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BASED ON LATEST PATTERN

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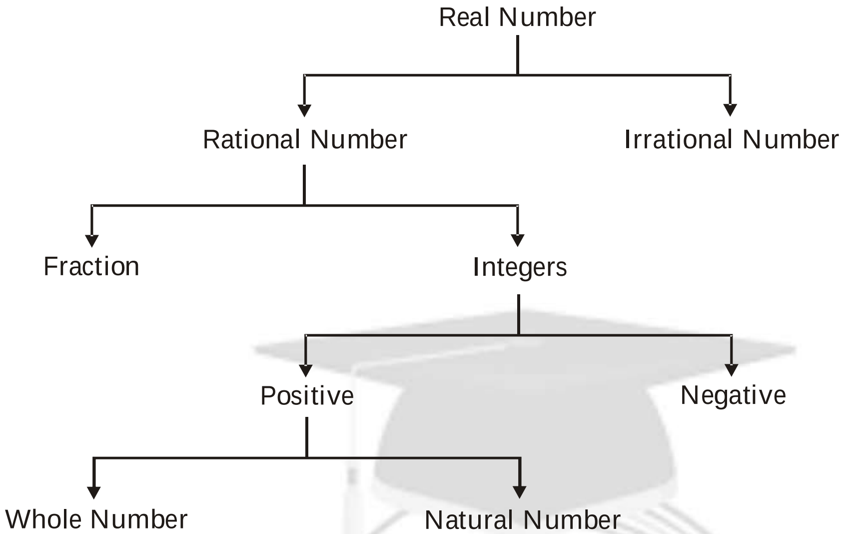
CONTENT

15 Practice Sets (Based on Latest Pattern)

Chapter 1

Number System and Simplifications



Natural Numbers → 1, 2, 3, ... ∞

Whole Numbers → 0, 1, 2, 3 .... ∞

Integers → $-\infty$ ....–3, –2, –1, 0, 1, 2 .... ∞

Rational Numbers → Integers and Fractions.

Fraction: Any number that can be represented in the form of p / q, where p & q are integers & q is not equal to zero is called a rational number.

Irrational Number → Any real number that cannot be expressed as a ratio of integers, i.e as a fraction.

Example: $\sqrt{5}$, $\sqrt{8}$

Prime Number: A number which has exactly two factors 1 & itself is called a prime number.

Prime numbers from 1 – 100 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97 i.e. there are total 25 prime numbers up to 100.

Some results on Prime Numbers:

(i) Up to 100 total prime numbers = 25

(ii) Up to 50 total prime numbers = 15

(iii) Sum of two prime numbers is always even except 2.

(iv) Sum of three prime numbers is even if and only if one number is 2.

(v) All prime numbers are odd except 2.

(vi) 2 is only even prime number.

(vii) Each prime number has two factors 1 & itself so 1 is not prime number.

(viii) Smallest prime number of three digit is 101

(ix) Largest prime number of three digit is 997

(x) If square of any prime number (except 2 and 3) is divided by 24 then remainder is always 1.

Example: $\frac{1}{24}\times\left(11^2,\ 13^2,\ 17^2,\ 19^2,\ 23^2\right)$ = (remainder 1 in each case).

Composite No.: A number which has more than two factor is a composite number.

Example: 4, 6, 8, 9 ....

Note: 1 is neither prime nor composite number, 2 is only even prime number.

Co-Prime No.: The pair of numbers which have no common factor other than one are called co-prime numbers.

Example: (4, 5), (15, 8)

Tests of Divisibility:

(i) Divisibility by 2 : A number is divisible by 2, if its unit place is any of 0, 2, 4, 6, 8

(ii) Divisibility by 3 : A number is divisible by 3 only when the sum of its digits is divisible by 3.

(iii) Divisibility by 4 : A number is divisible by 4 if the number formed by its last two digits is divisible by 4.

(iv) Divisibility by 5 : A number is divisible by 5 if its unit digit is 5 or 0.

(v) Divisibility by 6 : A number is divisible by 6 if it is divisible by both 2 & 3.

(vi) Divisibility by 8 : A number is divisible by 8 when the number formed by its last 3 digits is divisible by 8.

(vii) Divisibility by 9 : A number is divisible by 9 if the sum of its digits is divisible by 9.

(viii) Divisibility by 10 : A number is divisible by 10 only when its unit digit is zero.

(ix) Divisibility by 11 : A number is divisible by 11, if the difference of the sum of its digits at odd places & the sum of its digits at even places is either 0 or a number divisible by 11.

Some results on division:

(i) $(x^n - a^n)$ is divisible by $(x - a)$ for all value of n.

(ii) $(x^n - a^n)$ is divisible by $(x + a)$ for even value of n.

(iii) $(x^n + a^n)$ is divisible by $(x + a)$ for odd value of n.

$$\text{Dividend} = (\text{Divisor} \times \text{Quotient}) + \text{Remainder}$$

Some Results on Numbers:

(i) The product of four numbers which are consecutive natural numbers is always divisible by 24.

Example: $\frac{101 \times 102 \times 103 \times 104}{24}$ or $\frac{7 \times 8 \times 9 \times 10}{24}$

(ii) The difference of square of two consecutive natural numbers is always equal to sum of those numbers.

Example: $9^2 - 8^2 = 9 + 8$, $119^2 - 118^2 = 119 + 118$

(iii) The difference of square of two consecutive odd numbers is always divisible by 8.

Example: $11^2 - 9^2 = 121 - 81 = 40$

$\frac{40}{8} = 5.$

(iv) The difference of square of two consecutive even numbers is always divisibly by 4.

Example: $10^2 - 8^2 = 100 - 64 = 36$

$\frac{36}{4} = 9.$

(v) Any digit repeated 6 times is divisible by 7, 11, 13 & 37.

Example: 5 5 5 5 5 5 or 2 2 2 2 2 2
are divisible by 7, 11, 13 & 37.

(vi) Any two digit number repeated 2 times is always divisible by 101.

Example: 3 4 3 4 or 5 6 5 6 is divisible by 101.

(vii) If P is prime number & a is an integer then $(a^P - a)$ is always divisible by P.

Example: $(5^{11} - 5)$ is divisible by 11.

(viii) If n is an odd number then $(2^{2n} + 1)$ is always divisible by 5.

(ix) If n is an even number, then $(2^{2n} - 1)$ is always divisible by 5.

(x) The product of three consecutive natural numbers is always divisible by 6.

Example: $\frac{1}{6} \times (8 \times 9 \times 10)$ or $\frac{1}{6} \times (11 \times 12 \times 13)$

(xi) The product of three consecutive natural numbers starting with even number is always divisible by 24.

Example: $\frac{1}{24} \times (\underset{\substack{\downarrow \\ \text{even}}}{8} \times 9 \times 10)$ or $\frac{1}{24}(\underset{\substack{\downarrow \\ \text{even}}}{18} \times 19 \times 20)$

(xii) Any number written in the form $9\,(10^n - 1)$ is always divisible by 3 & 9 both.

(xiii) Any natural number of the form $(n^3 - n)$ is always divisible by 6.

Unit digit : $3^4 = 81 = 1$ i.e. 1 is unit digit

$3215 \times 5163 \times 7298$

product of unit digits = $5 \times 3 \times 8 = 120$, i.e. unit digit is zero.

The unit digit of the numbers in following forms is:

$5^n = 5$	$4^{odd} = 4$	$9^{odd} = 9$
$6^n = 6$	$4^{even} = 6$	$9^{even} = 1$
$0^n = 0$		
$1^n = 1$		

Example :

(i) $234^{567} + 566^{133}$

Unit digit = 4 + 6 = 10 = 0

(ii) $249^{33} + 250^{34} + 251^{35}$

unit digit = 9 + 0 + 1 = 10 = 0

Remaining digit : (2, 3, 7, 8)

- $212^{79} \Rightarrow 2^{79/4} = 2^3 = 8$
- $378^{41925} \Rightarrow 8^{25/4} = 8^1 = 8$
- $473^{2188} \Rightarrow 3^{88/4} = 3^4 = 81 = 1$
- In case remainder is zero, then power would be 4

  Example : $214^{2164} \Rightarrow 4^{64/4} = 4^4 = 256 = 6$

Testing of prime numbers

- Test whether 191 is prime or not

  Clearly 14 > $\sqrt{191}$

  Prime numbers up to 14 are 2, 3, 5, 7, 11, 13

  No one of these divides 191 exactly

  $\therefore$ 191 is a prime number.

- Test wether 221 is prime or not

  Clearly 15 is > $\sqrt{221}$

  Prime numbers up to 15 are 2, 3, 5, 7, 11, 13

  Clearly, 13 divides 221 exactly

  So, 221 is not prime.

(i) Sum of n natural numbers

$$1 + 2 + 3 + \ldots\ldots\ldots\ldots. + n = \frac{n(n+1)}{2}$$

(ii) Sum of squares of n natural numbers

$$1^2 + 2^2 + 3^2 + \ldots\ldots\ldots\ldots\ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

(iii) Sum of cube of n natural numbers

$$1^3 + 2^3 + 3^3 + \ldots\ldots\ldots\ldots\ldots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

Odd number : Those numbers which are not divisible by 2, are known as odd numbers

Example: 1, 3, 5, 7, ...

$n = \frac{t_n + 1}{2}$, where n = total number of term, t_n = last term.

Sum of Ist n odd numbers = n^2

Example : $1 + 3 + 5 + \ldots\ldots\ldots\ldots\ldots\ldots + 49$

$n = \frac{49+1}{2} = 25$, sum = $(25)^2$ (since, n = 25)

= 625

Example: Find the sum of the series

$51 + 53 + \ldots\ldots\ldots\ldots\ldots + 99$

$$= \frac{(\text{Last term} + \text{I}^{st}\text{ term}) \times (\text{Last term} - \text{Previous term of I}^{st}\text{ term})}{4} = \frac{(99+51)(99-49)}{4} = \frac{150 \times 50}{4} = 1875$$

Even Numbers: Those numbers which are divisible by 2 are known as even numbers.

Example : 2, 4, 6, 8, ...

$n = \frac{t_n}{2}$, Where n = total numbers of term, t_n = last term

sum of Ist n even numbers = n(n + 1)

Example : 2 + 4 + 6 + ... + 58

$n = \frac{58}{2} = 29$, sum = n(n + 1) = 29(29 + 1) = 870

Remainder Theorem:

1. When a_1, a_2, a_3 ..... a_n are divided by 'd' individually the respective remainders are R_1, R_2, R_3.... R_n and when $(a_1 + a_2 + a_3 ....a_n)$ is divided by 'd' the remainder can be obtained by dividing $(R_1 + R_2 + R_3 ....R_n)$ by 'd'

Example : Find remainder when 38 + 71 + 85 is divided by 16

$= \frac{38+71+85}{16} = \frac{6+7+5}{16}$

(Remainder obtained when numbers are individually divided by 16)

$= \frac{18}{16} \Rightarrow$ Remainder = 2

2. When a_1, a_2, a_3... a_n are divided by a divisor d the respective remainders obtained are R_1, R_2, R_3.....R_n, and the remainder when $(a_1 \times a_2 \times a_3 .... \times a_n)$ is divided by 'd' can be obtained by dividing $(R_1 \times R_2 \times R_3 ....R_n)$ by d.

Example : Find Remainder when 7^7 is divided by 4.

$\frac{7^7}{4} = \frac{7\times7\times7\times7\times7\times7\times7}{4} = \frac{3\times3\times3\times3\times3\times3\times3}{4}$ (Remainder obtained individually)

$= \frac{9\times9\times9\times3}{4} = \frac{1\times1\times1\times3}{4} \Rightarrow$ Remainder = 3

So we can say that remainders can be added as well as multiplied.

Some results on remainder

- For $\frac{nx}{n}$, Remainder = 0
- For $\frac{(nx+1)^n}{n}$, Remainder = 1
- For $\frac{(nx-1)^{even}}{n}$, Remainder = 1
- For $\frac{(nx-1)^{odd}}{n}$, Remainder = –1 or (n–1)

Where x and n are any positive integers.

Recurring Decimal : A decimal number in which a digit or a set of digits repeats regularly, over a constant period, is called a recurring decimal.

Example : 2.3333..... , 7.5555.... , 1.3333.... they are represented as $2.\bar{3}$, $7.\bar{5}$, $1.\bar{3}$

(i) Pure Recurring decimal : A decimal fraction in which all the figures occur repeatedly is called a pure recurring decimal e.g 7.4444.... , 2.1111.... , 3.4545...

(ii) Mixed Recurring decimal : A decimal number in which some of the digits do not recur is called a mixed recurring decimal e.g. 0.1777, .087373...

(iii) Non recurring decimal : A decimal number in which there is no regular pattern of repitition of digits after decimal point is called non-recurring decimal e.g. 3.24662676...

Fraction : The word fraction means a part of anything. It can be expressed in the from of $\frac{p}{q}$ where p and q are integers and 'q' is not equal to '0'.

Proper fraction : When the numerator is less than the denominator, then the fraction is called a proper fraction.

Example : $\frac{7}{12}, \frac{5}{17}, \frac{12}{43}$ etc.

Improper fraction : When the numerator is greater than the denominator, then the fraction is called an improper fraction.

Example : $\frac{17}{13}, \frac{18}{14}, \frac{45}{19}$ etc.

Like fraction : Fractions having same denominator are called like fractions.

Example : $\frac{1}{9}, \frac{5}{9}, \frac{7}{9}$ etc.

Unlike fraction : Fractions having different denominators are called unlike fractions.

Example : $\frac{14}{23}, \frac{17}{43}, \frac{53}{19}$ etc.

Compound fraction : It is a fraction of a fraction.

Example : $\frac{1}{3}$ of $\frac{5}{9}$, $\frac{7}{9}$ of $\frac{61}{53}$, $\frac{9}{13}$ of $\frac{7}{19}$

Complex fraction : In such a fraction, both the numerator and the denominator are fractions.

Example : $\dfrac{\frac{12}{13}}{\frac{17}{21}}, \quad \dfrac{\frac{5}{17}+\frac{13}{72}}{\frac{74}{43}+\frac{7}{9}}$

Mixed fraction : Those fractions which consist of a whole number and a proper fraction, are known as mixed fractions.

Example : $5\frac{7}{8}, 7\frac{4}{9}, 12\frac{13}{17}$ etc.

Continued fraction : It contains an additional fraction in the numerator or in the denominator.

Example : $12+\cfrac{1}{12+\cfrac{14}{65+\cfrac{2}{3}}}$

Decimal faction : In such a fraction, the denominator has power of 10.

Example: $0.45 = \frac{45}{100}, 0.7 = \frac{7}{10}, 0.000071 = \frac{71}{1000000}$ etc.

Types of Questions

1. A number when divided by 91 gives a remainder 17. When the same no is divided by 13, the remainder will be

Sol. $\frac{17}{13} = 4$ remainder

2. $(4^{61} + 4^{62} + 4^{63})$ is divisible by:

Sol. $4^{61}(1 + 4 + 4^2) = 4^{61} \times 21$

i.e. 21 is divisible by 3

3. Find the number of zeros in the product of 1 × 2 × 3 × .... × 99 × 100.

Sol. $\frac{100}{5} = 20$ and $\frac{20}{5} = 4$

i.e. total numbers of zeros = 20 + 4 = 24

4. Find the total number of zeros in the product of 1 × 2 × 3 × .... × 250.

Sol. $\frac{250}{5} = 50$, $\frac{50}{5} = 10$ and $\frac{10}{5} = 2$

i.e. total numbers of zeros = 50 + 10 + 2 = 62

5. Find the total number of zeros in the product of 51 × 52 × 53 × .... × 100.

Sol. $\frac{100}{5} = 20$, $\frac{20}{5} = 4$

and, $\frac{50}{5} = 10$, $\frac{10}{5} = 2$

So, total number of zeros = (20 + 4) – (10 + 2) = 12

6. Find the remainder in the following questions

(i) $\frac{5^{37}}{8}$ (ii) $\frac{2^{75}}{5}$

(iii) $\frac{517^{517}}{2}$ (iv) $\frac{2243^{165}}{5}$

(v) $\frac{7^{129}}{5}$ (vi) $\frac{8^{123}}{9}$

(vii) $\frac{2^{76}}{9}$ (viii) $\frac{19^{20} + 19^{40}}{20}$

(ix) $\frac{4^{75} + 4^{76}}{17}$ (x) $\frac{517^{517}}{5}$

Sol. (i) $\frac{5^{37}}{8} \Rightarrow \frac{(5^2)^{18} \times 5^1}{8} = \frac{25^{18} \times 5}{8} = \frac{1^{18} \times 5}{8} = 5$

(ii) $\frac{2^{75}}{5} \Rightarrow \frac{(2^4)^{18} \times 2^3}{5} = \frac{16^{18} \times 8}{5} = \frac{(1)^{18} \times 8}{5} = 3$

(iii) $\frac{517^{517}}{2} \Rightarrow \frac{1^{517}}{2} = 1$

(iv) $\frac{2243^{165}}{5} \Rightarrow \frac{3^{165}}{5} = \frac{(3^4)^{41} \times 3^1}{5} = 3$

(v) $\frac{7^{129}}{5} \Rightarrow \frac{2^{129}}{5} = \frac{(2^4)^{32} \times 2}{5} = 2$

(vi) $\frac{8^{123}}{9} \Rightarrow \frac{(-1)^{123}}{9} = 9 - 1 = 8$

(vii) $\frac{2^{76}}{9} \Rightarrow \frac{(2^3)^{25} \times 2}{9} = \frac{(-1)^{25} \times 2}{9} = \frac{-2}{9} = 7$

(viii) $\frac{19^{20} + 19^{40}}{20} \Rightarrow \frac{(-1)^{20} + (-1)^{40}}{20} = \frac{2}{20} = 2$

(ix) $\frac{4^{75} + 4^{76}}{17} \Rightarrow \frac{(4^2)^{37} \times 4 + (4^2)^{38}}{17}$

$= \frac{(-1)^{37} \times 4 + (-1)^{38}}{17} = \frac{-1 \times 4 + 1}{17} = \frac{-3}{17} = 14$

(x) $\frac{517^{517}}{5} \Rightarrow \frac{2^{517}}{5} = \frac{(2^4)^{129} \times 2^1}{5} = \frac{1^{129} \times 2}{5} = 2$

7. Find the unit digit in the following questions.

(i) $(124)^{372} + (124)^{373}$

(ii) $(4387)^{245} + (621)^{72}$

(iii) $25^{6521} + 36^{528} + 73^{54}$

(iv) $7^{71} \times 6^{63} \times 3^{65}$

(v) $(251)^{98} + (21)^{29} - (106)^{100} + (705)^{35} - 16^4 + 259$

Sol. (i) $(124)^{372} + (124)^{373} = 6 + 4$

$\Rightarrow$ unit digit = 0

(ii) $(4387)^{245} + (621)^{72} = (7)^1 + (1)^{72} = 7 + 1$

= 8 (unit digit).

