R/L, $1/RC, 1/\sqrt{LC}$ have dimensions of frequency.

  (iii) q^2/C, LI^2, qV, V^2C, all have dimensions of energy.

- If R is resistance, I is current, V is potential difference and t is time, then

  (i) VI, V^2/R, $I^2 R$ have the dimensions of power.

  (ii) $VI\,t$, $\dfrac{V^2t}{R}$, I^2Rt have the dimensions of energy.

- If m is mass, s is specific heat, $\Delta\theta$ is change in temperature, L is latent heat, P is pressure, V is volume, T is temperature and R is gas constant, then $ms\ \Delta\theta$; mL ; PV ; RT all have the same dimensions of energy [$\text{M}^1\ \text{L}^2\ \text{T}^{-2}$].
- The principle of homogeneity of dimensions is used in checking the correctness of formulae, and also in the derivation of formulae.
- Only like quantities having the same dimensions can be added to or subtracted from each other.
- The order of accuracy obtained in adopting atomic standards of length and time, as detailed earlier is 1 part in 10^9.

- Planck's constant has the same dimensional formula as angular momentum, *i.e.* $[M^1 L^2 T^{-1}]$.
- Thermal capacity, Boltzmann constant, entropy have the same dimesions, *i.e.* $[ML^2 T^{-2} K^{-1}]$.
- Work, energy, kinetic energy, potential energy, couple, moment of a force, torque, internal energy, heat energy, electron volt, kilowatt, hour, etc. all have the same dimensional formula $[M^1 L^2 T^{-2}]$.
- The three coefficients of elasticity, stress, pressure have the same dimensions, *i.e.* $[M^1 L^{-1} T^{-2}]$.
- Strain, refractive index, relative density, angle, solid angle, trigonometrical ratios, phase, relative permittivity and relative permeability-all are dimensionless.
- Surface tension, surface energy, spring constant and force gradient have the same dimensional formula, *i.e.* $[M^1L^0T^{-2}]$.
- Force, thrust and weight have the same dimensional formula, *i.e.* $[M^1 L^1 T^{-2}]$.
- Inertia and mass have the same dimensional formula.
- Relative velocity also has the dimensions of velocity.
- Distance travelled in *n*th second has the dimensions of velocity.

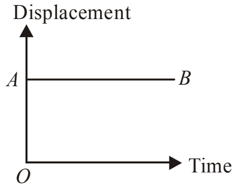
End

2

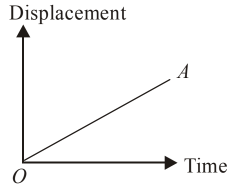
CHAPTER

kinematics and vectors

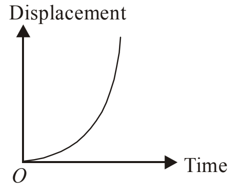
- Mechanics can be divided under two branches- (i) **statics** which deals with the study of stationary objects and (ii) **dynamics** which deals with the study of moving objects.
- An object is said to be at **rest** if it does not change its position with time with respect to its surroundings.
- An object is said to be in **motion** if it changes its position with time, with respect to its surroundings.
- **Rest and motion are relative**. It means an object observed in one frame of reference may be at rest, the same object can be in motion in another frame of reference.
- An object can be considered as a point object if during motion in a given time, it covers a distance much greater than its own size.
- The motion of an object is said to be **one dimensional motion** if only one out of the three coordinates specifying the position of the object changes with respect to time. In such a motion, an object moves along a straight line.
- The motion of an object is said to be **two dimensional motion** if two out of the three coordinates specifying the position of the object change with respect to time. In such a motion, the object moves in a plane.
- The motion of an object is said to be **three dimensional motion** if all the three coordinates specifying the position of the object change with respect to time. In such a motion, the object moves in a space.
- The **distance** travelled by an object is defined as the length of the actual path traversed by an object during motion in a given interval of time. Distance is a scalar quantity. Its value can never be zero or negative, during the motion of an object.
- The **displacement** of an object in a given interval of time is defined as the change in the position of the object along a particular direction during that time and is given by the straight line joining the initial position to final position. The displacement of an object can be positive, zero or negative. The displacement of an object between two positions has a unique value, which is the shortest distance between them. The magnitude of the displacement of an object in a given time interval can be equal or less than the actual distance travelled but never greater than the distance travelled.
- For a stationary body, the displacement-time graph is a straight line (AB) parallel to time axis.