(iii) $25^{6521} + 36^{528} + 73^{54} = 5 + 6 + (3)^2 = 5 + 6 + 9 = 20$

$\therefore$ unit digit = 0

(iv) $7^{71} \times 6^{63} \times 3^{65}$

$= 7^3 \times 6^3 \times 3^1 = 3 \times 6 \times 3$

= 4 (unit digit)

(v) $(251)^{98} + (21)^{29} - (106)^{100} + (705)^{35} - 16^4 + 259$

= 1 + 1 – 6 + 5 – 6 + 9 = 16 – 12

= 4 (unit digit)

Foundation

Questions

1. The sum of all those prime numbers which are less than 31 is
(a) 119 (b) 129
(c) 132 (d) 137

2. The sum of all even numbers between 21 and 51 is
(a) 518 (b) 540
(c) 560 (d) 596

3. Which of the following is one of the factors of the sum of first 25 natural numbers
(a) 26 (b) 24
(c) 13 (d) 12

4. The digit in the unit place of the product $(2464)^{1793} \times (615)^{317} \times (131)^{491}$ is
(a) 0 (b) 2
(c) 3 (d) 5

5. The digit in the unit place of $[(251)^{98} + (21)^{29} - (106)^{100} + (705)^{35}]$ is
(a) 1 (b) 4
(c) 5 (d) 6

6. Find the remainder value in the following expression
$$\frac{(23^2 + 29^2 + 31^2 + 37^2)}{24}$$
(a) 13 (b) 17
(c) 4 (d) 3

7. Find the value of given series
$1 - 2 + 3 - 4 + 5 - 6 + \ldots\ldots + 95 - 96 + 97 - 98$
(a) 49 (b) 53
(c) –49 (d) –53

8. Find the total number of zeros in the following series
$2 \times 4 \times 6 \times \ldots\ldots \times 248 \times 250$
(a) 31 (b) 37
(c) 39 (d) 43

9. 101 × 102 × 103 × 104 is a number which is always divisible by the greatest number in the given option.
(a) 6 (b) 24
(c) 48 (d) 16

10. Find the number of total prime numbers up to 100
(a) 27 (b) 23
(c) 25 (d) 26

11. When two numbers are separately divided by 33, the remainders are 21 and 28 respectively. If the sum of the two numbers is divided by 33, the remainder will be
(a) 10 (b) 12
(c) 14 (d) 16

12. In a question of division, the divisor is 7 times the quotient and 3 times the remainder. If remainder is 28, then the dividend is
(a) 588 (b) 784
(c) 823 (d) 1036

13. If 17^{200} is divided by 18, the remainder is
(a) 17 (b) 16
(c) 1 (d) 2

14. Which of the following numbers is not divisible by 18
(a) 54036 (b) 50436
(c) 34056 (d) 65043

15. It is given that $(2^{32} + 1)$ is exactly divisible by a certain number. Which one of the following is also definitely divisible by the same number.
(a) $2^{96} + 1$ (b) 7×2^{33}
(c) $2^{16} - 1$ (d) $2^{16} + 1$

16. The least number among $\frac{4}{9}, \sqrt{\frac{9}{49}}, 0.45$ and $(0.8)^2$ is
(a) $\frac{4}{9}$ (b) $\sqrt{\frac{9}{49}}$
(c) 0.45 (d) $(0.8)^2$

17. The number 0.121212 ... in the form $\frac{p}{q}$ is equal to
(a) $\frac{4}{11}$ (b) $\frac{2}{11}$
(c) $\frac{4}{33}$ (d) $\frac{2}{33}$

18. The least among the fraction $\frac{15}{16}, \frac{19}{20}, \frac{24}{25}, \frac{34}{35}$ is
(a) $\frac{34}{35}$ (b) $\frac{15}{16}$
(c) $\frac{19}{20}$ (d) $\frac{24}{25}$

19. If $1^3 + 2^3 + \ldots + 9^3 = 2025$, then the value of $(0.11)^3 + (0.22)^3 + \ldots + (0.99)^3$ is close to
(a) 0.2695 (b) 2.695
(c) 3.695 (d) 0.3695

20. Which of the following number is the greatest among all?
$0.9, 0.\overline{9}, 0.0\overline{9}, 0.\overline{09}$
(a) 0.9 (b) $0.\overline{9}$
(c) $0.0\overline{9}$ (d) $0.\overline{09}$

21. How many natural numbers divisible by 7 are there between 3 and 200?
(a) 27 (b) 28
(c) 29 (d) 36

22. The sum of three consecutive odd natural numbers is 87. The smallest of these numbers is
(a) 29 (b) 31
(c) 23 (d) 27

23. What will be the unit digit in 7^{105}?
(a) 5 (b) 7
(c) 9 (d) 1

24. Which one of the following will completely divide $5^{71} + 5^{72} + 5^{73}$?
(a) 150 (b) 160
(c) 155 (d) 30

25. When 2^{33} is divided by 10, the remainder will be
(a) 2 (b) 3
(c) 4 (d) 8

26. When a number is divided by 24, the remainder is 16. The remainder when the same number is divided by 12 is
(a) 3 (b) 4
(c) 6 (d) 8

27. The remainder when 3^{21} is divided by 5 is
(a) 1 (b) 2
(c) 3 (d) 4

28. A 4-digit number is formed by repeating a 2-digit number such as 1515, 3737, etc. Any number of this form is exactly divisible by
(a) 7 (b) 11
(c) 13 (d) 101

29. How many numbers less than 1000 are multiples of both 10 and 13?
(a) 9 (b) 8
(c) 6 (d) 7

30. What number should be divided by $\sqrt{0.25}$ to give the result as 25?
(a) 25 (b) 50
(c) 12.5 (d) 125

31. The smallest number that must be added to 803642 in order to obtain a multiple of 11 is
(a) 1 (b) 4
(c) 7 (d) 9

32. 1008 should be divided by which single digit number to get a perfect square?
(a) 9 (b) 4
(c) 8 (d) 7

33. $(1^2 + 2^2 + 3^2 + ... + 10^2)$ is equal to
(a) 380 (b) 385
(c) 390 (d) 392

34. Given that $1^2 + 2^2 + 3^2 + ... + n^2 = \frac{n}{6}(n+1)(2n+1)$, then, $10^2 + 11^2 + 12^2 + ... + 20^2$ is equal to
(a) 2616 (b) 2585
(c) 3747 (d) 2555

35. The largest natural number, which exactly divides the product of any four consecutive natural numbers, is
(a) 6 (b) 12
(c) 24 (d) 120

36. If $1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 = 441$. Then find the value of $2^3 + 4^3 + 6^3 + 8^3 + 10^3 + 12^3$
(a) 882 (b) 1323
(c) 1764 (d) 3528

37. The greatest fraction among $\frac{2}{3}, \frac{5}{6}, \frac{11}{15}$ and $\frac{7}{8}$ is
(a) $\frac{7}{8}$ (b) $\frac{11}{15}$
(c) $\frac{5}{6}$ (d) $\frac{2}{3}$

38. $0.4\overline{23}$ is equivalent to the fraction
(a) $\frac{491}{990}$ (b) $\frac{419}{990}$
(c) $\frac{49}{99}$ (d) $\frac{94}{99}$

39. 0.393939 ..... is equal to
(a) $\frac{39}{100}$ (b) $\frac{13}{33}$
(c) $\frac{93}{100}$ (d) $\frac{39}{990}$

40. If one-third of one-fourth of a number is 15, then three-tenth of the number is
(a) 35 (b) 36
(c) 45 (d) 54

Moderate

1. Which is the largest of the following fractions?

$\frac{2}{3}, \frac{3}{5}, \frac{8}{11}, \frac{7}{9}, \frac{11}{17}$

(a) $\frac{2}{3}$ (b) $\frac{11}{17}$

(c) $\frac{7}{9}$ (d) $\frac{3}{5}$

2. Which one of the group is in descending order?

(a) $\frac{7}{12}, \frac{9}{17}, \frac{13}{24}$ (b) $\frac{13}{24}, \frac{9}{17}, \frac{7}{12}$

(c) $\frac{9}{17}, \frac{13}{24}, \frac{7}{12}$ (d) $\frac{7}{12}, \frac{13}{24}, \frac{9}{17}$

3. $1\frac{1}{2} + 11\frac{1}{2} + 111\frac{1}{2} + 1111\frac{1}{2} + 11111\frac{1}{2} = ?$

(a) $12347\frac{1}{2}$ (b) $12346\frac{1}{2}$

(c) $12345\frac{1}{2}$ (d) $12344\frac{1}{2}$

4. $3\frac{1}{3} + 33\frac{1}{3} + 333\frac{1}{3} + 3333\frac{1}{3} + 33333\frac{1}{3} = ?$

(a) $37031\frac{2}{3}$ (b) $37037\frac{1}{3}$

(c) $37036\frac{2}{3}$ (d) $37032\frac{1}{3}$

5. $\frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \frac{1}{42} + \frac{1}{56} + \frac{1}{72} = ?$

(a) $\frac{5}{18}$ (b) $\frac{7}{18}$

(c) $\frac{11}{18}$ (d) $\frac{13}{18}$

6. $\frac{1}{5 \times 9} + \frac{1}{9 \times 13} + \frac{1}{13 \times 17} + ... \frac{1}{61 \times 65} = ?$

(a) $\frac{4}{45}$ (b) $\frac{3}{65}$

(c) $\frac{2}{35}$ (d) $\frac{3}{35}$

7. If $\frac{3}{2 + \frac{2}{2 + \frac{2}{2 + \frac{2}{3}}}} = x$, then find the value of x.

(a) 3.3 (b) 2.2

(c) 1.1 (d) 4.4

8. If $x + \frac{1}{2 + \frac{1}{3 + \frac{1}{4 + \frac{1}{5}}}} = 12$, then find the value of x.

(a) $\frac{1816}{157}$ (b) $\frac{2012}{153}$

(c) $\frac{1818}{151}$ (d) $\frac{1818}{157}$

9. Find the value of $5.\overline{12} + 3.\overline{21} + 4.\overline{31} = ?$

(a) $12\frac{64}{99}$ (b) $12\frac{74}{99}$

(c) $12\frac{77}{99}$ (d) $12\frac{84}{99}$

10. The difference of $5.\overline{76}$ and $2.\overline{3}$ is.

(a) $3.\overline{54}$ (b) $3.\overline{73}$

(c) $3.\overline{46}$ (d) $3.\overline{43}$

11. If x is a prime number and $-1 \le \frac{2x-7}{5} \le 1$, then the number of values of x is:

(a) 4 (b) 3

(c) 2 (d) 5

12. The sum of a natural number and its square equals the product of the first three prime numbers. The number is:

(a) 2 (b) 3

(c) 5 (d) 6

13. The rational number between $\frac{1}{2}$ and $\frac{3}{5}$ is:

(a) $\frac{2}{5}$ (b) $\frac{4}{7}$

(c) $\frac{2}{3}$ (d) $\frac{1}{3}$

14. What is the sum of two consecutive even numbers, the difference of whose square is 84?

(a) 38 (b) 34

(c) 42 (d) 46

15. The sum of all the natural numbers from 50 to 100 is:

(a) 5050 (b) 4275

(c) 4025 (d) 4005

16. The last digit of $(1001)^{2008} + 1002$ is:

(a) 0 (b) 3

(c) 4 (d) 6

17. The unit digit in the product $7^{71} \times 6^{63} \times 3^{65}$ is:
(a) 1 (b) 2
(c) 3 (d) 4

18. Unit's digit of the number $(22)^{23}$ is:
(a) 4 (b) 6
(c) 8 (d) 2

19. The digit in unit's place of the product $(2153)^{167}$ is:
(a) 1 (b) 3
(c) 7 (d) 9

20. If the sum of the digits of any integer lying between 100 and 1000 is subtracted from the number, the result always is:
(a) divisible by 2 (b) divisible by 9
(c) divisible by 5 (d) divisible by 6

21. In a division, the divisor is 10 times the quotient and 5 times the remainder. If the remainder is 46, then the dividend is:
(a) 4236 (b) 4306
(c) 4336 (d) 5336

22. If a and b are two odd positive integers, by which of the following integers is $(a^4 - b^4)$ always divisible.
(a) 3 (b) 6
(c) 8 (d) 12

23. A number, when divided by 136, leaves remainder 36. If the same number is divided by 17, the remainder will be:
(a) 9 (b) 7
(c) 3 (d) 2

24. A number, when divided by 899, leaves remainder 63. What will be the remainder if the same number is divided by 29 ?
(a) 3 (b) 1
(c) 5 (d) 0

25. The greatest fraction among $\frac{2}{3}, \frac{5}{6}, \frac{11}{15}$ and $\frac{7}{8}$ is:
(a) $\frac{7}{8}$ (b) $\frac{11}{15}$
(c) $\frac{5}{6}$ (d) $\frac{2}{3}$

26. If $(67^{67} + 67)$ is divided by 68. Then, the remainder is
(a) 1 (b) 67
(c) 63 (d) 66

27. $[2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2 + 9^2 + 10^2]$ is equal to
(a) 385 (b) 2916
(c) 540 (d) 384

28. If $1^3 + 2^3 + ... + 10^3 = 3025$. Then, 4 + 32 + 108 + ... + 4000 is equal to
(a) 12000 (b) 12100
(c) 12200 (d) 12400

29. Which of the following fractions is the smallest?
(a) $\frac{7}{6}$ (b) $\frac{7}{9}$
(c) $\frac{4}{5}$ (d) $\frac{5}{7}$

30. $0.\overline{001}$ is equal to
(a) $\frac{1}{1000}$ (b) $\frac{1}{999}$
(c) $\frac{1}{99}$ (d) $\frac{1}{9}$

Difficult

1. The sum of the squares of three consecutive natural numbers is 2030. Then, what is the middle number?
(a) 25 (b) 26
(c) 27 (d) 28

2. In a division, the divisor is 10 times the quotient and 5 times the remainder. If the remainder is 40, then the dividend is
(a) 240 (b) 440
(c) 4040 (d) 4000

3. If m and n are positive integers and (m – n) is an even number, then $(m^2 - n^2)$ will be always divisible by
(a) 4 (b) 6
(c) 8 (d) 12

4. Both the ends of a 99 digits number N are 2. N is divisible by 11, then all the middle digits are
(a) 1 (b) 2
(c) 3 (d) 4

5. The last 5 digits of the following expression will be
$(1\,!)^5 + (2\,!)^4 + (3\,!)^3 + (4\,!)^2 + (5\,!)^1 + (10\,!)^5 + (100\,!)^4 + (1000\,!)^3 + (10000\,!)^2 + (100000\,!)$
(a) 45939 (b) 00929
(c) 20929 (d) can't be determined

6. What fraction of $\frac{4}{7}$ must be added to itself to make the sum $1\frac{1}{14}$?
(a) $\frac{7}{8}$ (b) $\frac{1}{2}$
(c) $\frac{4}{7}$ (d) $\frac{15}{14}$

7. Find the sum of the first five terms of the following series $\frac{1}{1\times4}+\frac{1}{4\times7}+\frac{1}{7\times10}+...+\frac{1}{13\times16}$

(a) $\frac{9}{32}$ (b) $\frac{7}{16}$

(c) $\frac{5}{16}$ (d) $\frac{1}{210}$

8. The sum $(5^3 + 6^3 + ... + 10^3)$ is equal to
(a) 2295 (b) 2425
(c) 2495 (d) 2925

9. If $(10^{12} + 25)^2 - (10^{12} - 25)^2 = 10^n$, then the value of n is
(a) 20 (b) 14
(c) 10 (d) 5

10. The value of

$$\frac{3}{1^2\cdot 2^2}+\frac{5}{2^2\cdot 3^2}+\frac{7}{3^2\cdot 4^2}+\frac{9}{4^2\cdot 5^2}+\frac{11}{5^2\cdot 6^2}+\frac{13}{6^2\cdot 7^2}+\frac{15}{7^2\cdot 8^2}+\frac{17}{8^2\cdot 9^2}+\frac{19}{9^2\cdot 10^2}$$ is

(a) $\frac{1}{100}$ (b) $\frac{99}{100}$

(c) $\frac{101}{100}$ (d) 1

11. $\frac{1}{1+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+...+\frac{1}{\sqrt{99}+\sqrt{100}}$
is equal to
(a) 1 (b) 5
(c) 9 (d) 10

12. When simplified, the sum

$\frac{1}{2}+\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\frac{1}{30}+...+\frac{1}{n(n+1)}$ is equal to

(a) $\frac{1}{n}$ (b) $\frac{1}{n+1}$

(c) $\frac{2(n-1)}{n}$ (d) $\frac{n}{n+1}$

13. If $1^2 + 2^2 + 3^2 + ... + x^2 = \frac{x(x+1)(2x+1)}{6}$, then
$1^2 + 3^2 + 5^2 + ... + 19^2$ is equal to
(a) 1330 (b) 2100
(c) 2485 (d) 2500

14. $(1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + ... + 9^2 - 10^2)$ is equal to
(a) –55 (b) 55
(c) –56 (d) 56

15. The sum of the first 20 terms of the series

$\frac{1}{5\times 6}+\frac{1}{6\times 7}+\frac{1}{7\times 8}+...$ is

(a) 0.16 (b) 1.6
(c) 16 (d) 0.016

16. A divisor is 25 times the quotient and 5 times the remainder. The quotient is 16, the dividend is
(a) 6400 (b) 6480
(c) 400 (d) 480

17. Given that $3.718 = \frac{1}{0.2689}$. Then, $\frac{1}{0.0003718}$ is equal to
(a) 2689 (b) 2.689
(c) 26890 (d) 0.2689

18. Largest four digit number which when divided by 15 leaves a remainder of 12 and if the same number is divided by 8 it leaves the remainder 5. Such greatest possible number is:
(a) 9963 (b) 9957
(c) 9945 (d) 9999

19. Number of zeros at the end of the following expression $(5\,!)^{5!} + (10\,!)^{10!} + (50\,!)^{50!} + (100\,!)^{100!}$ is:
(a) 165 (b) 120
(c) 125 (d) None of these

20. A fraction in its lowest form is such that when it is squared and then its numerator is reduced by $\frac{1}{3}$rd and denominator is reduced to $\frac{1}{5}$th, it results as twice of the original fraction. Then the sum of numerator and denominator can be:
(a) 7 (b) 8
(c) 9 (d) 17

21. The value of the expression
$7777+7777\times 7777\times(5\div 77)\times(11\div 35)$:
(a) 1234321 (b) 12344321
(c) 7^{7777} (d) None of these

22. Find the last digit of $32^{32^{32}}$.
(a) 6 (b) 8
(c) 10 (d) 4

23. Find the last digit of $222^{888} + 888^{222}$.
(a) 8 (b) 4
(c) 0 (d) 6

24. Find the unit digit of 111! (fractorial 111).
(a) 0 (b) 2
(c) 3 (d) 4

25. Which is not the factor of $4^{6n} - 6^{4n}$ for any positive integer n?
(a) 5 (b) 25
(c) 7 (d) None of these

26. $19^n - 1$ is:
(a) always divisible by 9
(b) always divisible by 20
(c) is never divisible by 19
(d) only (a) and (c) are true

27. Find the remainder when $10^1 + 10^2 + 10^3 + 10^4 + 10^5 + ... + 10^{99}$ is divided by 6.
(a) 0 (b) 4
(c) 2 (d) 6

28. A number when divided by 5 gives a number which is 8 more than the remainder obtained on dividing the same number by 34. Such a least possible number is:
(a) 175 (b) 75
(c) 680 (d) does not exist

29. Total number of factors of a number is 24 and the sum of its 3 prime factors out of four, is 25. The product of all 4 prime factors of this number is 1365. Then such a greatest possible number can be :
(a) 17745 (b) 28561
(c) 4095 (d) can't be determined

30. How many numbers are there in the set S = {200, 201, 202, ..., 800} which are divisible by neither 5 nor 7?
(a) 411 (b) 412
(c) 410 (d) None of these

Previous Year (Memory Based)

1. I multiplied a natural number by 18 and another by 21 and added the products. Which one of the following could be the sum?
(a) 2007 (b) 2008
(c) 2006 (d) 2002

2. Out of six consecutive natural numbers, if the sum of first three is 27, what is the sum of the other three?
(a) 36 (b) 35
(c) 25 (d) 24

3. Which one of the following is a factor of the sum of first 25 natural numbers?
(a) 26 (b) 24
(c) 13 (d) 12

4. The sum of all the natural numbers from 51 to 100 is
(a) 5050 (b) 4275
(c) 4025 (d) 3775

5. The unit digit in the sum of $(124)^{376} + (124)^{375}$ is
(a) 5 (b) 4
(c) 2 (d) 0

6. The unit digit of the expression $25^{6527} + 36^{526} + 73^{54}$ is
(a) 6 (b) 5
(c) 4 (d) 0

7. The digit in the unit place of $[(251)^{98} + (21)^{29} - (106)^{100} + (705)^{35} - 16^4 + 259]$ is
(a) 1 (b) 4
(c) 5 (d) 6

8. If n is even, $(6^n - 1)$ is divisible by
(a) 37 (b) 35
(c) 30 (d) 6

9. 'a' divides 228 leaving a remainder 18. The biggest two digit value of 'a' is
(a) 21 (b) 35
(c) 30 (d) 70

10. $2^{16} - 1$ is divisible by
(a) 11 (b) 13
(c) 17 (d) 19

11. A certain number when divided by 175 leaves a remainder 132. When the same number is divided by 25, the remainder is
(a) 6 (b) 7
(c) 8 (d) 9

12. $(4^{61} + 4^{62} + 4^{63})$ is divisible by
(a) 3 (b) 11
(c) 13 (d) 17

13. The digit in the unit place of the product $(2464)^{1793} \times (615)^{317} \times (131)^{491}$ is
(a) 0 (b) 2
(c) 3 (d) 5

14. $(2^{71} + 2^{72} + 2^{73} + 2^{74})$ is divisible by
(a) 9 (b) 10
(c) 11 (d) 13

15. In a division problem, the divisor is 4 times the quotient and 3 times the remainder. If remainder is 4, the dividend is
(a) 36 (b) 40
(c) 12 (d) 30

16. If a number is divisible by both 11 and 13, then it must be necessarily
(a) divisible by (11 + 13)
(b) divisible by (13 – 11)
(c) divisible by (11 × 13)
(d) 429

17. A common factor of $(13^7 + 11^7)$ and $(13^5 + 11^5)$ is
(a) 24 (b) $13^5 + 11^5$
(c) $13^2 + 11^2$ (d) None of these

18. Sum of three consecutive even integers is 54. Find the least among them.

(a) 18 (b) 15
(c) 14 (d) 16

19. The unit digit in the product $(122)^{173}$ is
(a) 2 (b) 4
(c) 6 (d) 8

20. What least number of 5 digits is divisible by 41?
(a) 10045 (b) 10004
(c) 10041 (d) 41000

21. A number divided by 13 leaves a remainder 1 and if the quotient, is divided by 5, we get a remainder of 3. What will be the remainder if the number is divided by 65?
(a) 28 (b) 16
(c) 18 (d) 40

22. If p,q,r are in Geometric Progression, then which is true among the following?
(a) $q=\frac{p+r}{2}$ (b) $p^2=qr$
(c) $q=\sqrt{pr}$ (d) $\frac{p}{r}=\frac{r}{q}$

23. If $1 + 10 + 10^2 + ...$ upto n terms $=\frac{10^n-1}{9}$, then the sum of the series 4 + 44 + 444 + ... upto n terms is
(a) $\frac{4}{9}\left(10^n-1\right)-\frac{4n}{9}$ (b) $\frac{4}{81}\left(10^n-1\right)-\frac{4n}{9}$
(c) $\frac{40}{81}\left(10^n-1\right)-\frac{4n}{9}$ (d) $\frac{40}{9}\left(10^n-1\right)-\frac{4n}{9}$

24. The decimal fraction of $2.3\overline{49}$ is equal to
(a) $\frac{2326}{999}$ (b) $\frac{2326}{990}$
(c) $\frac{2347}{999}$ (d) $\frac{2347}{990}$

25. $(5^2 + 6^2 + 7^2 + ... + 10^2)$ is equal to
(a) 330 (b) 345
(c) 355 (d) 360

26. Two numbers are in the ratio 1 : 2 when 4 is added to each, the ratio becomes 2 : 3. Then, the numbers are
(a) 9 and 12 (b) 6 and 8
(c) 4 and 8 (d) 6 and 9

27. $[1^3 + 2^3 + 3^3 + ... + 9^3 + 10^3]$ is equal to
(a) 3575 (b) 2525
(c) 5075 (d) 3025

28. A number, when divided by 899, leaves remainder 63. What will be the remainder if the same number is divided by 29?
(a) 3 (b) 1
(c) 5 (d) 0

29. When 25^{25} is divided by 26, the remainder is
(a) 1 (b) 2
(c) 24 (d) 25

30. A number when divided by the sum of 555 and 445 gives two times their difference as quotient and 30 as the remainder. The number is
(a) 220030 (b) 22030
(c) 1220 (d) 1250

Foundation

Solutions

1. (b); The prime numbers Less than 31 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29
$\therefore$ required sum = 2 + 3 + 5 + 7 + 11 + 13 + 17 + 19 + 23 + 29 = 129

2. (b); Total even numbers from 1 to 50 = 25
Total even numbers from 1 to 20 = 10
Sum of even numbers = n(n + 1)
Required sum = sum of even numbers from 1 to 50 – sum of even numbers from 1 to 20
= 25(25 + 1) – 10(10 + 1)
= 25 × 26 – 10 × 11 = 540

3. (c); sum of first n natural numbers $=\frac{n(n+1)}{2}$
$\therefore$ sum of 1st 25 natural numbers
$=\frac{25\times(25+1)}{2}=25\times13$
i.e. 13 is one of the factor

4. (a); $(4)^{1793/4}\times5\times1$
4 × 5 × 1 = 20 So, unit digit is 0.

5. (a); 1 + 1 – 6 + 5 = 1

6. (c); If square of any prime number is divided by 24 then remainder is always 1.
so, $\frac{(1+1+1+1)}{24}=\frac{4}{24}$ i.e 4 is unit digit.

7. (c); (1 + 3 + 5 + ... + 97) – (2 + 4 + 6 + ... + 98)
$n_1=\frac{97+1}{2}=49$, $n_2=\frac{98}{2}=49$
sum $= n_1^2 - n_2(n_2 + 1) = 49^2 - 49 \times 50 = -49$

8. (a); $\frac{250}{2} = 125$, $\frac{125}{5} = 25$, $\frac{25}{5} = 5$, $\frac{5}{5} = 1$
i.e. required numbers of zero = 25 + 5 + 1 = 31

9. (b); 24

10. (c); 25

11. (d); Required remainder = $\frac{(21+28)}{33} = 16$

12. (d); Let quotient = x
divisor = 7x also divisor = 3 × (remainder)
= 3 × 28 = 84
7x = 84, x = 12
Dividend = Divisor × Quotient + Remainder
= 84 × 12 + 28 = 1036

13. (c); Since it is form of $\frac{a^n}{a+1}$
i.e. $\frac{17^{200}}{17+1}$
$\therefore$ Remainder = 1, Since n is even positive integer

14. (d); A number is exactly divisible by 18 if it is divisible by 2 and 9 both.
since, 65043 is not divisible by 2, so it is not divisible by 18.

15. (a); by checking option
$2^{96} + 1 = (2^{32})^3 + 1^3 = (2^{32} + 1)(2^{64} - 2^{32} + 1)$

16. (b); Decimal equivalent of fractions
$\frac{4}{9} = 0.44; \sqrt{\frac{9}{49}} = \frac{3}{7} = 0.43$
$(0.8)^2 = 0.64$
$\therefore$ Least number = $0.43 = \sqrt{\frac{9}{49}}$

17. (c); Expression = 0.121212 ...
$= 0.\overline{12} = \frac{12}{99} = \frac{4}{33}$
[Since, 12 is repeating after decimal]

18. (b); Decimal equivalent of fractions
$\frac{15}{16} = 0.94$, $\frac{19}{20} = 0.95$, $\frac{24}{25} = 0.96$, $\frac{34}{35} = 0.97$
$\therefore$ Least fraction = $\frac{15}{16}$

19. (b); Given, $1^3 + 2^3 + \ldots 9^3 = 2025$
Then, $(0.11)^3 + (0.22)^3 + \ldots + (0.99)^3$
$= \left(\frac{11}{100}\right)^3 + \left(\frac{22}{100}\right)^3 + \ldots + \left(\frac{99}{100}\right)^3$
$= \left(\frac{11}{100}\right)^3 \left(1^3 + 2^3 + \ldots + 9^3\right)$
$= \frac{1331}{1000000} \times 2025$
[$\because$ $1^3 + 2^3 + \ldots + 9^3 = 2025$]
$= \frac{2695275}{1000000} = 2.695275 \approx 2.695$

20. (b); Decimal equivalent of fractions
$0.9 = \frac{9}{10}$, $0.\overline{9} = \frac{9}{9} = 1$, $0.0\overline{9} = \frac{9}{90} = \frac{1}{10}$
and $0.\overline{09} = \frac{9}{99} = \frac{1}{11}$
$\therefore$ $0.\overline{9}$ is greatest.

21. (b); Natural numbers between 3 and 200
= 200 – 3 = 197
Now divide 197 by 7

```
    28
 7)197
   14
   ---
    57
    56
   ---
     1
```

So 28 natural numbers are there

22. (d); Let the consecutive odd no. are x, x + 2, x + 4
x + x + 2 + x + 4 = 87
3x + 6 = 87
x = $\frac{81}{3}$ = 27
so, smallest number is 27.

23. (b); 7^{105}
Cyclicity of 7 is 4.
So $\frac{105}{4}$ = Remainder is 1.
7^1 = Unit digit

24. (c); $5^{71} + 5^{72} + 5^{73}$
$5^{71}(1 + 5 + 5^2)$
$5^{71} \times 31$
$5^{70} \times 155$
so 155 divides the expression completly

25. (a); We know that $2^1 = 2, 2^2 = 4, 2^3 = 8, 2^4 = 16$
Remainder $= \frac{33}{4} = 1$.
Unit's digit in 2^{33} = unit digit in 2^1
Hence units digit = 2
Remainder on division by 10 = 2.

26. (b); Remainder = 16
Divisor = 24
Let number = x
x = 24y + 16 where y is quotient.
Since 24 is a multiple of 12

Remainder = $\frac{16}{12} = 4$

27. (c); $\frac{3^{21}}{5}$

$\frac{(3^4)^5 \times 3}{5} = \frac{(81)^5 \times 3}{5}$

$= \frac{1^5 \times 3}{5}$

so, remainder = 3

28. (d); Let the two digit number be xy
xy xy = xy × 100 + xy
= xy (100 + 1) = 101 xy

29. (d); Numbers which are multiple of both 10, 13 will be multiple of 130 also
Numbers less then 1000 which are multiple of both 10 and 13

$= \frac{1000}{130} = 7$

30. (c); $\frac{x}{\sqrt{0.25}} = 25$

x = 25 × (0.5) = 12.5

31. (c); Required number = (Sum of digits at odd places) – Sum of digits at even place)
= (2 + 6 + 0) – (8 + 3 + 4) = – 7
smallest number to be added = 7

32. (d); Factor of 1008
= 2 × 2 × 2× 2 × 3 × 3 × 7
so number is divided by 7 to make it perfect square.

33. (b); $1^2 + 2^3 + 3^2 ...... + 10^2$

$= \frac{n(n+1)(2n+1)}{6} = \frac{10 \times 11 \times 21}{6} = 385$

34. (b); Sum of squares from 1 to 20 – Sum of squares from 1 to 9

$= \frac{20 \times 21 \times 41}{6} - \frac{9 \times 10 \times 19}{6} = 2870 - 285 = 2585$

35. (c); Let four consecutive natural numbers are
1, 2, 3, 4
1 × 2 × 3 × 4 = 24
So 24 is a natural number which divides four consecutive natural number completely

36. (d); Given, $1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 = 441$
$2^3 + 4^3 + 6^3 + 8^3 + 10^3 + 12^3$
$= 2^3(1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3)$
$= 2^3 \times 441 = 3528$

37. (a); $\frac{2}{3}, \frac{5}{6}, \frac{11}{15}$ and $\frac{7}{8}$

Using cross multiplication method.

$\frac{2}{3} \times \frac{5}{6} = 12 < 15$

So, $\frac{5}{6} > \frac{2}{3}$

$\frac{5}{6} \times \frac{11}{15} = 75 > 66$

So , $\frac{5}{6}$ is greater than $\frac{11}{15}$

$\frac{5}{6} \times \frac{7}{8} = 40 < 42$

So $\frac{7}{8}$ is the greatest fraction.

38. (b); $0.4\overline{23} = \frac{423 - 4}{990} = \frac{419}{990}$

39. (b); 0.393939 ....

$= 0.\overline{39} = \frac{39}{99} = \frac{13}{33}$

40. (d); Let number = y.
According to question

$\frac{1}{3} \times \frac{1}{4} y = 15$, y = 180

so, $\frac{3}{10} y = \frac{3}{10} \times 180 = 54$

Moderate

1. (c); $\frac{2}{3}\times\frac{3}{5}$, 10 > 9. Taking greater of these two fractions and the next one: $\frac{2}{3}\times\frac{8}{11}$, 22 < 24. Taking greater of these two fractions and the next one: $\frac{8}{11}\times\frac{7}{9}$, 72 < 77. Taking greater of these two fractions and the next one: $\frac{7}{9}\times\frac{11}{17}$, 119 > 99. $\frac{7}{9}$ is the largest

2. (d); $\frac{7}{12}\times\frac{13}{24}$ 168 > 156; $\frac{13}{24}\times\frac{9}{17}$ 221 > 216

$\frac{7}{12} > \frac{13}{24}$ and $\frac{13}{24} > \frac{9}{17}$

Hence descending order = $\frac{7}{12} > \frac{13}{24} > \frac{9}{17}$

3. (a); $1\frac{1}{2}+11\frac{1}{2}+111\frac{1}{2}+1111\frac{1}{2}+11111\frac{1}{2}$

= [1 + 11 + 111 + 1111 + 11111] + $\left[\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}\right]$

$= 12345 + 2\frac{1}{2} = 12347\frac{1}{2}$

4. (c); $3\frac{1}{3}+33\frac{1}{3}+333\frac{1}{3}+3333\frac{1}{3}+33333\frac{1}{3}$

= [3 + 33 + 333 + 3333 + 33333] + $\left[\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}\right]$

$= 37035 + 1\frac{2}{3} = 37036\frac{2}{3}$

5. (b); $\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\frac{1}{30}+\frac{1}{42}+\frac{1}{56}+\frac{1}{72}$

$= \frac{1}{2\times3}+\frac{1}{3\times4}+.....+\frac{1}{8\times9}$

$= \frac{1}{2}-\frac{1}{3}+\frac{1}{3}-\frac{1}{4}+.....+\frac{1}{8}-\frac{1}{9}$

$= \frac{1}{1}\left[\frac{1}{2}-\frac{1}{9}\right] = \frac{7}{18}$

6. (b); $\frac{1}{5.9}+\frac{1}{9.13}+\frac{1}{13.17}+.........\frac{1}{61.65} = ?$

Using formula:

$\frac{+1}{\text{Difference of denominator value}}\left[\frac{1}{\text{First value}}-\frac{1}{\text{Last value}}\right]$

$= \frac{1}{4}\left[\frac{1}{5}-\frac{1}{65}\right] = \frac{1}{4}\left[\frac{13-1}{65}\right] = \frac{1}{4}\left[\frac{12}{65}\right] = \frac{3}{65}$

7. (c); $x = \cfrac{3}{2+\cfrac{2}{2+\cfrac{2}{\left.2+\frac{2}{3}\right\}}}} = \frac{8}{3}$

$= \cfrac{3}{2+\cfrac{2}{\left.2+\cfrac{2}{\frac{8}{3}}\right\}}} = 2+\frac{2}{1}\times\frac{3}{8} = 2+\frac{3}{4} = \frac{11}{4}$

$= \cfrac{3}{2+\cfrac{2}{\frac{11}{4}}} = \cfrac{3}{2+\frac{2}{1}\times\frac{4}{11}}$

$= \cfrac{3}{\left.2+\frac{2}{1}\times\frac{4}{11}\right\}} = 2+\frac{8}{11} = \frac{30}{11}$

$= \cfrac{3}{\frac{30}{11}} = \frac{3}{1}\times\frac{11}{30} = \frac{11}{10} = 1.1$

8. (a); $x+\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{4+\frac{1}{5}}}} = 12$

$12 = x+\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{\frac{21}{5}}}} = x+\cfrac{1}{2+\cfrac{1}{3+\frac{5}{21}}}$

$$12 = x + \frac{1}{2 + \frac{1}{\frac{68}{21}}} = x + \frac{1}{\frac{157}{68}} = x + \frac{68}{157}$$

$$x = 12 - \frac{68}{157}$$

157x = 1884 – 68 = 1816

$$x = \frac{1816}{157}$$

9. (a); $5.\overline{12} + 3.\overline{21} + 4.\overline{31} = 5\frac{12}{99} + 3\frac{21}{99} + 4\frac{31}{99}$

$$= (5+3+4) + \frac{64}{99} = 12\frac{64}{99}$$

10. (d); $5.\overline{76} - 2.\overline{3} = 5\frac{76}{99} - 2\frac{3}{9} = 3\frac{43}{99} = 3.\overline{43}$

11. (b); Given, $-1 \le \frac{2x-7}{5} \le 1$

$\Rightarrow\ -5 \le 2x - 7 \le 5$ ... (i)

$\Rightarrow\ -5 + 7 \le 2x - 7 + 7 \le 5 + 7$

[by adding 7 in eq. (i)]

$\Rightarrow\ 2 \le 2x \le 12$

$\Rightarrow\ 1 \le x \le 6$

So, number of values of x = 3 (2, 3 and 5)

12. (c); Let the required number be x.