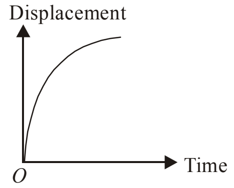
- When a body is moving with a constant velocity, then displacement-time graph will be a straight line.



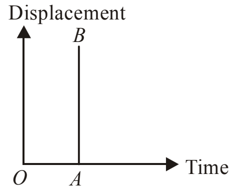
- When a body is moving with a constant acceleration, the displacement-time graph is a parabola.



- When a body is moving with a constant retardation, the displacement-time graph is a curve which bends downwards.



- When a body is moving with infinite velocity, the time-displacement curve is a straight line *AB* parallel to displacement axis. But such motion of a body is never possible.



- The **speed** of an object is defined as the time rate of change of position of the object in any direction,
  i.e. speed = distance travelled/(time taken).
  Speed is a scalar quantity. It can be zero or positive but never negative.

$$v = (s/t) \text{ m/s}.$$

- An object is said to be moving with a **uniform speed**, if it covers equal distances in equal intervals of time, howsoever small these intervals may be.
- An object is said to be moving with a **variable speed** if it covers equal distances in unequal intervals of time or unequal distances in equal intervals of time, howsoever small these intervals may be.
- The **average speed** of an object for the given motion is defined as the ratio of the total distance travelled by the object to the total time taken.

  i.e. Average speed = $\dfrac{\text{total distance travelled}}{\text{total time taken}}$
- **Average speed-harmonic mean :** When a body travels equal distance with speeds v_1 and v_2, the average speed (v) is the harmonic mean of the two speeds.

$$\frac{2}{v} = \frac{1}{v_1} + \frac{1}{v_2}.$$

- **Average speed and average of speeds:** When a body travels for equal time with speeds v_1 and v_2, the average speed v is the arithmetic mean of the two speeds.

$$v = \frac{v_1 + v_2}{2}.$$

  This speed is same as average of speeds.
- The speed of an object at a given instant of time is called its **instantaneous speed**.

$$v = \frac{ds}{dt}$$

- The **velocity** of an object is defined as the time rate of change of displacement of the object.

  i.e. velocity = displacement/time taken.

$$\text{velocity } (\vec{v}) = \frac{\text{displacement}}{\text{time}} = \frac{d\vec{x}}{dt} \qquad \text{or,} \qquad \vec{v} = \frac{\vec{x}_2 - \vec{x}_1}{t_2 - t_1} \text{ m/s}.$$

  where $\vec{x}_1$ and $\vec{x}_2$ are the displacement of an object at instants t_1 and t_2.

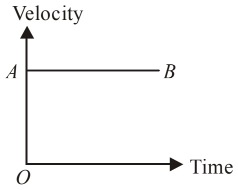
  The velocity is a vector quantity. The velocity of an object can be positive, zero or negative.
- If an object undergoes equal displacements in equal intervals of time, it is said to be moving with a **uniform velocity**. If an object undergoes unequal displacements in equal intervals of time or equal displacements in unequal intervals of time, it is said to be moving with a **variable velocity**.
- The average velocity of an object is equal to the ratio of the total displacement, to the total time interval for which the motion takes place.

  Average velocity = $\dfrac{\text{total displacement}}{\text{total time}}$.
- The velocity of an object at a given instant of time is called **instantaneous velocity**.

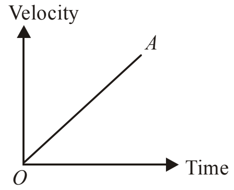
  $\vec{v} = d\vec{x}/dt$

  When a body is moving with a uniform velocity, its instantaneous velocity = average velocity = uniform velocity.