According to the question,

$x^2 + x = 2 \times 3 \times 5$

$\Rightarrow\ x^2 + x - 30 = 0$

$\Rightarrow\ x^2 + 6x - 5x - 30 = 0$

$\Rightarrow\ x(x + 6) - 5(x + 6) = 0$

$\Rightarrow\ (x - 5)(x + 6) = 0$

$\therefore\ x = 5$

13. (b); Required number between $\frac{1}{2}$ and $\frac{3}{5}$

$$\Rightarrow \frac{\frac{1}{2} + \frac{3}{5}}{2}$$

$$\Rightarrow \frac{5+6}{20} = \frac{11}{20} \approx \frac{4}{7}$$

14. (c); Let two consecutive even numbers are x and (x+2).

$\therefore$ According to the question,

$(x+2)^2 - x^2 = 84$

$\Rightarrow\ x^2 + 4x + 4 - x^2 = 84$

$\Rightarrow\ 4x = 84 - 4 = 80$

$\Rightarrow\ x = \frac{80}{4} = 20$

Two numbers are 20 and 22.

$\therefore$ The required sum = 20 + 22 = 42

15. (d); Required sum = (sum of natural numbers from 1 to 100) – (sum of natural numbers from 1 to 49.)

Sum of [1 + 2 + 3 + 4 + ... + 100]

$$= \frac{n(n+1)}{2} = \frac{100(101)}{2} = 5050$$

and sum of [1 + 2 + 3 + 4 ..... + 49]

$$= \frac{n(n+1)}{2} = \frac{50(49)}{2} = 1045$$

Hence, sum of [50 + 51 + 52 + 53 + ... + 100]

= 5050 – 1045 = 4005

16. (b); Given, $(1001)^{2008} + 1002$

Unit digit of $(1001)^{2008} = 1$

Last digit of 1002 = 2

$\therefore$ The last digit = 1 + 2 = 3

17. (d); Given, $7^{71} \times 6^{63} \times 3^{65}$

Then, $7^1 = 7$, $7^2 = 49$, $7^3 = 343$, $7^4 = 2401$

$\therefore$ Unit digit of $(7)^{71} = 3$

$3^1 = 3$, $3^2 = 9$, $3^3 = 27$, $3^4 = 81$

Unit digit of $(3)^{65} = 3$

Unit digit of $(6)^{63} = 6$

$\therefore$ Product = 3 × 6 × 3 = 54

$\therefore$ Unit digit = 4

18. (c); $2^1 = 2$, $2^2 = 4$, $2^3 = 8$, $2^4 = 16$, $2^5 = 32$

Unit digit repeats itself after 4 powers.

Remainder of $\frac{23}{4} = 3$

$\therefore\ (22)^{23} = (22)^3 = 2^3 = 8$

Unit digit = 8.

19. (c); Given, $(2153)^{167}$

Then, remainder of $\frac{167}{4} = 3$

$\therefore$ Unit digit of 3^3 (i.e., 27) = 7

20. (b); Such number is always divisible by 9. To make it clear, you can take some example.

Example:

496 – (4 + 9 + 6) = 477,

which is divisible by 9.

971 – (9 + 7 + 1) = 954,

which is divisible by 9.

21. (d); According to the question,
Divisor = 5 × Remainder
= 5 × 46 = 230

Quotient = $\frac{230}{10} = 23$

Dividend = Divisor × Quotient + Remainder
Dividend = 230 × 23 + 46 = 5290 + 49 = 5336
∴ Dividend = 5336

22. (c); Given, a and b are odd positive integers.
$a^4 - b^4 = (a^2 + b^2)(a + b)(a - b)$
Let two positive odd integers be 1 and 3.
∴ Required number
$= (1^2 + 3^2)(3 + 1)(3 - 1) = 80$
Which is divisible by 8.

23. (d); Required remainder = $\frac{\text{Last remainder}}{\text{New divisor}}$

Required remainder = $\frac{36}{17} = 2\frac{2}{17}$

∴ Remainder = 2

24. (c); Remainder = $\frac{\text{Last remainder}}{\text{New divisor}}$

Remainder = $\frac{63}{29} = 2\frac{5}{29} = 5$

25. (a); Decimal equivalent of fractions

$\frac{2}{3} = 0.67$; $\frac{5}{6} = 0.83$

$\frac{11}{15} = 0.73$; $\frac{7}{8} = 0.875$

∴ Greatest fractions is 7/8.

26. (d); $67^{67} = (68 - 1)^{67}$ when divided by 68, leaves remainder $(-1)^{67} = -1$
∴ Required remainder = – 1 + 67 = 66

27. (d); We know that,
Sum of squares of 1st n natural numbers

$= \frac{n(n+1)(2n+1)}{6}$

Required sum = (Sum of squares of natural numbers from 1 to 10) – 1^2

$= \frac{10(10+1)(2\times10+1)}{6} - 1^2 = \frac{10\times11\times21}{6} - 1$

= 385 – 1 = 384

28. (b); Here, $1^3 + 2^3 + ... + 10^3 = 3025$
Now, 4 + 32 + 108 + ... + 40000
= 4(1 + 8 + 27 + ... + 1000)
$= 4(1^3 + 2^3 + 3^3 + ... + 10^3)$
= 4 × 3025 = 12100

29. (d); To find the smallest fraction first we have to find the decimal equivalent of fractions

$\frac{7}{6} = 1.166$, $\frac{7}{9} = 0.777$, $\frac{4}{5} = 0.8$ and $\frac{5}{7} = 0.714$

Therefore, the smallest number is $\frac{5}{7}$.

30. (b); $0.\overline{001} = \frac{1}{999}$

Difficult

1. (b); Let the three consecutive natural numbers be x, x + 1 and x + 2.
According to the question,
$x^2 + (x + 1)^2 + (x + 2)^2 = 2030$
$\Rightarrow x^2 + x^2 + 2x + 1 + x^2 + 4x + 4 = 2030$
$\Rightarrow 3x^2 + 6x + 5 = 2030$
$\Rightarrow 3x^2 + 6x - 2025 = 0$
$\Rightarrow x^2 + 2x - 675 = 0$
$\Rightarrow x^2 + 27x - 25x - 675 = 0$
$\Rightarrow x(x + 27) - 25(x + 27) = 0$
$\Rightarrow (x - 25)(x + 27) = 0$
$\Rightarrow x = 25$ or -27
∴ Three consecutive natural numbers are 25, 26 and 27
Now, required number = 26

2. (c); Let quotient = x
Then, divisor = 10x
and remainder = $\frac{\text{Divisor}}{5} = 2x$
According to the question,
Remainder = 40
$\Rightarrow$ 2x = 40
∴ x = 20
Now, Dividend = Divisor × Quotient + Remainder
$= x \times 10x + 40 = 10x^2 + 40 = 4000 + 40 = 4040$

3. (a); Given, m and n are positive integers and m-n is an even number.
Let, m – n = 2p ... (i)
where, 2p is the even difference
So, it is clear that both m and n may be either odd or even
So, m + n = 2q ... (ii)

where, 2q is the even sum of the numbers. Then, on multiplying Eqs. (i) and (ii), we get

(m – n) (m + n) = 2p × 2q

$\Rightarrow$ $m^2 - n^2 = 4pq$

$\therefore$ $m^2 - n^2$ will be divisible by 4.

4. (d); Since, the middle digits are given to be same.

$\therefore$ Let the 99 digits numbers be

$$2\underbrace{x\,x\ldots x\,x}_{97\text{ digits}}2$$

Sum of digits at odd places

$$= 2+\underbrace{x+x+\ldots x+x}_{48\text{ digits}}+2 = 4 + 48x$$

(there are 99 digits in all, 50 at odd places and 49 at even places)

Sum of digits at even places

= x + x + ... + 49 terms = 49x

Difference between the sum of digits at odd and even places

= 4 + 48x – 49x = 4 – x

Now, 4 – x = 0 or a multiple of 11

4 – x = 0 $\Rightarrow$ x = 4

5. (b);

$(1\,!)^5 = 1$

$(2\,!)^4 = 16$

$(3\,!)^3 = 216$

$(4\,!)^2 = 576$

$(5\,!)^1 = 120$

The last 5 digit of $(10\,!)^5$ = 00000

The last 5 digit of $(100\,!)^4$ = 00000

$(1000\,!)^3$ = 00000

$(10000\,!)^2$ = 00000

$(100000\,!)^1$ = 00000

Thus the last 5 digits of the given expression

= 00929

[$\because$ 1 + 16 + 216 + 576 + 120 + 00000 + 00000 + 00000 + 00000 + 00000 = 00929]

6. (a); Let the fraction be x.

According to the question,

$$\frac{4x}{7}+\frac{4}{7}=\frac{15}{14} \Rightarrow \frac{4x}{7}=\frac{15}{14}-\frac{4}{7}$$

$$=\frac{15-8}{14}=\frac{7}{14}=\frac{1}{2} \Rightarrow x=\frac{1}{2}\times\frac{7}{4}=\frac{7}{8}$$

$\therefore$ $\frac{7}{8}$ must be added.

7. (c); $\frac{1}{1\times4}+\frac{1}{4\times7}+\frac{1}{7\times10}+\frac{1}{10\times13}+\frac{1}{13\times16}$

$$=\left(1-\frac{1}{4}+\frac{1}{4}-\frac{1}{7}+\frac{1}{7}-\frac{1}{10}+\frac{1}{10}-\frac{1}{13}+\frac{1}{13}-\frac{1}{16}\right)\times\frac{1}{3}$$

$$=\left(1-\frac{1}{16}\right)\times\frac{1}{3}=\frac{15}{16}\times\frac{1}{3}=\frac{5}{16}$$

8. (d); Required sum = [Sum of cubes of 1 to 10 natural numbers] – [Sum of cubes of natural numbers from 1 to 4]

$$=\left[\frac{10\times(10+1)}{2}\right]^2-\left[\frac{4(4+1)}{2}\right]^2$$

$$=\left[\frac{10\times11}{2}\right]^2-\left[\frac{4\times5}{2}\right]^2$$

= 3025 – 100 = 2925

9. (b); Given,

$(10^{12} + 25)^2 - (10^{12} - 25)^2 = 10^n$ ... (i)

$(a + b)^2 - (a - b)^2 = 4ab$

$\therefore$ $(10^{12} + 25)^2 - (10^{12} - 25)^2$

$= 4 \times 10^{12} \times 25$ ... (ii)

On comparing Eqs. (i) and (ii), we get

$10^n = 4 \times 10^{12} \times 25 = 10^{14}$

i.e., $10^n = 10^{14}$

$\therefore$ n = 14

10. (b); Expression

$$=\frac{3}{1^2\cdot2^2}+\frac{5}{2^2\cdot3^2}+\frac{7}{3^2\cdot4^2}+\ldots+\frac{17}{8^2\cdot9^2}+\frac{19}{9^2\cdot10^2}$$

On arranging

$$=\left(\frac{1}{1^2}-\frac{1}{2^2}\right)+\left(\frac{1}{2^2}-\frac{1}{3^2}\right)+\left(\frac{1}{3^2}-\frac{1}{4^2}\right)+\ldots$$

$$+\left(\frac{1}{8^2}-\frac{1}{9^2}\right)+\left(\frac{1}{9^2}-\frac{1}{10^2}\right)$$

$$=\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{2^2}-\frac{1}{3^2}+\frac{1}{3^2}-\frac{1}{4^2}+\ldots+\frac{1}{8^2}-\frac{1}{9^2}+\frac{1}{9^2}-\frac{1}{10^2}$$

$$=1-\frac{1}{10^2}=1-\frac{1}{100}=\frac{100-1}{100}=\frac{99}{100}$$

11. (c); Let

$$\frac{1}{1+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\ldots+\frac{1}{\sqrt{99}+\sqrt{100}}$$

$$\frac{}{1+\sqrt{2}}=\frac{}{\sqrt{2}+1}\times\frac{\sqrt{2}-}{\sqrt{2}-1}=\sqrt{2}-1$$

$$\frac{1}{\sqrt{2}+\sqrt{3}}=\frac{\sqrt{3}-\sqrt{2}}{\left(\sqrt{3}-\sqrt{2}\right)\left(\sqrt{3}-\sqrt{2}\right)}=\sqrt{3}-\sqrt{2}$$

$\therefore$ Given expression = $\sqrt{2}-1+\sqrt{3}-\sqrt{2}$

$+\sqrt{4}-\sqrt{3}+...+\sqrt{100}-\sqrt{99}$

$=\sqrt{100}-1=10-1=9$

12. (d); Expression

$$=\frac{1}{2}+\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\frac{1}{30}+...+\frac{1}{n(n+1)}$$

$$=\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+\frac{1}{4\times5}+...+\frac{1}{n(n+1)}$$

$$=\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\left(\frac{1}{4}-\frac{1}{5}\right)+...$$

$$+\frac{1}{n}-\frac{1}{n+1}$$

$$=1-\frac{1}{n+1}=\frac{n}{n+1}$$

13. (a); Sum of squares of n terms $=\frac{n(n+1)(2n+1)}{6}$

Required sum = (sum of squares of natural numbers from 1 to 20) – 2^2 × (sum of squares of natural numbers from 1 to 10)

$$=\frac{20(20+1)(40+1)}{6}-\frac{2^2\times(10)(10+1)\times(20+1)}{6}$$

$$=\frac{20\times21\times41}{6}-\frac{4\times10\times11\times21}{6}$$

= 2870 – 1540 = 1330

14. (a); Taking in pairs.

[$\because$ $(a^2-b^2)=(a-b)(a+b)$]

$(1^2-2^2)+(3^2-4^2)+...+(9^2-10^2)$

$=(1+2)(1-2)+(3+4)(3-4)+...$

$+(9+10)(9-10)$

$=-3-7-11-15-19=-55$

15. (a); First term = $\frac{1}{5\times6}=\frac{1}{5}-\frac{1}{6}$

Second term = $\frac{1}{6\times7}=\frac{1}{6}-\frac{1}{7}$

20th term of series $=\frac{1}{24\times25}=\frac{1}{24}-\frac{1}{25}$

$\therefore$ Required sum

$$=\left(\frac{1}{5}-\frac{1}{6}\right)+\left(\frac{1}{6}-\frac{1}{7}\right)+...+\left(\frac{1}{24}-\frac{1}{25}\right)$$

$$=\frac{1}{5}-\frac{1}{25}=\frac{5-1}{25}=\frac{4}{25}=0.16$$

16. (b); According to the question,

Divisor = 25 × Quotient

Divisor = 25 × 16 = 400

Also, divisor = 5 × Remainder

$\therefore$ Remainder = $\frac{400}{5}=80$

$\therefore$ Dividend = divisor × Quotient + Remainder

= 16 × 400 + 80 = 6400 + 80 = 6480

17. (a); Given, $3.718=\frac{1}{0.2689}$

Then, $\frac{1}{0.0003718}=0.2689\times10000=2689$

18. (b); Let the smallest possible number be x, then

x = 15k + 12 and x = 8l + 5

$\Rightarrow$ 15k + 12 = 8l + 5

$\Rightarrow$ 15k + 7 = 8l

$\Rightarrow$ $l=\frac{15k+7}{8}$,

l must be an integer putting k = 1, 2, 3, ... etc.

But at k = 7, we get a number which on being divided by 8, gives 'l' as an integer.

So, x = 15 × 7 + 12, x = 117

The next higher numbers

= (L.C.M. of divisors) m + 117

= (L.C.M. of 15 and 8) m + 117 = 120m + 117

So consider the highest possible value of m such that 120m + 117 $\leq$ 9999 (largest possible number of four digit)

Thus at m = 82, the value of 120m + 117 = 9957, which is the required number.

19. (b); The number of zeros at the end of $(5!)^{5!}$ = 120

[$\because$ 5! = 120 and thus $(120)^{120}$ will give 120 zeros]

and the number of zeros at the end of the $(10!)^{10!}$, $(50!)^{50!}$ and $(100!)^{100!}$ will be greater than 120.

Now since the number of zeros at the end of the whole expression will depend on the number which has least number of zeros at the end of the number among other given numbers.

So, the number of zeros at the end of the given expression is 120.

20. (b); Let the fraction be $\frac{x}{y}$, then

$$\left(\frac{x}{y}\right)^2=\frac{x^2}{y^2}$$

then $\frac{\frac{2}{3}x^2}{\frac{1}{5}y^2} = \frac{10x^2}{3y^2}$

Thus $\frac{10x^2}{3y^2} = 2\frac{x}{y} \Rightarrow \frac{x}{y} = \frac{3}{5}$

Hence $x + y = 3 + 5 = 8$

21. (d); 7777 + 7777 × 7777 × (5 ÷ 77) × (11 ÷ 35)

$= 7777 + 7777 \times 7777 \times \frac{5}{77} \times \frac{11}{35}$

= 7777 + 1111 × 1111 = 7777 + 1234321 = 1242098

22. (a); The last digit of $32^{32^{32}}$ is same as $2^{32^{32}}$

But $2^{32^{32}} = 2^{32\times32\times32\times\ldots\times32 \text{ times}}$

$\Rightarrow 2^{32^{32}} = 2^{4\times8\times(32\times32\times\ldots\times31 \text{ times})}$

$\Rightarrow 2^{32^{32}} = 2^{4n}$,

where n = 8 × (32 × 32 × ... × 31 times)

Again $2^{4n} = (16)^n \Rightarrow$ unit digit is 6, for every $n \in N$

Hence, the required unit digit = 6.

23. (c); The last digit of the expression will be same as the last digit of $2^{888} + 8^{222}$.

Now the last digit of 2^{888} is 6 and the last digit of the 8^{222} is 4.

Thus the last digit of $2^{888} + 8^{222}$ is 0 (zero), since 6 + 4 = 10.

24. (a); 111! = 1 × 2 × 3 × 4 × 5 × ... × 100 × 111

Since there is a product of 5 and 2 hence it will give zero as the unit digit.

Hence the unit digit of 111! is 0 (zero).

25. (d); $4^{6n} - 6^{4n} = (64)^{2n} - (36)^{2n} = (64^n + 36^n)(64^n - 36^n)$

For n = 1, 3, 5, ... etc. $(64^n + 36^n)$ is divisible by 100 and all its factors. Also $(64^n - 36^n)$ is divisible by 28 and all its factors.

Again for n = 2, 4, 6, ... etc. $(64^n - 36^n)$ is always divisible by 100 and all its factors. Also it is divisible by 28 and all its factors.

26. (d); $19^n - 1$ is divisible by 18 = (19 – 1) when n is even or odd. So (a) is correct.

$19^n - 1$ is divisible by 20 only when n is even so (b) is wrong.

$19^n - 1$ is never divisible by 19 which is correct. Thus (d) is the most appropriate answer.

27. (a); The remainder when 10^1 is divided by 6 is 4

The remainder when 10^2 is divided by 6 is 4

The remainder when 10^3 is divided by 6 is 4

The remainder when 10^4 is divided by 6 is 4

The remainder when 10^5 is divided by 6 is 4

Thus the remainder is always 4.

So, the required remainder

$= \frac{4+4+4+\ldots 99 \text{ times}}{6} = \frac{396}{6}$

Thus the remainder is zero.

28. (b); Let the number be N then

N = 34Q + R, ... (i)

where Q is any quotient

Again N = 5D and D is also a quotient

but D = R + 8

so N = 5 (R + 8) ... (ii)

$\therefore$ 5 (R + 8) = 34Q + R

5R + 40 = 34Q + R

$\Rightarrow$ 34Q – 40 = 4R

$\Rightarrow$ 17Q – 2R = 20

So the minimum possible value of Q = 2 and the corresponding value of R = 7

So N = 34 × 2 + 7

N = 75

Hence (b) is correct Choice.

29. (a); Since the product of 4 prime factors = 1365

= 3 × 5 × 7 × 13

and the sum of the 3 prime factors

= 25 = (5 + 7 + 13)

Now, total number of factors of the required number N = 24

$= 2^3 \times 3 \Rightarrow (1 + 1)(1 + 1)(1 + 1)(2 + 1)$

Let N can be expressed as $N = 3^p \times 5^q \times 7^r \times 13^s$

Thus, for N to be greatest possible number in the above expressed form, the power of the greatest prime factors will be greater.

So, $N = 3 \times 5 \times 7 \times 13^2$

= 105 × 169 = 17745

30. (a); Total numbers in the set = (800 – 200) + 1 = 601

Number of numbers which are divisible by 5

$= \frac{(800-200)}{5} + 1 = 121$

Number of numbers which are divisible by 7

$= \frac{(798-203)}{7} + 1 = 86$

Number of numbers which are divisible by both 5 and 7

$= \left(\frac{770-210}{35}\right) + 1 = 17$

$\therefore$ Number of numbers which are either divisible by 5 or 7 or both

= (121 + 86) – 17 = 190

Thus the number of numbers in the given set which are neither divisible by 5 nor by 7

= 601 – 190 = 411.

Hence (a) is correct option.

Previous Year (Memory Based)

1. (a); Let the natural numbers be x and y.
$\therefore$ Required sum = 18x + 21y = 3 (6x + 7y)
Hence, the sum is divisible by 3.
Out of the given options, only 2007 is completely divisible by 3.

2. (a); Let first three consecutive natural numbers be x, x + 1 and x + 2.
According to the question,
x + x + 1 + x + 2 =27
$\Rightarrow$ 3x + 3 = 27 $\Rightarrow$ x = 8
First three consecutive numbers 8, 9 and 10.
$\therefore$ Sum of next 3 consecutive numbers
= 11 + 12 + 13 = 36

3. (c); Sum of first n natural number = $\frac{n(n+1)}{2}$
Sum of first 25 natural numbers.
$\therefore$ 1 + 2 + 3 + ... + 25
$= \frac{25(25+1)}{2} = 25 \times 13$
$\therefore$ The factor 13 is one of numbers

4. (d); Required sum = (sum of natural numbers from 1 to 100) – (sum of natural numbers from 1 to 50.)
sum of [1 + 2 + 3 + 4 + ... + 100]
$= \frac{n(n+1)}{2} = \frac{100(101)}{2} = 5050$
and sum of [1 + 2 + 3 + 4 + ... + 50]
$= \frac{n(n+1)}{2} = \frac{50(51)}{2} = 1275$
Hence, sum of [51 + 52 + 53 + ... + 100]
= 5050 – 1275 = 3775

5. (d); Given, $(124)^{376} + (124)^{375}$
$4^1 = 4$; $4^2 = 16$; $4^3 = 64$
Remainder on dividing 376 by 4 = 0
$\therefore$ Unit digit of $(124)^{376} = 6$
Remainder on dividing 375 by 4 = 3
$\therefore$ Unit digit of $(124)^{375} = 4$
$\therefore$ Sum = 6 + 4 = 10
Hence, unit digit = 0.

6. (d); Unit digit in the expansion of $5^{6527} = 5$
(5 repeats for every power increased)
36^{526} = Unit digit in $6^{526} = 6$
(6 repeats for every power increased)
Now, $3^1 = 3, 3^2 = 9, 3^3 = 27 ; 3^4$
$= 81, 3^5 = 243$ ...
Now, remainder of $\frac{54}{4} = 2$
$\therefore$ Unit digit = $(3)^2 = 9$
Required digit = Sum of (9 + 5 + 6) = 20
Hence, required unit digit = 0

7. (b); Unit digit of given numbers
$(251)^{98} = ...1$
$(21)^{29} = ...1$
$(106)^{100} = ...6$
$(705)^{35} = ...5$
$(16)^4 = ...6$
259 = ... 9
$\therefore$ Required answer = 1 + 1 – 6 + 5 – 6 + 9 = 4
Hence, unit digit = 4

8. (b); We have, $(6^n - 1)$
If n is even, then taking n = 2,
$6^n - 1 = 6^2 - 1 = 36 - 1 = 35$
Here, number 35 is divisible by 35.
Hence, for any even value of n, $(6^n - 1)$ is divisible by 35.

9. (d); Given, divisor = a
and remainder = 18
We know that,
Dividend = (Divisor × Quotient) + Remainder
Through opotins
228 = (70 × 3) + 18
Hence, biggest two digit value = 70

10. (c); Expression = $2^{16} - 1 = (2^8)^2 - 1$
$= (2^8 + 1)(2^8 - 1) = (256 + 1)(256 - 1)$
= 257 × 255
which is exactly divisible by 17.
(since, 17 × 15 = 255)

11. (b); Dividend = (Divisor × Quotient) + Remainder
= 175 × q + 132 = 25 × 7 × q + 25 × 5 + 7
= 25(7q + 5) + 7
It is clear that when the number is divided by 25, remainder will be 7.

12. (a); Given, $4^{61} + 4^{62} + 4^{63} = 4^{61}(1 + 4 + 4^2) = 4^{61} \times 21$
Here, 21 is divisible by 3.

13. (a); Given, $(2464)^{1793} \times (615)^{317} \times (131)^{491}$
Then, unit digit of $(2464)^{1793}$
Remainder of $\frac{1793}{4} = 1$
Hence, unit digit of $(2464)^{1793} = 4$
Unit digit of $(615)^{317} = 5$
Unit digit of $(131)^{491} = 1$
so, unit digit = 4 × 5 × 1 = 20 = 0

14. (b); Expression = $(2^{71} + 2^{72} + 2^{73} + 2^{74})$
$= 2^{71}(1 + 2 + 4 + 8) = 2^{71} \times 15 = 2^{71} \times 3 \times 5$
Which is exactly divisible by 10.

15. (b); Given, remainder = 4
According to the question,
Divisor = 3 × Remainder
$\Rightarrow$ Divisor = 3 × 4 = 12
Again, divisor = 4 × Quotient
$\Rightarrow$ 4 × Quotient = 12
$\Rightarrow$ Quotient $= \frac{12}{4} = 3$
Dividend = Quotient × Divisor + Remainder
= 3 × 12 + 4 = 40

16. (c); If a number is divisible by two numbers separately, then it should be divisible by their product.

17. (a); $(x^n + y^n)$ is exactly divisible by (x + y) when, n is odd.
Here, x = 13, y = 11
and n = 5, 7.
$\therefore$ The common factor
= x + y = 13 + 11 = 24

18. (d); Let three consecutive even integers be x, x + 2 and x + 4, respectively.
According to the question.
x + x + 2 + x + 4 = 54
$\Rightarrow$ 3x + 6 = 54 $\Rightarrow$ 3x = 48
$\therefore$ x = 16
$\therefore$ The least even number = 16

19. (a); We know that
$2^1 = 2, 2^2 = 4,$
$2^3 = 8$
$2^4 = 16$
$2^5 = 32$
2 repeats at unit's place after power of 4.
Now, $(122)^{173} = (122^4)^{43}.122$
Unit digit in $(122)^{173}$ = Unit digit in
$(122^4)^{43}$ × Unit digit in 122
= Unit digit in (6 × 2) = 2

20. (b); The least number of 5-digits = 10000

```
41) 10000 (243
     82
    ----
    180
    164
    ----
    160
    123
    ----
     37
```

$\therefore$ Required number
= 10000 + (41 – 37) = 10004

21. (d); Let the number be x and quotient be y.
$\therefore$ Case I $\frac{x}{13} = y\frac{1}{13}$
Case II Now, quotient is divided by 5 and remainder is 3.
$\therefore \quad \frac{y}{5} = 1\frac{3}{5}$
$\therefore \quad y = (5 \times 1) + 3 = 8$
and $\frac{x}{13} = 8\frac{1}{13}$
$x = (8 \times 13) + 1 = 105$
Now, $\frac{105}{65} = 1\frac{40}{65}$
Remainder = 40

22. (c); Since, p, q, r are in geometric progression.
$\therefore \quad q^2 = pr$
Then, $q = \sqrt{pr}$

23. (c); Expression = 4 + 44 + 444 + ... to n terms
= 4 (1 + 11 + 111 + ... to n terms)
multiplying and dividing by 9.
$$= \frac{4}{9}(9 + 99 + 999 + \ldots \text{ to n terms})$$
On rearranging
$$= \frac{4}{9}[(10 - 1) + (100 - 1) + (1000 - 1) + \ldots \text{ n terms}]$$
$$= \frac{4}{9}[(10 + 10^2 + 10^3 + \ldots \text{ n terms}) - n]$$
$$= \frac{4}{9}[(10(1 + 10 + 10^2 + \ldots \text{ n terms}) - n]$$
$$= \frac{40}{9}.\frac{(10^n - 1)}{9} - \frac{4}{9}n$$
[Sum of n terms of the GP given as $\left(\frac{10^n - 1}{9}\right)$]
$$= \frac{40}{81}(10^n - 1) - \frac{4}{9}n$$

24. (b); Expression $= 2.3\overline{49} = 2 + 0.3\overline{49}$
$$= 2 + \frac{(349 - 3)}{990} = 2\frac{346}{990} = \frac{2326}{990}$$

25. (c); Given, $(5^2 + 6^2 + 7^2 + ... + 10^2)$

Sum of squares of first n natural numbers

$$= \frac{n(n+1)(2n+1)}{6}$$

Required sum = (sum of squares of natural numbers from 1 to 10) – (sum of squares of natural numbers from 1 to 4)

$$= \frac{10\times(10+1)(20+1)}{6} - \frac{4\times(4+1)(8+1)}{6}$$

$$= \frac{10\times11\times21}{6} - \frac{4\times5\times9}{6}$$

= 385 – 30 = 355

26. (c); Let the two numbers be x and y, respectively

$\therefore \quad \frac{x}{y} = \frac{1}{2}$ (given)

or $\quad y = 2x$ ... (i)

Now, when 4 is added to each number the ratio becomes 2 : 3.

Then, $\frac{x+4}{y+4} = \frac{2}{3}$

On solving,

3x + 12 = 2y + 8

3x +12 = 2(2x) + 8 [from Eq. (i)]

3x + 12 = 4x + 8

$\Rightarrow$ x = 4

Now, from Eq. (i)

y = 2x

i.e., y = 2 × 4 = 8

$\therefore$ The numbers are 4 and 8.

27. (d); Sum of cubes of 1st n natural numbers

$$= \left\{\frac{n(n+1)}{2}\right\}^2$$

$\therefore \quad 1^3 + 2^3 + 3^3 + ... + 10^3$

$$= \left[\frac{10\times(10+1)}{2}\right]^2 \quad [\because n = 10]$$

$$= \left(\frac{10\times11}{2}\right)^2 = (55)^2$$

= 55 × 55 = 3025

28. (c); Remainder = $\frac{\text{Last remainder}}{\text{New divisor}}$

Remainder = $\frac{63}{29} = 2\frac{5}{29} = 5$

29. (d); $\frac{(25)^{25}}{26} = \frac{(25^2)^{12}\times25}{26}$

$$= \frac{(625)^{12}\times25}{26} = \frac{(1)^{12}\times25}{26}$$

$\therefore$ Remainder = 25

30. (a); According to the question,

Divisor = 555 + 445 = 1000

Dividend = ?

Quotient = (555 – 445) × 2 = 110 × 2 = 220

Remainder = 30

Dividend = (Divisor × Quotient) + Remainder

= (1000 × 220) + 30 = 220000 + 30 = 220030

Chapter 2

LCM and HCF

Factors and Multiples: If a divides b exactly, we say that a is a factor of b and also we say that b is multiple of a.

i.e. 7 is a factor of 14, 8 is factor of 24 e.t.c. or 14 is multiple of 7, 24 is multiple of 8.

HCF/G.C.D/G.C.M: The HCF of two or more than two numbers is the greatest number that divides each of them exactly.

Method of finding HCF

(i) Factorization method: Express each one of the given numbers as the product of prime factors. The product of common prime factors with least power gives HCF.

Example: Find the HCF of 42, 63 and 140, $42 = 7 \times 2 \times 3$, $63 = 7 \times 3 \times 3$, $140 = 7 \times 5 \times 2 \times 2$, So HCF = 7

(i) Division Method: Suppose we have find the HCF of two given numbers. Divide the larger number by the smaller one. Now divide the divisor by the remainder . Repeat this process till remainder is zero. The last divisor is required HCF.

Example: Find the HCF of 148 and 185

```
148)185(1
    148
    ---
     37)148(4
        148
        ---
        X X
        ---
```

i.e. HCF = 37.

LCM : The least number which is exactly divisible by each one of the given number is called their LCM.

Methods

(1) Factorization method: Resolve each of the given number into a product of prime factors. LCM is the product of terms of highest power of all factors.

Product of two numbers = their LCM × their HCF

Co-prime: Two numbers are said to be co-prime if their HCF is 1.

i.e. 4 and 3 arc co-prime numbers.

HCF and LCM of fractions

(i) $\text{HCF} = \dfrac{\text{HCF of numerators}}{\text{LCM of denominators}}$ (ii) $\text{LCM} = \dfrac{\text{LCM of numerators}}{\text{HCF of denominators}}$

Important Results

(i) Product of two numbers = HCF of the numbers × LCM of the numbers

(ii) The greatest number which divides the number x, y and z leaving remainders a, b and c respectively.
= HCF of (x – a) (y – b) (z – c)

(iii) The least number which when divided by x, y and z leaves the remainder a, b and c respectively, is given by [LCM of (x, y, z) + K], where K = (x – a) = (y – b) = (z – c)

(iv) The least number which divided by x, y and z leaves the same remainder k in each case, is given by [LCM of (x, y, z) + K]

(v) The greatest number that will divide x, y and z leaving the same remainder in each case, is given by [HCF of (x –y), (y – z), (z – x)]

(vi) When the HCF of each pair of n given numbers is a and their LCM is b, then product of these numbers is given by $(a)^{n-1} \times b$ or $(HCF)^{n-1} \times LCM$

Types of Questions

1. HCF of $\frac{2}{3}$, $\frac{4}{5}$ and $\frac{6}{7}$

Sol. HCF of fractions = $\frac{\text{HCF of numerators}}{\text{LCM of denominators}}$

$= \frac{HCF\ (2,4,6)}{LCM\ (3,5,7)} = \frac{2}{105}$.

2. The HCF (GCD) of a, b is 12 and a and b are positive integers and a > b > 12. The smallest value of (a, b) are respectively

Sol. Given HCF of a, b = 12

Let the numbers be 12x and 12 y, where x and y are co-prime.

But given a > b > 12

i.e. a = 36

b = 24

3. The LCM of three different numbers is 120. Which of the following cannot be HCF

(i) 4 (ii) 12 (iii) 35 (iv) 8

Sol. LCM of three no is = 120

Now, factors of 120 = 2 × 2 × 2 × 3 × 5,

Hence HCF can be 4, 8, 12

But 35 can't be HCF.

4. Two numbers are in the ratio 3 : 4. If their LCM is 84, then the greater number is

Sol. Let the number be 3x and 4x

LCM of 3x and 4x = 12x

LCM = 84

12x = 84

x = 7

Greatest Number = 4 × 7 = 28

5. A rectangular piece of cloth has dimentions 16 m and 12m. What is the least number of equal square that can be cut out of this cloth?

Sol. HCF of 16 and 12 = 4

No of pieces = $\frac{16 \times 12}{4 \times 4} = 3 \times 4 = 12$

6. Find the largest number, which divides 34, 90, 104, leaving the same remainder in each case.

Sol. Difference between numbers = 90 – 34, 104 – 90 and 104 – 134

= 56, 14 and 70.

HCF of 56, 14 and 70 = 14

i.e. 14 is largest number.

7. Which is the smallest multiple of 7, which when divided by 6, 9, 15 and 18 respectively leaves 4 as remainder in each case.