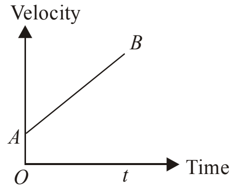
- When a body is moving with a constant velocity, the velocity-time graph is a straight line *AB* parallel to time axis.



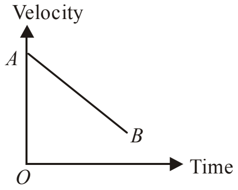
- When a body is moving with a constant acceleration and its initial velocity is zero, the velocity-time graph is an oblique straight line, passing through origin.



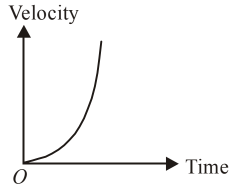
- When a body is moving with a constant acceleration and its initial velocity is not zero, the velocity-time graph is an oblique straight line *AB* not passing through origin.



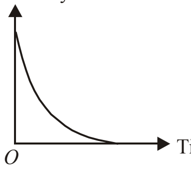
- When a body is moving with a constant retardation and its initial velocity is not zero, the velocity-time graph is an oblique straight line *AB*, not passing through origin.



- When a body is moving with increasing acceleration, the velocity-time graph is a curve which bend upwards.



- When a body is moving with decreasing acceleration, the velocity-time graph is a curve is similar to the one shown here.



- Slope of displacement-time graph gives **average velocity**.
- Slope of velocity-time graph gives **average acceleration**.
- The area of velocity-time graph with time axis gives total distance covered by the body.
- When a body is dropped freely from the top of the tower and another body is projected horizontally from the same point, both will reach the ground at the same time.
- In case of motion under gravity, motion is independent of the mass of the body, *i.e.* if a heavy and light body are dropped from the same height, they reach the ground simultaneously with the same speed because time taken by the body to reach the ground $= \sqrt{2h/g}$.
- **Relative velocity :** The relative velocity of one body with respect to another body is the velocity with which one body moves with respect to another body. If $\vec{v}_A$ and $\vec{v}_B$ are the velocities of two bodies A and B, and θ is the angle between them, then relative velocity of body A with respect to B is given by

  $$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$$

  where, $$|\vec{v}_{AB}| = \sqrt{|\vec{v}_A|^2 + |\vec{v}_B|^2 + 2|\vec{v}_A||\vec{v}_B|\cos(180° - \theta)}$$

  $$= \sqrt{|\vec{v}_A|^2 + |\vec{v}_B|^2 - 2|\vec{v}_A||\vec{v}_B|\cos\theta}$$

  and $$\tan\beta = \frac{|\vec{v}_B|\sin(180° - \theta)}{|\vec{v}_A| + |\vec{v}_B|\cos(180° - \theta)} = \frac{|\vec{v}_B|\sin\theta}{|\vec{v}_A| - |\vec{v}_B|\cos\theta}$$

  Here, β is the angle which $\vec{v}_{AB}$ makes with the direction of $\vec{v}_A$.
- If the two bodies A and B are moving in opposite directions with velocities $\vec{u}$ and $\vec{v}$ then, relative velocity of A with respect to B is equal to $(\vec{u} + \vec{v})$.
- If the two bodies A and B are moving with velocities $\vec{u}$ and $\vec{v}$ in the same direction, then relative velocity of A with respect to B is equal to $(\vec{u} - \vec{v})$.
- If rain drops are falling vertically with a velocity $\vec{v}$ and a person is walking horizontally with a velocity $\vec{u}$, then he should hold an umbrella at angle θ with vertical given by $\tan\theta = \frac{|\vec{u}|}{|\vec{v}|}$, to prevent himself from being wet.
- The **acceleration** of an object is defined as the time rate of change of velocity of the object,

  i.e. acceleration = change in velocity/time taken.

  Acceleration is a vector quantity. Acceleration is positive, if the velocity is increasing and is negative if velocity is decreasing. The negative acceleration is called retardation or deceleration.

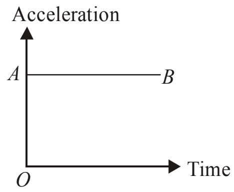
$$\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$$

- If the velocity of an object changes by equal amounts in equal intervals of time, it is said to be moving with a **uniform acceleration**. If the velocity of an object changes by unequal amounts in equal intervals of time, it is said to be moving with a **variable acceleration**.
- The **average acceleration** of an object for a given motion is defined as the ratio of the total change in velocity of the object during motion to the total time taken.

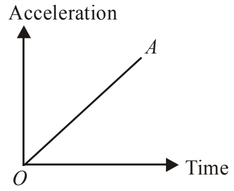
  i.e. average acceleration = $\dfrac{\text{total change in velocity}}{\text{total time taken}}$
- The acceleration of an object at a given instant or at a given point of motion is called its **instantaneous acceleration**. It is defined as the first time derivative of velocity at a given instant or it is also equal to the second time derivative of the position of the object at a given instant.

  i.e. instantaneous acceleration,

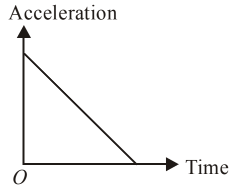
  $$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{x}}{dt^2}$$
- Negative acceleration is known as **retardation**. It indicates that velocity of the object is decreasing with respect to time.
- When a body is moving with constant acceleration, the acceleration-time graph is a straight line *AB* parallel to time axis.



- When a body is moving with constant increasing acceleration, the acceleration-time graph is a straight line *OA*.



- When a body is moving with constant decreasing acceleration, the acceleration-time graph is a straight line.



- **Equations of motion** for uniformly accelerated motion along a straight line.

  (i) $v = u + at$ (ii) $s = ut + \frac{1}{2}at^2$

  (iii) $v^2 = u^2 + 2as$ (iv) $s_n = u + \frac{a}{2}(2n - 1)$

  where u is initial velocity, v is final velocity, a is uniform acceleration, s is distance travelled in time t, s_n is distance covered in n^{th} second. These equations are not valid if the acceleration is non-uniform.

- **Scalars :** These are those quantities which have only magnitudes but no direction. For example mass, length, time, speed, work, temperature, etc.

- **Vectors :** These are those quantities which have magnitude as well as direction. For example displacement, velocity, acceleration, force, momentum, etc.

- **Tensor :** A physical quantity which has different values in different directions at the same point is called a tensor. Pressure, stress, modulii of elasticity, moment of inertia, radius of gyration, refractive index, wave velocity, dielectric constant, conductivity, resistivity and density are a few examples of tensor. Magnitude of tensor is not unique.

- **Representation of a vector :**
  - A vector is represented by a straight line.
  - It carries an arrow head at one extremity.
  - The arrow head represents direction of vector.
  - The length of line represents magnitude of vector.
  - Tail is the point from which the vector originates.
  - Head is a point at which the vector ends.

- **Addition of vectors - Properties :**

  (a) Vector addition is commutative, *i.e.* $\vec{A} + \vec{B} = \vec{B} + \vec{A}$.

  (b) Vector addition is associative. It means that

  $\vec{A} + (\vec{B} + \vec{C}) = \vec{B} + (\vec{C} + \vec{A}) = \vec{C} + (\vec{A} + \vec{B})$.

  (c) Vector addition is distributive. It means that

  $m(\vec{A} + \vec{B}) = m\vec{A} + m\vec{B}$

  $(m + n)\vec{A} = m\vec{A} + n\vec{A}$

  (d) The maximum value of vector addition is $(A + B)$.

  (e) The minimum value of vector addition is $(A - B)$.

- **Rotation of a vector :**

  (a) If the frame of reference is rotated or translated, the given vector does not change. The components of the vector may, however, change.

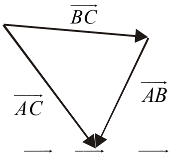
  (b) If a vector is rotated through an angle θ, which is not an integral multiple of 2π, the vector changes.