Sol. LCM of 6, 9, 15 and 18 = 90

Remainder = 4 , but number is also divisible by 7 so required number

= 90k + 4 ... (i)

By putting

k = 1, 2, 3, 4 ...... and checking if it is divisible by 7

Required number = 90k + 4 = 90 × 4 + 4 = 364

Foundation

Questions

1. LCM of $\frac{2}{3}, \frac{4}{9}, \frac{5}{6}$ is
(a) $\frac{20}{3}$ (b) $\frac{10}{3}$
(c) $\frac{20}{27}$ (d) $\frac{8}{27}$

2. The HCF of two numbers is 8. Which one of the following can never be their LCM?
(a) 24 (b) 48
(c) 56 (d) 60

3. The LCM of two numbers is 520 and their HCF is 4. If one of the numbers is 52, then the other number is
(a) 40 (b) 42
(c) 50 (d) 52

4. The HCF of two numbers is 96 and their LCM is 1296. If one of the numbers is 864, the other is
(a) 132 (b) 135
(c) 140 (d) 144

5. The product of two numbers is 216. If the HCF is 6, then their LCM is
(a) 72 (b) 60
(c) 48 (d) 36

6. Two numbers are in the ratio 3 : 4. If their HCF is 4, then their LCM is
(a) 48 (b) 42
(c) 36 (d) 24

7. The LCM of two numbers, which are multiples of 12 is 1056. If one of the numbers is 132, the other number is
(a) 12 (b) 72
(c) 96 (d) 132

8. Two numbers are in the ratio 3 : 4. The product of their HCF and LCM is 2028. The sum of the numbers is
(a) 68 (b) 72
(c) 86 (d) 91

9. Two numbers are in the ratio 3 : 4. If their LCM is 240, the smaller of the two number is
(a) 100 (b) 80
(c) 60 (d) 50

10. The HCF of two numbers is 16 and their LCM is 160. If one of the number is 32, then the other number is
(a) 48 (b) 80
(c) 96 (d) 112

11. The product of two numbers is 4107. If the HCF of the numbers is 37, the greater number is
(a) 185 (b) 111
(c) 107 (d) 101

12. The ratio of two numbers is 3 : 4 and their HCF is 5. Their LCM is
(a) 80 (b) 48
(c) 120 (d) 60

13. The LCM of two numbers is 1820 and their HCF is 26. If one number is 130. Then, the other number is
(a) 70 (b) 1690
(c) 364 (d) 1264

14. The HCF of two numbers 12906 and 14818 is 478. Their LCM is
(a) 400086 (b) 200043
(c) 60012 (d) 800172

15. What is the greatest number which will divide 110 and 128 leaving a remainder 2 in each case?
(a) 8 (b) 18
(c) 28 (d) 38

16. The smallest perfect square divisible by each of 6, 12 and 18 is
(a) 196 (b) 144
(c) 108 (d) 36

17. When a number is divided by 15, 20 and 35, each time the remainder is 8. Then, that smallest number is
(a) 428 (b) 427
(c) 328 (d) 388

18. Find the HCF of 35 and 30.
(a) 5 (b) 6
(c) 7 (d) 8

19. The HCF and the product of two numbers are 15 and 6300 respectively. The number of possible pairs of the numbers are.
(a) 4 (b) 3
(c) 2 (d) 1

20. The LCM of two numbers is 12 times their HCF. The sum of the HCF and the LCM is 403. If one of the numbers is 93, then other number is
(a) 124 (b) 128
(c) 134 (d) 138

21. Find the least number exactly divisible by 12, 15, 20, 27.
(a) 450 (b) 540
(c) 230 (d) 640

22. The largest number which divides 25, 73 and 97 to leave the same remainder in each case, is:
(a) 24 (b) 23
(c) 21 (d) 6

23. Find the largest 5 digits number which is exactly divisible by 12, 15, 18, 27.
(a) 90000 (b) 99999
(c) 99010 (d) 99900

24. Find the least number which when divided by 20, 25, 35, 40 leaves remainders 14, 19, 29 and 34 respectively.
(a) 1220 (b) 1394
(c) 1365 (d) 1470

25. 3 different containers contain 496l, 403l and 713l mixture of milk and water. What biggest measure of a container can measure all the 3 quantities exactly?.
(a) 31 l (b) 41 l
(c) 51 l (d) 52 l

26. Three bells ring simultaneously at 11am. They ring at regular intervals of 20 min, 30 min and 40 min, respectively. The time when all the three ring together next is:
(a) 2 pm (b) 1 pm
(c) 1:15 pm (d) 1:30 pm

27. Four runners started running simultaneously from a point on a circular track. They took 200 s, 300 s, 360 s and 450 s to complete one round. After how much time do they meet at the strarting point for the first time?
(a) 1800 s (b) 3600 s
(c) 2400 s (d) 4800 s

28. The LCM of two positive integers is twice the larger number. The difference of the smaller number and the HCF of the two numbers is 4. The smaller number is:
(a) 12 (b) 6
(c) 8 (d) 10

29. Six bells commence tolling together at intervals of 2, 4, 6, 8, 10 and 12 s, respectively. In 30 min., how many times do they toll together?
(a) 16 (b) 15
(c) 10 (d) 4

30. Rajesh is incharge of buying bread rolls and buns for a party. There are 10 buns in each box of buns and 8 bread rolls in each box of bread rolls. Rajesh wants to buy exactly the same number of buns and bread rolls. What is the smallest number of boxes he should buy for buns alone?
(a) 10 (b) 8
(c) 4 (d) 5

Moderate

1. Three tankers contain 403 l, 434 l, 465 l of diesel, respectively. Then, the maximum capacity of container that can measure the diesel of the three containers in exact number of times is
(a) 31 l (b) 62 l
(c) 41 l (d) 84 l

2. A, B and C start together from the same point to travel around a circular path of 30 km in circumference. A and B are travelling in the same direction and C in the opposite direction. If A travels 5 km, B travels 7 km and C travels 8 km in an hour, then they all come together again after:
(a) 25 h (b) 30 h
(c) 15 h (d) 20 h

3. The traffic lights at three different road crossings change after 24 s, 36 s and 54 s, respectively. If they all change simultaneously at 10 : 15 : 00 am, then at what time will they again change simultaneously?
(a) 10 : 16 : 54 am (b) 10 : 18 : 36 am
(c) 10 : 17 : 02 am (d) 10 : 22 : 12 am

4. Sum of two numbers is 384, HCF of the numbers is 48. The difference of the number is
(a) 100 (b) 192
(c) 288 (d) 336

5. The sum of two numbers is 45. Their difference is $\frac{1}{9}$ of their sum. Their LCM is
(a) 200 (b) 250
(c) 100 (d) 150

6. The ratio of the sum to the LCM of two natural numbers is 7 : 12. If their HCF is 4, then the smaller number is
(a) 20 (b) 16
(c) 12 (d) 8

7. If the students of a class can be grouped exactly into 6 or 8 or 10, then the minimum number of students in the class must be
(a) 60 (b) 120
(c) 180 (d) 240

8. Three numbers are in the ratio 2 : 3 : 4 and their HCF is 12. The LCM of the numbers is
(a) 144 (b) 192
(c) 96 (d) 72

9. The greatest common divisor of $3^{3^{333}} + 1 \text{and} 3^{3^{334}} + 1$ is
(a) $3^{3^{333}} + 1$ (b) 20
(c) 2 (d) 1

10. A milk vendor has 21 l of cow milk, 42 l of toned milk and 63 l of double toned milk. If he wants to pack them in cans, so that each can contains same number of litres of milk and does not want to mix any two kinds of milk in a can, then the least number of cans required as
(a) 3 (b) 6
(c) 9 (d) 12
11. There are 24 peaches, 36 apricots and 60 bananas and they have to be arranged in several rows in such a way that every row contains the same number of fruits of only one type. What is the minimum number of rows required for this to happen?
(a) 12 (b) 9
(c) 10 (d) 6
12. Three numbers which are coprimes to one another are such that the product of the first two is 551 and that of the last two is 1073. The sum of the three numbers is
(a) 75 (b) 81
(c) 85 (d) 89
13. Three sets of English, Mathematics and Science books containing 336, 240 and 96 books, respectively have to be stacked in such a way that all the books are stored subject–wise and the height of each stack is the same. Total number of stacks will be
(a) 14 (b) 21
(c) 22 (d) 48
14. The sum of two numbers is 36 and their HCF and LCM are 3 and 105, respectively. The sum of the reciprocals of two numbers is
(a) $\frac{2}{35}$ (b) $\frac{3}{25}$
(c) $\frac{12}{35}$ (d) $\frac{2}{25}$
15. What is the least number of square tiles required to pave the floor of a room 15 m 17 cm long and 9 m 2 cm broad?
(a) 840 (b) 841
(c) 820 (d) 814
16. The HCF (GCD) of a, b is 12. a, b are positive integers and a > b > 12. The smallest values of (a, b) are respectively.
(a) 12, 24 (b) 24, 12
(c) 24, 36 (d) 36, 24
17. If $P = 2^3 \times 3^{10} \times 5$ and $Q = 2^5 \times 3 \times 7$, then HCF of P and Q is
(a) 2 . 3. 5.7 (b) 3.2^3
(c) $2^2 .3^7$ (d) $2^5 . 3^{10} . 5.7$
18. The sum of two numbers is 84 and their HCF is 12. Total number of such pairs of numbers is
(a) 2 (b) 3
(c) 4 (d) 5
19. The sum of two numbers is 36 and their HCF is 4. How many pairs of such numbers are possible?
(a) 1 (b) 2
(c) 3 (d) 4
20. The HCF of two numbers 12908 and 14808 is 672. Their LCM is
(a) 284437 (b) 200043
(c) 60012 (d) 800172
21. Two jars of capacity 50 l and 80 l are filled with oil. What must be the capacity of a mug that can completely measure the oil of the two jars?
(a) 5 l (b) 15 l
(c) 10 l (d) 20 l
22. The traffic lights at three different road crossings change after every 48 sec, 72 sec, and 108 sec respectively. If they all change simultaneously at 8 : 20 hours, then at what time will they again change simultaneously?
(a) 8 : 27 : 12 hours (b) 8 : 25 : 14 hours
(c) 8 : 24 : 12 hours (d) 8 : 29 : 12 hours
23. The LCM of two numbers is 2310 and their HCF is 30. If one of the number is 7 × 30, Find the other number.
(a) 320 (b) 330
(c) 340 (d) 350
24. What is the least multiple of 7, which when divided by 2, 3, 4, 5 and 6 leaves the remainders 1, 2, 3, 4 and 5 respectively?
(a) 117 (b) 119
(c) 121 (d) 123
25. The least number, which when divided by 12, 15, 20 or 54 leaves a remainder 4 in each case is:
(a) 450 (b) 454
(c) 540 (d) 544
26. The maximum number of students among whom 1001 pens and 910 pencils can be distributed in such a way that each student gets same number of pens and same number of pencils, is
(a) 91 (b) 910
(c) 1001 (d) 1911
27. A, B and C start running at the same time and from the same point in the same direction in a circular stadium. A completes a round in 252 s, B in 308 s and C in 198s. After what time will they meet again at the starting point?
(a) 26 min 18 s (b) 42 min 36 s
(c) 45 min (d) 46 min 12 s
28. Three men step-off together from the same spot. Their steps measures 63 cm, 70 cm and 77 cm, respectively. The minimum distance each should cover, so that all can cover the distance in complete steps, is
(a) 9630 cm (b) 9360 cm
(c) 6930 cm (d) 6950 cm

29. Find the largest number of four digits such that on dividing by 15, 18, 21 and 24 the remainders are 11, 14, 17 and 20, respectively.
(a) 6557 (b) 7556
(c) 5675 (d) 7664

30. The total number of integers between 100 and 200, which are divisible by both 9 and 6 is
(a) 5 (b) 6
(c) 7 (d) 8

Difficult

1. The largest possible length of a tape which can measure 525 cm, 1050 cm and 1155 cm length of cloths in a minimum number of attempts without measuring the length of a fraction of the tape's length is
(a) 25 (b) 105
(c) 75 (d) None of these

2. There are three drums with 1653 litre 2261 litre and 2527 litre of petrol. The greatest possible size of the measuring vessel with which we can measure up the petrol of any drum while every time the vessel must be completely filled is:
(a) 31 (b) 27
(c) 19 (d) 41

3. Mr. Baghwan wants to plant 36 mango trees, 144 orange trees and 234 apple trees in his garden. If he wants to plant the equal no. of trees in every row, but the rows of mango, orange and apple trees will be separate, then the minimum number of rows in his garden is :
(a) 18 (b) 23
(c) 36 (d) Can't be determined

4. Find the least possible perfect square number which is exactly divisible by 6, 40, 49 and 75
(a) 176400 (b) 15000
(c) 175600 (d) 16500

5. Three bells in the Bhootnath temple toll at the interval of 48, 72 and 108 seconds individually. If they have tolled all together at 6 : 00 AM then at what time will they toll together after 6 : 00 AM?
(a) 6 : 07 : 15 AM (b) 6 : 07 : 12 AM
(c) 4 : 04 : 12 AM (d) 6 : 06 : 12 AM

6. What is the least possible number which when divided by 18, 35 and 42 leaves 2, 19 and 26 as the remainders respectively?
(a) 400 (b) 740
(c) 614 (d) 621

7. Find the HCF of 0.0005, 0.005, 0.15, 0.175, 0.5 and 3.5.
(a) .0005 (b) .005
(c) .05 (d) .5

8. The least number which when divided by 2, 3, 4, 5 and 6 leaves the remainder 1 in each case. If the same number is divided by 7 it leaves no remainder. The number is:
(a) 231 (b) 301
(c) 371 (d) 441

9. Three bells, toll at interval of 36 sec, 40 sec and 48 sec respectively. They start ringing together at particular time. They next toll together after :
(a) 6 minutes (b) 12 minutes
(c) 18 minutes (d) 24 minutes

10. Mr. Black has three kinds of wine, of the first kind 403 litres, of the second 434 litres and of the third 465 litres. What is the least number of full corks of equal size in which these can be stored without mixing?
(a) 31 (b) 39
(c) 42 (d) 51

11. Abhishek, Bobby and Charlie start from the same point and travel in the same direction round an Island 6 km in circumference. Abhishek travels at the rate of 3km / hr , Bobby at the rate of $2\frac{1}{2}$ km / hr and Charlie at the rate of $1\frac{1}{4}$ km / hour. In how many hours will they come together again?
(a) 6 hrs (b) 12 hrs
(c) 24 hrs (d) 15 hrs

12. The LCM of two numbers is 4 times their HCF. The sum of LCM and HCF is 125. If one of the numbers is 100, then the other number is
(a) 5 (b) 25
(c) 100 (d) 125

13. LCM of two numbers is 120 and their HCF is 10. Which of the following can be the sum of those two numbers?
(a) 140 (b) 80
(c) 60 (d) 70

14. A fraction becomes $\frac{1}{6}$ when 4 is subtracted from its numerator and 1 is added to its denominator. If 2 and 1 are respectively added to its numerator and the denominator, it becomes $\frac{1}{3}$. Then, the LCM of the numerator and denominator of the said fraction, must be

(a) 14 (b) 350
(c) 5 (d) 70

15. From a point on a circular track 5 km long, A, B and C started running in the same direction at the same time with speeds of $2\frac{1}{2}$ km / h, 3 km / h and 2 km / h, respectively. Then, on the starting point all three will meet again after:
(a) 30 h (b) 6 h
(c) 10 h (d) 15 h

16. The HCF of two numbers is 15 and their LCM is 300. If one of the numbers is 60, the other is
(a) 50 (b) 75
(c) 65 (d) 100

17. HCF and LCM of two numbers are 7 and 140, respectively. If the numbers are between 20 and 45, the sum of the numbers is
(a) 70 (b) 77
(c) 63 (d) 56

18. The number of integers in between 100 and 600, which are divisible by 4 and 6 both is
(a) 40 (b) 42
(c) 41 (d) 50

19. The least number, which when divided by 18, 27 and 36 separately leaves remainders 5, 14 and 23 respectively, is
(a) 95 (b) 113
(c) 149 (d) 77

20. The largest number of five digits which, when divided by 16, 24, 30, and 36 leaves the same remainder 10 in each case, is
(a) 99279 (b) 99370
(c) 99269 (d) 99530

Previous Year (Memory Based)

1. The HCF and LCM of two numbers are 44 and 264 respectively. If the first number is divided by 2, then quotient is 44. The other number is
(a) 147 (b) 528
(c) 132 (d) 264

2. The ratio of two numbers is 5 : 6 and their LCM is 480, then their HCF is
(a) 20 (b) 16
(c) 6 (d) 5

3. Product of two coprime numebers is 117, then their LCM is
(a) 9 (b) 13
(c) 39 (d) 117

4. The LCM of two numbers is 48. The numbers are in the ratio 2 : 3. The sum of the numbers is
(a) 28 (b) 32
(c) 40 (d) 64

5. The ratio of two numbers is 4 : 5 and their HCF is 8. Then, their LCM is
(a) 130 (b) 140
(c) 150 (d) 160

6. If the HCF and LCM of two consecutive (positive) even numbers is 2 and 84 respectively, then the sum of the numbers is
(a) 30 (b) 26
(c) 14 (d) 34

7. The HCF and LCM of two numbers are 8 and 48 respectively. If one of the numbers is 24, then the other number is
(a) 48 (b) 36
(c) 24 (d) 16

8. The product of the LCM and the HCF of two numbers is 24. If the difference of the numbers is 2, then the greater of the number is
(a) 3 (b) 4
(c) 6 (d) 8

9. The HCF and product of two numbers are 15 and 6300, respectively. The number of possible pairs of the numbers is
(a) 4 (b) 3
(c) 2 (d) 1

10. The LCM of two numbers is 20 times their HCF. The sum of HCF and LCM is 2520. If one of the numbers is 480, the other number is
(a) 400 (b) 480
(c) 520 (d) 600

11. HCF of $\frac{2}{3}, \frac{4}{5}$ and $\frac{6}{7}$ is
(a) $\frac{48}{105}$ (b) $\frac{2}{105}$
(c) $\frac{1}{105}$ (d) $\frac{24}{105}$

12. The LCM and the HCF of the numbers 28 and 42 are in the ratio
(a) 6 : 1 (b) 2 : 3
(c) 3 : 2 (d) 7 : 2

13. If the LCM and HCF of two expressions are $(x^2 + 6x + 8)(x + 1)$ and $(x + 1)$ respectively and one of the expression is $x^2 + 3x + 2$, then find the other.
(a) $x^2 + 5x + 4$ (b) $x^2 - 5x + 4$
(c) $x^2 + 4x + 5$ (d) $x^2 - 4x + 5$

14. Two numbers are in the ratio 3 : 4. Their LCM is 84. Then, the greater number is
(a) 21 (b) 24
(c) 28 (d) 84

15. Two numbers, both greater than 29, have HCF 29 and LCM 4147. The sum of the numbers is
(a) 966 (b) 696
(c) 669 (d) 666

16. The HCF and LCM of two numbers are 11 and 385, respectively. The numbers are
(a) 55 and 57 (b) 55 and 77
(c) 44 and 77 (d) 22 and 770

17. The HCF and LCM of two 2 digit numbers are 16 and 480, respectively. The numbers are
(a) 40, 48 (b) 60, 72
(c) 64, 80 (d) 80, 96

18. Find the greatest number which will exactly divide 200 and 320.
(a) 10 (b) 20
(c) 16 (d) 40

19. The greatest number which can divide 1356, 1868, 2764 leaving the same remainder in each case is
(a) 260 (b) 64
(c) 124 (d) 128

20. What is the least number which, when divided by 5, 6, 7 and 8 gives the remainder 3 but is divisible by 9?
(a) 1463 (b) 1573
(c) 1683 (d) 1793

21. The greatest number of four digits which when divided by 12, 16 and 24 leave remainder 2, 6 and 14 respectively is
(a) 9974 (b) 9970
(c) 9807 (d) 9998

22. What least number must be subtracted from 1936, so that the resulting number when divided by 9, 10 and 15 will leave in each case the same remainder 7?
(a) 37 (b) 36
(c) 39 (d) 30

23. The largest number, which divides 25, 73 and 97 to leave the same remainder in each case, is
(a) 24 (b) 23
(c) 21 (d) 6

24. The least multiple of 13, which on dividing by 4, 5, 6, 7 and 8 leaves remainder 2 in each case, is
(a) 2520 (b) 842
(c) 2522 (d) 840

25. What is the greatest number that will divide 307 and 330 leaving remainders 3 and 7, respectively?
(a) 19 (b) 16
(c) 17 (d) 23

26. Let the least number of six digits which when divided by 4, 6, 10 and 15 leaves in each case same remainder 2 be N. The sum of digits of N is
(a) 3 (b) 5
(c) 4 (d) 6

27. The smallest square number divisible by 10, 16 and 24 is:
(a) 900 (b) 1600
(c) 2500 (d) 3600

28. The greatest number, which divides 989 and 1327 leaving remainders 5 and 7 respectively, is
(a) 8 (b) 16
(c) 24 (d) 32

29. Find the largest number of four digits such that on dividing by 15, 18, 21 and 24 the remainders are 11, 14, 17 and 20 respectively.
(a) 6557 (b) 7556
(c) 5675 (d) 7664

30. The greatest number, that divides 122 and 243 leaving respectively, 2 and 3 as remainders is
(a) 12 (b) 24
(c) 30 (d) 120

Foundation

Solutions

1. (a); Given, fractions $\frac{2}{3}, \frac{4}{9}$ and $\frac{5}{6}$

$$\text{LCM of fractions} = \frac{\text{LCM of numerators}}{\text{HCF of denominators}}$$

$$= \frac{\text{LCM}(2,4,5)}{\text{HCF}(3,9,6)} = \frac{20}{3}$$

2. (d); HCF of two numbers is 8. This means 8 is a factor common to both the numbers.
∴ LCM must be multiple of 8. By going through the options, 60 cannot be the LCM since it is not a multiple of 8.

3. (a); HCF × LCM = Product of two numbers.
4× 520 = 52 × Second number

$$\therefore \quad \text{Second number} = \frac{4 \times 520}{52} = 40$$

4. (d); HCF × LCM
= First number × Second number
$\therefore$ Second number $= \frac{96 \times 1296}{864} = 144$

5. (d); HCF × LCM = Product of two numbers.
Then,
$LCM = \frac{216}{6} = 36$

6. (a); Let the numbers be 3x and 4x
HCF = x = 4
LCM of 3x and 4x = 12x = 12 × 4 = 48

7. (c); Given, first number = 132
Since, each of the two numbers is a multiple of 12 (given),
$\therefore$ HCF = 12 and LCM = 1056 (given)
LCM × HCF = First number × Second number
$\therefore$ Second number $= \frac{1056 \times 12}{132} = 96$

8. (d); Let the numbers be 3x and 4x respectively.
HCF×LCM
= First number × Second number
$\therefore$ 2028 = 3x × 4x
$x^2 = \frac{2028}{12} = 169$, x = 13
Numbers are = 39, 52
Sum of numbers = 39 + 52 = 91

9. (c); Let the numbers be 3x and 4x.
LCM of 3x and 4x = 12x
But given, LCM = 240
$\therefore$ 12x = 240
$\Rightarrow$ $x = \frac{240}{12} = 20$
$\therefore$ Smaller number = 3x = 3 × 20 = 60

10. (b); HCF × LCM = Product of two number
$\therefore$ Second number $= \frac{16 \times 160}{32} = 80$

11. (b); HCF × LCM = Product of two numbers
$\therefore$ $LCM = \frac{4107}{37} = 111$, ab × HCF = LCM
where a, b are prime factors $ab = \frac{111}{37} = 3$
Prime number pairs (3, 1)
Numbers = 3 × HCF, 1 × HCF
$\therefore$ Numbers are 111 and 37.
Hence, 111 is the greater number

12. (d); The ratio of two numbers is 3 : 4 and their HCF is 5. Their LCM is
Given that the numbers are in ratio of 3 : 4,
First number = 3 × 5 =15
Second number = 4 × 5 = 20
LCM × HCF = Product of two numbers
$\therefore$ $LCM = \frac{15 \times 20}{5} = 60$

13. (c); We know that,
LCM × HCF = Product of two numbers
$\therefore$ Second number $= \frac{26 \times 1820}{130} = 364$

14. (a); We know that,
HCF × LCM = Product of two numbers
$\therefore$ LCM = $= \frac{12906 \times 14818}{478}$
LCM = 400086

15. (b); Required number
= HCF of {(110 – 2) and (128 – 2)}
= HCF of 108 and 126
By Division Method

108)126(1
108
18)108(6
108
×

$\therefore$ Greatest number = 18

16. (d); The LCM of 6, 12 and 18 = 36
36 is a perfect square of 6.

17. (a); The smallest number
= [LCM of (15, 20, 35) +k]

3	15, 20, 35
5	5, 20, 35
	1, 4, 7

LCM = 4 ×7 × 5 × 3= 420
Smallest number = 420 + 8 = 428

18. (a); 30 —— 5 × 3 × 2
35 —— 5 × 7
So, HCF = 5

19. (c); Let two numbers be 15x and 15y.Where x and y are co-prime to each other.
$\therefore$ 15x × 15y = 6300
x × y = $\frac{6300}{15 \times 15}$
x × y = 28
Factors of 28 are 1, 2, 4, 7, 28
$\therefore$ 28 = 1 × 28 or 4 × 7
$\therefore$ There are only two possible pairs

20. (a); HCF + LCM = 403
HCF + 12 HCF = 403
(It is given that LCM = 12 × HCF)
13 × HCF = 403
HCF = 31
$\because$ Product of two number is equal to product of their LCM and their HCF.
$\therefore$ 93 × other number = 31 × 12 × 31
Other number = $\frac{31 \times 12 \times 31}{93} = 124$

21. (b); We need to find the LCM of 12, 15, 20, 27.
$12 = 2^2 \times 3$
$15 = 3 \times 5$
$20 = 2^2 \times 5$
$27 = 3^3$
LCM = Product of highest powers of factors
$= 2^2 \times 3^3 \times 5 = 540$

22. (a); Let x be the remainder then (25 – x), (73 – x) and (97 – x) will be exactly divisible by the required number.
Required number = HCF of (73 – x)– (25 – x), (97 – x) – (73 – x) and (97 – x) – (25 – x)
= HCF of (73 – 25), (97 – 73) and (97 – 25)
= HCF of 48, 24 and 72 = 24

23. (d); LCM of 12, 15, 18, 27 = 540
Largest number of 5 digits = 99999
On dividing 99999 by 540, remainder = 99
$\therefore$ Required number = 99999 – 99 = 99900

24. (b); 20 – 14 = 25 – 19 = 35 – 29 = 40 – 34 = 6
Required number = (LCM of 20, 25, 35, 40) – 6
= 1400 – 6= 1394

25. (a); To find the biggest measure, we have to find the HCF of 496, 403 and 713.
HCF of 496, 403 and 713 = 31

26. (b); Required time = LCM of 20, 30 and 40

10	20, 30, 40
2	2, 3, 4
	1, 3, 2

LCM = 10 × 2 × 3 × 2 = 120
Hence, the bells will simultaneously ring after 2h i.e., at 1 pm

27. (a); Required time = LCM of (200, 300, 360, 450) s.

10	200, 300, 360, 450
5	20, 30, 36, 45
3	4, 6, 36, 9
2	4, 2, 12, 3
2	2, 1, 6, 3
3	1, 1, 3, 3
	1, 1, 1, 1

$\therefore$ LCM = 10 × 5 × 3 × 2 × 2 × 3 = 1800 s

28. (c); Let the numbers be ax and bx, where x is the HCF and bx > ax.
$\therefore$ LCM = abx
abx = 2bx
$\Rightarrow$ a = 2
Again, ax – x = 4
Putting the value of a, we get
2x – x = 4
$\Rightarrow$ x = 4
$\therefore$ Smaller number = ax = 2 × 4 = 8

29. (a); All the 6 bells ring together will be LCM of (2, 4, 6, 8, 10 and 12)

2	2, 4, 6, 8, 10, 12
2	1, 2, 3, 4, 5, 6
3	1, 1, 3, 2, 5, 3
	1, 1, 1, 2, 5, 1

$\therefore$ They will ring together after
= 2 × 2 × 2 × 3 × 5 = 120 s
i.e., they will ring together after 2 min
$\therefore$ Number of time they will ring together in 30 min = $1 + \frac{30}{2} = 1 + 15 = 16$ times

30. (c); Smallest number of boxes for buns alone
$= \frac{\text{LCM of 10 and 8}}{\text{Number of buns in a box}} = \frac{40}{10} = 4$

Moderate

1. (a); Capacity of three containers containing diesel is 403 l, 434 l and 465 l, respectively.

Now, maximum capacity of the container that can measure the diesel of the three containers exactly

= HCF of quantity of three containers

= HCF (403, 434, 465)

```
403)434(1
    403
    ---
    31)403(13
       403
       ---
        ×
```

```
Again, 31)465(15
          465
          ---
           ×
```

So, Capacity of container = 31 L

2. (b); Time taken by A = $\frac{30}{5}$ h

Time taken by B = $\frac{30}{7}$ h

Time taken by C = $\frac{30}{8}$ h

Time taken by all to come together

$= \text{LCM of } \frac{30}{5}, \frac{30}{7} \text{ and } \frac{30}{8}$

$= \frac{\text{LCM of } 30, 30 \text{ and } 30}{\text{HCF of } 5, 7 \text{ and } 8} = \frac{30}{1} = 30\text{h}$

3. (b); LCM of 24, 36 and 54

2	24, 36, 54
2	12, 18, 27
2	6, 9, 27
3	3, 9, 27
3	1, 3, 9
3	1, 1, 3
	1, 1, 1

Required time = LCM of 24, 36 and 54 s = 216 s

$= \frac{216}{60} = 3\frac{36}{60}\text{min}$

= 3 min 36 s

∴ Time when they will change simultaneously

= 10 : 15 : 00 + 3 min 36 s

= 10 : 18 : 36 am

4. (c); Let the numbers be 48a and 48b, where a and b are coprimes,

∴ 48a + 48b = 384

⇒ 48 (a +b) = 384

⇒ $a + b = \frac{384}{48} = 8$ ... (i)

Possible valid pairs of a and b satisfying this condition are (1, 7) and (3, 5).

∴ Numbers are 48 × 1 = 48

and 48 ×7 = 336 or

or, 48 × 3 = 144

and 48 × 5 = 240

∴ Required difference = 336 – 48 = 288

or 240 – 144 = 96

5. (c); Let the numbers be a and b.

According to the question,

a + b = 45 ... (i)

Again, $a - b = \frac{1}{9}(a+b)$

⇒ $a - b = \frac{1}{9} \times 45$ ⇒ a – b = 5 ... (ii)

Adding equ. (i) and (ii), we get

2a = 50

$a = \frac{50}{2} = 25$

On putting the value of a Eq. (i) we get

25 + b= 45

⇒ b = 45 – 25 = 20,

a = 25 and b = 20

∴ LCM of 25 and 20.

5	25, 20
5	5, 4
	1, 4

LCM = 5 ×5 × 4 = 100

6. (c); Let the numbers be 4a and 4b where a and b are coprimes.

LCM = 4ab

∴ $\frac{(4a+4b)}{4ab} = \frac{7}{12}$

⇒ $\frac{1}{a} + \frac{1}{b} = \frac{7}{12} = \frac{1}{3} + \frac{1}{4}$

⇒ a = 3, b = 4

∴ First number = (4 × 3) = 12

Second number = (4 × 4) = 16

∴ Smaller number = 12

7. (b); Minimum number of students = LCM of 6, 8, 10.

2	6, 8, 10
2	3, 4, 5
2	3, 2, 5
	3, 1, 5

LCM = 2 × 2 × 2 × 3 × 5 = 120
Hence, the minimum number of students = 120

8. (a); Let the number be 2x, 3x and 4x, respectively.
∴ HCF = x = 12
∴ Numbers 2 × 12 = 24, 3 × 12 = 36, 4 × 12 = 48
LCM of 24, 36, 48

2	24, 36, 48
2	12, 18, 24
2	6, 9, 12
2	3, 9, 6
3	3, 9, 3
3	1, 3, 1
	1, 1, 1

∴ LCM = 2 × 2 × 2 × 2 × 3 × 3 = 144

9. (d); Given,

$3^{3^{333}} + 1$ and $3^{3^{334}} + 1$ or $27^{333} + 1^{333}$ and $27^{334} + 1^{334}$

Now, $x^m + a^m$ is divisible by (x + a) when m is odd.
$27^{333} + 1^{333}$ is divisible by (27 + 1) =28
Similarly, $27^{334} + 1^{334}$ is never divisible by (x+ a)
So, the greatest common divisor between $\left(3^{3^{333}} + 1\right)$ and $\left(3^{3^{334}} + 1\right)$ is 1.

10. (b); Maximum quantity in each can
= HCF of (21, 42 and 63) L = 21 L
By Division Method

$$21\overline{)42}(2$$
42
×
$$21\overline{)63}(3$$
63
×

HCF = 21 L
∴ Least number of cans

$$= \frac{21}{21} + \frac{42}{21} + \frac{63}{21} = 1 + 2 + 3 = 6 \text{ cans.}$$

11. (c); To find the minimum number of rows, we determine the HCF of 24, 36 and 60.
∴ HCF of 24, 36 and 60 = 12
Thus, 12 fruits are there in a row.

$$\therefore \text{ Number of rows } = \frac{24}{12} + \frac{36}{12} + \frac{60}{12}$$
$$= 2 + 3 + 5 = 10$$

12. (c); Let the numbers be p, q and r which are coprime to one another.
Now, pq = 551 and qr = 1073
q = HCF of 551 and 1073

$$551\overline{)1073}(1$$
551
$$522\overline{)551}(1$$
522
$$29\overline{)522}(18$$
522
×

$\therefore$ q = 29 $\quad \therefore$ $p = \frac{551}{29} = 19$

and $r = \frac{1073}{29} = 37$

∴ Sum of three numbers.
= 19 + 29 + 37 = 85

13. (a); Number of books in each stack = HCF of (336, 240, 96)

$$240\overline{)336}(1$$
240
$$96\overline{)240}(2$$
192
$$48\overline{)96}(2$$
96
×

∴ Number of books in each stack = 48

$$\therefore \text{ Total number of stacks } = \frac{336}{48} + \frac{240}{48} + \frac{96}{48}$$
$$= 7 + 5 + 2 = 14$$

14. (c); Givn, HCF = 3, LCM = 105
Now, let the numbers be 3a and 3b,
∴ 3a + 3b = 36
⇒ a + b = 12 ... (i)
and LCM 3ab = 105 ... (ii)
Dividing Eq. (i) by Eq. (ii), we have

$$\frac{a}{3ab} + \frac{b}{3ab} = \frac{12}{105}$$

$$\Rightarrow \frac{1}{3a} + \frac{1}{3b} = \frac{4}{35} \quad \Rightarrow \frac{1}{a} + \frac{1}{b} = \frac{12}{35}$$

15. (d); Length of the floor = 15 m 17 cm = 1517 cm
Breadth of the floor = 9 m 2 cm = 902 cm
Area of the floor = $1517 \times 902 \text{ cm}^2$
The number of square tiles will be least, when the size of each tile is maximum.
∴ Size of each tile = HCF of 1517 and 902

902)1517(1
902
615)902(1
615
287)615(2
574
41)287(7
287
×

HCF = 41

∴ Required number of tiles

$$= \frac{\text{Area of the floor}}{\text{Area of each square tile}} = \frac{1517 \times 902}{41 \times 41} = 814$$

16. (d); Given, HCF of a and b = 12

Let the numbers be 12x and 12y, where x and y are coprime.

But given a > b > 12

Smallest coprime pair for the above condition = (3, 2)

∴ a = 36 and b = 24

17. (b); Given,

$P = 2^3 \times 3^{10} \times 5$, $Q = 2^5 \times 3 \times 7$

[∵ 2^3 and 3 is common factor to both P and Q]

$HCF = 2^3 \times 3$

18. (b); HCF = 12

Numbers = 12a and 12b where, a and b are coprimes

∴ 12a + 12b = 84 ⇒ 12 (a +b) = 84

$$\Rightarrow a + b = \frac{84}{12} = 7$$

∴ Possible pairs of numbers satisfying this condition = (1, 6), (2, 5) and (3, 4)

Hence, pairs of required numbers = 3.

19. (c); HCF of two numbers = 4

Hence, the numbers can be expressed as 4a and 4b, where a and b are coprime,

4a + 4b = 36, a +b = 9

Now, possible pairs satisfying above condition are (1, 8), (4, 5), (2, 7).

∴ 3 pairs are possible

20. (a); We know that,

HCF × LCM = Product of two numbers

$$\therefore \text{LCM} = \frac{12908 \times 14808}{672} = 284437$$

21. (c); Factors of 50 = $5^2 \times 2$

Factors of 80 = $5^1 \times 2^4$

H.C.F. of 50 & 80 = $5^1 \times 2^1$ = 10 l

The capacity of the mug must be 10 l

22. (a); LCM of 48, 72 and 108 = 432

The traffic lights will change simultaneously after 432 seconds or 7 m 12 secs.

∴ They will change simultaneously at

= 8 : 20 hours + 7 m + 12 sec. = 8 : 27 : 12 hrs.

23. (b); Product of two numbers = H.C.F. × L.C.M.

7 × 30 × Second number = 30 × 2310

$$\text{Second number} = \frac{30 \times 2310}{7 \times 30} = 330$$

24. (b); LCM of 2, 3, 4, 5 and 6 = 60

Other numbers divisible by 2, 3, 4, 5, 6 are 60k, where k is a positive integer. Since 2 – 1 = 1, 3 – 2 = 1, 4 – 3 = 1, 5 – 4 = 1 and 6 – 5 = 1, the remainder in each case is less than the divisor by 1. Now, the required number is to be divisible by 7. Hence, we must choose the least value of k which will make (60k – 1) divisible by 7. Putting k equal to 1, 2, 3 etc. in succession, we find that k should be 2

∴ The required number = 60k – 1

= 60 × 2 – 1 = 119

25. (d); Required number

= (LCM of 12, 15, 20 & 54) + 4

= 540 + 4 = 544

26. (a); Required number of students = HCF of 1001 and 910

910)1001(1
910
91)910(10
910
0

Hence, HCF = 91

27. (d); Required time = LCM of 252, 308 and 198 s.