- **Direction cosines of vector $\vec{A}$:**

  $\cos\alpha = \frac{A_x}{A}$, $\cos\beta = \frac{A_y}{A}$, $\cos\gamma = \frac{A_z}{A}$

  $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$.

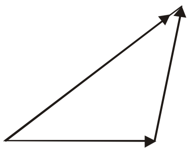
- Graphically a vector A is represented by a directed segment of a straight line, whose direction is that of the vector it represents and whose length corresponds to the magnitude $|\vec{A}|$ of $\vec{A}$.
- A **unit vector** of a given vector $\vec{A}$ is a vector of unit magnitude and has the same direction as that of the given vector. A unit vector of $\vec{A}$ is written as $\hat{A}$, where $\hat{A} = \vec{A}/|\vec{A}|$. A unit vector is unitless and dimensionless vector and represents direction only.
- The symbol $\hat{i}, \hat{j}, \hat{k}$ represent unit vectors along x, y and z directions of coordinate axes respectively.
- **Null vector** is a vector which has zero magnitude and an arbitrary direction. It is represented by $\vec{0}$ and is also known as **zero vector**. Velocity of a stationary object, acceleration of an object moving with uniform velocity and resultant of two equal and opposite vectors are the examples of null vector.
- **Equal vectors :** Two vectors are said to be equal if they have equal magnitude and same direction.
- A **negative vector** of a given vector is a vector of same magnitude but acting in a direction opposite to that of the given vector. The negative vector of $\vec{A}$ is represented by $-\vec{A}$.
- A vector whose initial point is fixed is called a **localised vector** and whose initial point is not fixed is called **non-localised vector**.
- **Multiplication of a vector by a real number :** When a vector $\vec{A}$ is multiplied by a real number n, it becomes another vector $n\vec{A}$. Its magnitude becomes n times the magnitude of $\vec{A}$. Its direction is same or opposite as that of $\vec{A}$, according as n is positive or negative real number. The unit of $n\vec{A}$ is the same as that of $\vec{A}$.
- **Multiplication of a vector by a scalar :** When a vector $\vec{A}$ is multiplied by a scalar S, it becomes a vector $S\vec{A}$, whose magnitude is S times the magnitude of $\vec{A}$ and it acts along the direction of $\vec{A}$. The unit of $S\vec{A}$ is different from the unit of vector $\vec{A}$.
- For the **addition of two vectors**, represent these two vectors by arrowed lines using the same suitable scale. Displace the second vector such that its tail coincides with the head of the first vector. Then the single vector, drawn from the tail of the first vector to the head of the second vector represents the resultant vector.



$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

- **Triangle law of vectors :** If two vectors acting simultaneously at a point are represented in magnitude and direction by two sides of a triangle taken in the same order, then their resultant is represented by the third side of the triangle taken in the opposite order.

According to triangle law of vector addition.



$$\vec{P} + \vec{Q} = \vec{R} \quad \text{or} \quad \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$$

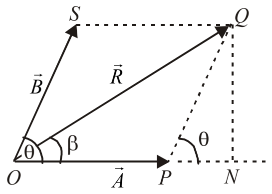
or $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \overrightarrow{AC} + \overrightarrow{CA}$

(adding vector $\overrightarrow{CA}$ to both sides of the equation).

or $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \overrightarrow{AC} - \overrightarrow{AC}$ $\left[\because \overrightarrow{CA} = -\overrightarrow{AC}\right]$

$= 0.$

- **Parallelogram law of vectors:** It states that if two vectors acting on a particle at the same time be represented in magnitude and direction by the two adjacent sides of a parallelogram drawn from a point, their resultant vector is represented in magnitude and direction by the diagonal of the parallelogram drawn from the same point.



If $\vec{R}$ is the resultant of $\vec{A}$ and $\vec{B}$, then

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta} \text{ and } \tan\beta = \frac{B\sin\theta}{A + B\cos\theta}$$

Note : The magnitude of the resultant vector is maximum if the two vectors are acting in the same direction and is minimum if the two vectors are acting in the opposite directions.

- **Polygon law of vectors :** It states that if number of vectors acting on a particle at a time are represented in magnitude and direction by the various sides of an open polygon taken in same order, their resultant vector is represented in magnitude and direction by the closing side of the polygon taken in opposite order. In fact polygon law of vectors is the outcome of triangle law of vectors.

- **Subtraction of vectors :** Subtraction of a vector $\vec{B}$ from a vector $\vec{A}$ is defined as the addition of vector $-\vec{B}$ (negative of vector $\vec{B}$) to vector $\vec{A}$.

  Thus, $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$

- **Rectangular components of a vector in a plane**

  When a vector is splitted into two component vectors at right angles to each other, the component vectors are called rectangular components of a vector. If $\vec{A}$ makes an angle θ with x-axis and $\vec{A}_x$ and $\vec{A}_y$ are the rectangular components of $\vec{A}$ along x-axis and y-axis respectively, then

  $\vec{A} = \vec{A}_x + \vec{A}_y = A_x\hat{i} + A_y\hat{j}$

  Here, $A_x = A\cos\theta$ and $A_y = A\sin\theta$

  $\therefore\ A^2(\cos^2\theta + \sin^2\theta) = A_x^2 + A_y^2$

  or, $A = (A_x^2 + A_y^2)^{1/2}$ and $\tan\theta = A_y/A_x$.

- **Dot product of two vectors :** The dot product of two vectors $\vec{A}$ and $\vec{B}$ is denoted by $\vec{A}\cdot\vec{B}$ and is given by $\vec{A}\cdot\vec{B} = AB\cos\theta$, where θ is the smaller angle between $\vec{A}$ and $\vec{B}$. The dot product of two vectors is a scalar.

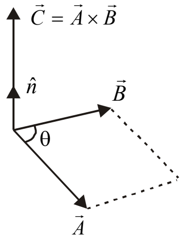
- **Geometrical interpretation of dot product of two vectors :** It is the product of the magnitude of one vector with the magnitude of the component of other vector in the direction of first vector.

- $\vec{A}\cdot\vec{A}=A^2$ or $A=(\vec{A}\cdot\vec{A})^{1/2}$
- $\hat{i}\cdot\hat{i}=\hat{j}\cdot\hat{j}=\hat{k}\cdot\hat{k}=1$ and $\hat{i}\cdot\hat{j}=\hat{j}\cdot\hat{k}=\hat{k}\cdot\hat{i}=0$
- **Dot product in cartesian co-ordinates**

  $\vec{A}\cdot\vec{B}=(A_x\hat{i}+A_y\hat{j}+A_z\hat{k})\cdot(B_x\hat{i}+B_y\hat{j}+B_z\hat{k})$

  $=A_xB_x+A_yB_y+A_zB_z.$

  $\vec{A}\cdot\vec{B}=\vec{B}\cdot\vec{A}$.



- **Cross product of two vectors :** The cross product of two vectors $\vec{A}$ and $\vec{B}$ is denoted by $\vec{A}\times\vec{B}$. It is a vector whose magnitude is equal to the product of the magnitudes of the two vectors and sine of the smaller angle between them. If θ is smaller angle between $\vec{A}$ and $\vec{B}$, then $\vec{A}\times\vec{B}=\vec{C}=AB\sin\theta\,\hat{n}$,

  where $\hat{n}$ is a unit vector in the direction of $\vec{C}$.
- **Geometrical interpretation of vector product of two vectors:** The magnitude of vector product of two vectors is equal
  (i) to the area of the parallelogram whose two sides are represented by two vectors.
  (ii) to twice the area of a triangle whose two sides are represented by the two vectors.
- **Properties of cross product**

  (i) Cross product of two parallel vectors is zero. So, $\vec{A}\times\vec{A}=0$.

  (ii) $\hat{i}\times\hat{i}=0=\hat{j}\times\hat{j}=\hat{k}\times\hat{k}$

  (iii) $\hat{i}\times\hat{j}=\hat{k},\ \hat{j}\times\hat{k}=\hat{i},\ \hat{k}\times\hat{i}=\hat{j}$

  (iv) $\vec{A}\times\vec{B}=-\vec{B}\times\vec{A}$
- **Cross product in cartesian co-ordinates:**

  $\vec{A}\times\vec{B}=(A_x\hat{i}+A_y\hat{j}+A_z\hat{k})\times(B_x\hat{i}+B_y\hat{j}+B_z\hat{k})$

  $$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix}$$

  $=(A_yB_z-A_zB_y)\hat{i}-(A_xB_z-B_xA_z)\hat{j}+(A_xB_y-A_yB_x)\hat{k}$
- **Direction of vector cross product :**

  (a) When $\vec{C}=\vec{A}\times\vec{B}$, the direction of $\vec{C}$ is at right angles to the plane containing the vectors $\vec{A}$ and $\vec{B}$. The **direction** is determined by the right hand screw rule and the right hand thumb rule.

  (b) **Right hand screw rule :** Rotate a right handed screw from first vector $(\vec{A})$ towards second vector $(\vec{B})$. The direction in which the right handed screw moves gives the direction of vector $\vec{C}$.

  (c) **Right hand thumb rule :** Curl the fingers of your right hand from $\vec{A}$ to $\vec{B}$. Then the direction of the erect thumb will point in the direction of $\vec{A}\times\vec{B}$.

Angular variables

- Angular displacement is the angle which the position vector sweeps out in a given interval of time. It is represented by θ. In SI, the unit of angular displacement is radian (rad). It is a dimensionless quantity.
- **Angular velocity :** The rate of change of angular displacement is called the angular velocity. It is represented as ω. In SI, the unit of angular velocity is radian per second (rad sec^{-1}), its dimensional formula is $[M^0L^0T^{-1}]$.

  Average angular velocity = $\dfrac{\Delta\theta}{\Delta t}$.

  Instantaneous angular velocity $= \lim\limits_{\Delta t\to\infty} \dfrac{\Delta\theta}{\Delta t} = \dfrac{d\theta}{dt} = \omega$.

  Relation between angular and linear velocity $v = r\omega$.
- **Time period :** The time taken to complete one revolution is called the time period. It is denoted by T.
- **Angular acceleration :** The rate of change of angular velocity is called angular acceleration. It is denoted as α and its unit is radian sec^{-2} (rad sec^{-2}). Dimensions are $[T^{-2}]$.

$$\vec{\alpha} = \frac{d\vec{\omega}}{dt} = \frac{d^2\vec{\theta}}{dt^2}.$$

- Relation between linear acceleration and angular acceleration

$$a = r\alpha.$$

- **Frequency :** The frequency is defined as the number of revolutions completed per second. It is denoted by υ.

$$\upsilon = \frac{1}{T}. \text{ Its unit is } s^{-1}.$$

Kinematical equations in circular motion

- (i) The angular velocity of the object after time t is given by $\omega = \omega_0 + \alpha t$.

  (ii) The angular displacement of the object after time t is given by

  $\theta = \omega_0 t + \dfrac{1}{2}\alpha t^2$.

  (iii) If the object attains angular velocity ω, while it covers angular displacement θ, then

  $\omega^2 - \omega_0^2 = 2\alpha\theta$.

CIRCULAR MOTION

- In physics, circular motion is movement of an object with constant speed around in a circle in a circular path or a circular orbit.
- Circular motion involves acceleration of the moving object by a centripetal force which pulls the moving object towards the centre of the circular orbit. Without this acceleration, the object would move inertially in a straight line, according to Newton's first law of motion. Circular motion is accelerated even though the speed is constant, because the velocity of the moving object is constantly changing.

- Examples of circular motion are: an artificial satellite orbiting the earth in geosynchronous orbit, a stone which is tied to a rope and is being swung in circles (cf. hammer throw), a racecar turning through a curve in a racetrack, an electron moving perpendicular to a uniform magnetic field, a gear turning inside a mechanism.
- A special kind of circular motion is when an object rotates around itself. This can be called **spinning motion**.
- Circular motion is characterized by an orbital radius r, a speed v, the mass m of the object which moves in a circle, and the magnitude F of the centripetal force. These quantities are all related to each other through the equations for circular motion.
- The centripetal force can be tension of the string, gravitational force, electrostatic force or Lorentzian. But the centrifugal force $= m\omega^2 r$ or mv^2/r in stable rotation, is equal to the centripetal force in magnitude and acts outwards.
- A centripetal force of magnitude $\frac{mv^2}{r}$ is needed to keep the particle in uniform circular motion.
- Centrifugal force is the force acting away from the centre and is equal in magnitude to the centripetal force.
- For a safe turn the co-efficient of friction between the road and the tyre should be, $\mu_s \geq \frac{v^2}{rg}$
  where v is the velocity of the vehicle
  r is the radius of the circular path
- Angle of banking, $\tan\theta = \frac{v^2}{rg}$ this θ depends on the speed, v and radius of the turn r.
- A cyclist provides himself the necessary centripetal force by leaning inward on a horizontal track.
- The maximum permissible speed for the vehicle is much greater than the optimum value of the speed on a banked road. It is because, friction between road and the tyre of the vehicle also contributes to the required centripetal force.
- Roads are usually banked for the average speed of vehicle passing over them. If μ is the coefficient of friction between the tyres and the road the safe value limit is $v = \sqrt{\frac{rg\tan\theta + \mu}{1 - \mu\tan\theta}}$.
- In case of vertical circle the minimum velocity v, the body should possess at the top so that the string does not slack, is $\sqrt{gr}$.
- The magnitude of velocity at the lowest point with which body can safely go round the vertical circle of radius r is $\sqrt{5gr}$.
- Tension in the string at lowest point $T = 6Mg$.
- The tangent at every point of the circular motion gives the direction of motion in circular motion at that point.

- A car some times overturns while taking a turn. When it overturns it is the inner wheel, which leaves the ground first.
- A car when passes a convex bridge exerts a force on it which is equal to $Mg - \frac{Mv^2}{r}$.
- The driver of a car should brake suddenly rather than taking sharp turn to avoid accident, when he suddenly sees a broad wall in front of him.

Uniform horizontal circular motion

- The instantaneous velocity and displacement act along tangent to the circle at a point.
- The centripetal acceleration and the centripetal force act along radius towards the centre of circle.
- The centripetal force and displacement are at right angles to each other. Hence the work done by the centripetal force is zero.
- Kinetic energy of a particle performing uniform circular motion, in horizontal plane, remains constant.
- The instantaneous velocity of particle and the centripetal acceleration are at right angles to each other. Hence the magnitude of velocity does not change but the direction of velocity changes continuously. It is thus a case of **uniformly accelerated motion**.
- Centripetal acceleration is also called radial acceleration as it acts along radius of circle.
- Momentum of the particle changes continuously along with the velocity.
- The centripetal force does not increase the kinetic energy and angular momentum of the particle moving in a circular path.

Non-uniform horizontal circular motion

- If the magnitude of the velocity of the particle in horizontal circular motion changes with respect to time, the motion is known as non-uniform circular motion.
- The acceleration of particle is called tangential acceleration. It acts along the tangent to the circle at a point. It changes the magnitude of linear velocity of the particle.
- Tangential acceleration $\vec{f}_T$ and angular acceleration $\vec{\alpha}$ are related as $\vec{f}_T = \vec{r} \times \vec{\alpha}$ where $\vec{r}$ denotes radius vector .
- Centripetal acceleration f_C and tangential acceleration f_T act at right angles to each other.

  $$\therefore \; f^2 = f_C^2 + f_T^2 = \left(\frac{v^2}{r}\right)^2 + f_T^2.$$

- $\tan\phi = \frac{f_T}{f_C} = \frac{r\alpha}{v^2/r} = \frac{r^2\alpha}{v^2}.$

PROJECTILE MOTION

- Anything thrown in space and then allowed to move under the effect of gravity alone is called **projectile**.