2	252, 308, 198
2	126, 154, 99
7	63, 77, 99
9	9, 11, 99
11	1, 11, 11
	1, 1, 1

∴ LCM = 2 × 2 × 7 × 9 × 11

$$= 2772 \text{ s} = \frac{2772}{60}$$

$$= 46\frac{1}{5} \text{ min} = 46 \text{ min } 12 \text{ s}$$

28. (c); Minimum distance each should cover, so that all can cover the distance in complete steps

= LCM of (63, 70, 77) = 6930 cm

29. (b); LCM of 15, 18, 21, 24

2	15, 18, 21, 24
2	15, 9, 21, 12
2	15, 9, 21, 6
3	15, 9, 21, 3
3	5, 3, 7, 1
5	5, 1, 7, 1
7	1, 1, 7, 1
	1, 1, 1, 1

LCM = 2 × 2 × 2 × 3 × 3 × 5 × 7 = 2520

Largest number of four digit = 9999

$$2520\overline{)9999}(3$$
$$\frac{7560}{2439}$$

Required number = 9999 – 2439 – 4 = 7556

$$\text{Where, } 4 = \begin{Bmatrix} 15-11=4 \text{ or} \\ 18-14=4 \text{ or} \\ 21-17=4 \text{ or} \\ 24-20=4 \end{Bmatrix}$$

30. (b); LCM of 9 and 6 = 18
Total numbers from 1 to 200 divisible by 18 = 11
Total numbers from 1 to 100 divisible by 18 = 5
∴ Required numbers from 100 to 200 divisible by 18 = 11 – 5 = 6

Difficult

1. (b); The largest possible length of the tape = HCF of 525, 1050, 1155 = 105
Hence (b) is the correct answer.

2. (c); The maximum capacity of the vessel = HCF of 1653, 2261 and 2527 = 19
Hence, (c) is the correct option.

3. (b); Minimum number of rows means max. number of trees per row, also equal number of trees per row is required so we need to find the HCF of 36, 144 and 234 to find the maximum number of trees in a row.
Thus HCF of 36, 144 and 234 = 18
Thus the number of rows =

$$= \frac{\text{Total no. of trees}}{\text{No. of trees in a row}} = \frac{36+144+234}{18} = 23$$

Hence (b) is correct answer.

4. (a); The required number must be divisible by the given numbers so it can be the LCM or its multiple number.
Now the LCM of 6, 40, 49 and 75
= 2 × 2 × 2 × 3 × 5 × 5 × 7 × 7
But the required number is a perfect square
Thus the LCM must be multiplied by 2 × 3 = 6.
Thus the required number
= (2 × 2 × 2 × 3 × 5 × 5 × 7 × 7) × (2 × 3)
= 2 × 2 × 2 × 2 × 3 × 3 × 5 × 5 × 7 × 7
= 176400

5. (b); The three bells toll together only at the LCM of the times they toll individually.
Thus the LCM of 48, 72 and 108 is 432 seconds.
Therefore all the bells will toll together at 6 : 07 : 12 AM
(∵ 432 seconds = 7 minutes 12 seconds)

6. (c); Since the difference between the divisors and the respective remainders is same.
Hence the least possible number
= (LCM of 18, 35 and 42) – 16
= 630 – 16 [∵ (18 – 2) = (35 – 19) = (42 – 26) = 16]
= 614

7. (a);

0.0005	⇒	5
0.0050	⇒	50
0.1500	⇒	1500
0.1750	⇒	1750
0.5000	⇒	5000
3.5000	⇒	35000

Then the HCF of 5, 50, 1500, 1750, 5000 and 35000 is 5.
So the HCF of the given number is 0.0005 (since there are four digits in all the adjusted (or equated) decimal places.)

8. (b); The required number = (LCM of 2, 3, 4, 5, 6) K + 1 = 7 l = 60K +1 =7l

$$\Rightarrow \frac{60k+1}{7} = l$$

Now put the least possible value of k such that l must be a positive integer. Hence at k = 5, l is an integer. Thus, the required value is 60 × 5 +1 = 301.

9. (b); The required time = LCM of 36, 40 and 48
= 720 seconds = 12 minutes
Hence, (b) is the right choice.

10. (c); Minimum number of corks =

$$\frac{403+434+465}{\text{HCF of }(403, 434, 465)} = \frac{1302}{31} = 42$$

11. (c); Time taken for each of three persons is respectively $\frac{6}{3}, \frac{6}{2\frac{1}{2}}$ and $\frac{6}{1\frac{1}{4}}$ hrs

i.e., $\frac{2}{1}, \frac{12}{5}$ and $\frac{24}{5}$ hrs.

So, it is required to find the LCM of

$\frac{2}{1}, \frac{12}{5}, \frac{24}{5} = \frac{24}{1} = 24\text{hr}$

Hence, (c) is the right choice.

12. (b); Let LCM be x and HCF be y.
According to the question,
LCM = 4 × HCF
$\Rightarrow$ x = 4y
According to the question,
LCM + HCF = 125
x + y = 125
Putting the value of x, we get
5y = 125
$\Rightarrow$ y = 25
$\therefore$ HCF = 25 amd LCM = 4 × 25 = 100
We know that, HCF × LCM
= First number × Second number

Second number $= \frac{\text{HCF} \times \text{LCM}}{\text{First number}}$

$= \frac{100 \times 25}{100} = 25$

13. (d); Let the numbers be 10a and 10b, where a and b are coprime
$\therefore$ LCM of 10a and 10b = 10ab
$\Rightarrow$ 10ab = 120 $\Rightarrow$ ab = 12
Possible pairs = (3, 4) or (1, 12)
Sum of the numbers :
(a, b) = (3, 4) :
(3×10)+ (4×10) = 30 + 40 = 70
(a, b) = (1, 12) :
(1 × 10) + (12 × 10)
= 10 + 120 = 130

14. (d); Let the original fraction be $\frac{x}{y}$.

According to the question,

$\frac{x-4}{y+1} = \frac{1}{6} \Rightarrow 6x - 24 = y + 1$

$\Rightarrow$ 6x– y= 25 ... (i)

Again, according to the question,

$\frac{x+2}{y+1} = \frac{1}{3}$

$\Rightarrow$ 3x + 6 = y +1

$\Rightarrow$ 3x – y = – 5 ... (ii)

On subtracting Eq. (ii) from Eq. (i), we get

$6x - y = 25$
$3x - y = -5$
$3x = 30$

$x = \frac{30}{3} = 10$

on putting the value of x in Eq. (i), we get
6 × 10 – y =25
$\Rightarrow$ – y = 25 – 60 $\Rightarrow$ – y = – 35
$\Rightarrow$ y =35
$\therefore$ x = 10 and y = 35
LCM of 10 and 35

5	10, 35
	2, 7

LCM = 2 × 7 × 5 = 70

15. (c); A makes one complete round in 5 / (5 / 2)

= 2 h $\left(\because \text{Speed} = \frac{\text{Distance}}{\text{Time}}\right)$

B completes in $\frac{5}{3}$h and C completes in $\frac{5}{2}$h .

Hence, the required time

= LCM of $\left(2, \frac{5}{3} \text{ and } \frac{5}{2}\right)$h

$= \frac{\text{LCM of } 2,5,5}{\text{HCF of } 3,2} = \frac{10}{1} = 10\text{h}$

Hence, A, B and C will meet after 10 h.

16. (b); By the Technique
First number × Second number = HCF × LCM

$\therefore$ Second number $= \frac{15 \times 300}{60} = 75$

17. (c); Let the numbers be 7a and 7b, where a and b are coprime. Now, LCM of 7a and 7b = 7ab
$\therefore$ 7ab = 140

ab = $\frac{140}{7} = 20$

Now, required values of a and b whose product is 20 are 4 and 5.
Numbers are 28 and 35 and they lie between 20 and 45.
Sum of the numbers = 28 + 35 = 63

18. (c); We know that
Numbers divisible by 4 and 6 will be multiples of the LCM of 4 and 6 i.e., 12
Now, numbers from 1 to 600 divisible by

$$12 = \frac{600}{12} - 1 = 49$$

(minus 1 because 600 is excluded)

Now, numbers divisible by 12 from 1 to 100

$$= \frac{100}{12} = 8$$

∴ Numbers divisible by between 100 and 600 = 49 – 8 = 41

19. (a);

2	18, 27, 36
2	9, 27, 18
3	9, 27, 9
3	3, 9, 3
3	1, 3, 1
	1, 1, 1

LCM = 2 × 2 ×3 × 3 × 3 = 108

Required number

= [LCM of (18, 27, 36)]-k = 108 –13=95

{where, k = 18 – 5 or 27 – 14 or 36 – 23 = 13}

20. (b); LCM of 16, 24, 30 and 36.

2	16, 24, 30, 36
2	8, 12, 15, 18
2	4, 6, 15, 9
2	2, 3, 15, 9
3	1, 3, 15, 9
3	1, 1, 5, 3
5	1, 1, 5, 1
	1, 1, 1, 1

LCM = 2 × 2 × 2 × 3 × 2 × 5 × 3 = 720

We know that largest five digit number is 99999

```
720)99999(138
     720
     2799
     2160
      6399
      5760
       639
```

Required number = (99999 – 639) + 10 = 99370

Previous Year (Memory Based)

1. (c); Here, first number = 2 ×44 = 88,
HCF = 44 and LCM = 264.
By the formula
1st number × 2nd number = HCF × LCM
⇒ 88 × 2nd number = 44 × 264
⇒ 2nd number $= \frac{264 \times 44}{88} = 132$
⇒ 2nd number = 132

2. (b); Let the common ratio = x
Then, numbers are 5x and 6x
Now, HCF of these two numbers is x
By the technique
LCM × HCF = Product of two numbers
⇒ 480 ×x = 5x ×6x ⇒ $480x = 30x^2$
∴ x = 16 ∴ HCF is 16

3. (d); We know that,
LCM of two coprimes is equal to their product,
Hence, LCM = 117

4. (c); If the numbers be 2x and 3x, then LCM of 2x and 3x = 6x
LCM = 48
∴ 6x = 48 ⇒ $x = \frac{48}{6} = 8$
∴ The numbers are (8 ×2 = 16) and (8 × 3= 24), respectively.
∴ Sum = 16 + 24 = 40

5. (d); Let the numbers be 4x and 5x.
∴ HCF = 8 = x
First number = 8 × 4 = 32
Second number = 8 × 5 = 40
∴ LCM of 32, 40 = 160

6. (b); Let the number be 2a and 2b, where a and b are coprime
∴ LCM = 2ab
2ab = 84
ab = 42 = 6 × 7
∴ Numbers are 12 and 14.
∴ Sum 12 + 14 = 26

7. (d); By the technique
HCF × LCM = First number × Second number
∴ Second number = $= \frac{8 \times 48}{24} = 16$

8. (c); Let the larger number be a.
Smaller number = a – 2
HCF × LCM = Product of two numbers
24 = a(a – 2)
⇒ $a^2 - 2a - 24 = 0$
⇒ $a^2 - 6a + 4a - 24 = 0$
⇒ a(a– 6)+4(a – 6)= 0
⇒ a = 6, - 4
But a ≠ –4 ∴ a = 6

9. (c); Let the numbers are ax and bx
where x = HCF = 15
ATQ,
ax × bx = 6300

$$ab = \frac{6300}{225} = 28$$

∴ possible pairs ⇒ 28 × 1, 7 × 4
Hence there are two pairs.

10. (d); Let LCM be x and HCF be y.
According to the question.
LCM = 20 × HCF
i.e., x = 20 y
and x + y = 2520
Putting the value of x, we get
20y +y = 2520
⇒ 21y = 2520

$$\Rightarrow \quad y = \frac{2520}{21} = 120$$

∴ LCM =x = 120 × 20 = 2400
LCM × HCF = Product of two numbers
2400 × 120 = 48 × x, x = 600

11. (b); Given fractions = $\frac{2}{4}, \frac{4}{5}$ and $\frac{6}{7}$

HCF of fractions = $\frac{\text{HCF of numerators}}{\text{LCM of denominators}}$

$$= \frac{\text{HCF}(2,4,6)}{\text{LCM}(3,5,7)} = \frac{2}{105}$$

12. (a); LCM of 28 and 42

2	28, 42
2	14, 21
7	7, 21
	1, 3

∴ LCM = 2 ×2 ×7 ×3= 84
HCF of 28 and 42
By Division method

$$28\overline{)42}(1 \quad 28 \quad 14\overline{)28}(2 \quad 28 \quad 0$$

∴ HCF = 14

$$\therefore \quad \text{Ratio} = \frac{\text{LCM}}{\text{HCF}} = \frac{84}{14} = \frac{6}{1} = 6:1$$

13. (a); LCM = $(x^2+6x+8)(x+1)$
or (x+4) (x+2) (x+1)
HCF = (x+1)
1st expression = $x^2+ 3x+2$
or (x+1) (x+2)
As we know that,
product of two expressions = LCM × HCF
(x+1) (x+2) × 2nd expression
= (x+4)(x+2)(x+1)(x+1)
2nd expressions

$$= \frac{(x+4)(x+2)(x+1)(x+1)}{(x+1)(x+2)}$$

= $(x + 4)(x + 1)= x^2 + 4x + x + 4 = x^2 + 5x + 4$

14. (c); Let the numbers be 3x and 4x.
LCM of 3x and 4x = 12x
Now, LCM = 84
Then, 12x = 84

$$\Rightarrow \quad x = \frac{84}{12} = 7$$

∴ Greatest number = 4x = 4 ×7 = 28

15. (b); Let the number be 29a and 29b, respectively where a and b are coprimes
LCM of 29a and 29b = 29ab
Now, 29ab = 4147

$$\therefore \quad ab = \frac{4147}{29} = 143$$

Thus, ab = 11 × 13
First number = (29 ×11) = 319
Second number = (29 ×13) = 377
∴ Sum of numbers = 319 + 377 = 696

16. (b); Factors of 11 and 385 are
11 = 11 × 1, 385 = 11 ×5 × 7
∴ LCM = 11 × 5 × 7 = 385
HCF = 11
First number = 11 × 5 = 55
Second number = 11 × 7 = 77
⇒ (11, 385) or (55, 77)

17. (d); HCF of the two digit numbers = 16
Hence, let the numbers be 16a and 16b.
where, a and b are coprimes.
Now, HCF × LCM = Product of two numbers.
⇒ 16a × 16b = 16 × 480

$$\Rightarrow \quad ab = \frac{16 \times 480}{16 \times 16} = 30$$

Possible pairs of a and b satisfying the condition ab = 30 are (3, 10), (5, 6), (1, 30), (2, 15). Since the

numbers are of 2 digit each.
Hence, required pair is (5, 6).
First number = 16 × 5 = 80
Second number = 16 × 6 = 96

18. (d); Greatest number that can exactly divide 200 and 320 = HCF of 200 and 320 = 40

200)320(1
200
120)200(1
120
80)120(1
80
40) 80(2
80
×

Hence the greatest number is 40.

19. (d); Required number = HCF of {(1868–1356), (2764–1868), (2764–1356)}
= HCF of (512, 896, 1408)

512)896(1
512
384)512(1
384
128)384(3
384
×

Hence, required number is 128.

20. (c); LCM of 5, 6, 7, 8 = 35 × 24 =840
Required number = 840x + 3, such that it is exactly divisible by 9.
By hit and Trial
for x = 2, it is divisible by 9.
Required number = 840x + 3 = 840 ×2 + 3 = 1683
(these type of questions can be solved with the help of given options)
(Out of all the given options, only 1683 is divisible by 9.)

21. (a); LCM of 12, 16 and 24

2	12, 16, 24
2	6, 8, 12
3	3, 4, 6
2	1, 4, 2
2	1, 2, 1
	1, 1, 1

= 2 × 2 × 2 × 2 × 3 = 48

48)9999(208
96
399
384
15

∴ Greatest four digit numbers divisible by 48
9999–15= 9984
∴ Required number = 9984–10 = 9974
(10 is the difference of each remainder)

22. (c); LCM of 9, 10 and 15

2	9, 10, 15
3	9, 5, 15
3	3, 5, 5
5	1, 5, 5
	1, 1, 1

∴ LCM = 2 × 3 × 3 × 5 = 90

90)1936(21
180
136
90
46

∴ Required number = 46 – 7 = 39

23. (a); Required number
= HCF of [| 25 – 73 | , | 73–97 | , | 97–25 |]
= HCF of {48, 24, 72}
HCF = 2 × 2 × 2 × 3= 24
∴ HCF = 24
∴ Largest number = 24

24. (c); LCM of 4, 5, 6, 7 and 8 = 840
Required number = 840x +2
By Hit and Trial
Putting x = 3
we get = 840x + 2 = 840 × 3+ 2 = 2522
2522 is a multiple of 13.

25. (a); Greatest number
= HCF of [(307 – 3), (330 – 7)]
= HCF of (304, 323)

304)323(1
304
19)304(16
304
×

∴ Required number = 19

26. (b); Least six digit number is 100000
LCM of 4, 6, 10, 15

2	4, 6, 10, 15
2	2, 3, 5, 15
3	1, 3, 5, 15
5	1, 1, 5, 5
	1, 1, 1, 1

∴ LCM = 2 × 2 × 3 × 5= 60

60)100000(1666
60
400
360
400
360
400
360
40

∴ Required number

= 100000 + (60 – 40) +2 = 100022

∴ Sum of the digits of

N = 1 +0 + 0 + 0 + 2 + 2 = 5

27. (d); LCM of 10, 16, 24

2	10, 16, 24
2	5, 8, 12
2	5, 4, 6
	5, 2, 3

∴ LCM of $2^2 \times 2^2 \times 5 \times 3$

[∵ powers must be equal for number to be perfect square]

∴ Required number

$= 2^2 \times 2^2 \times 5^2 \times 3^2 = 4 \times 4 \times 25 \times 9 = 3600$

28. (c); By the technique

Required number

= HCF of [(989 – 5), (1327 – 7)]

= HCF of (984, 1320)

$$\begin{array}{l} 984)1320(1 \\ \quad 984 \\ \quad 336)984(2 \\ \qquad 672 \\ \qquad 312)336(1 \\ \qquad\quad 312 \\ \qquad\quad 24)312(13 \\ \qquad\qquad 312 \\ \qquad\qquad \times \end{array}$$

∴ HCF = 24

∴ Required number = 24

29. (b); LCM of 15, 18, 21, 24

2	15, 18, 21, 24
2	15, 9, 21, 12
2	15, 9, 21, 6
3	15, 9, 21, 3
3	5, 3, 7, 1
5	5, 1, 7, 1
7	1, 1, 7, 1
	1, 1, 1, 1

LCM = 2 × 2 × 2 × 3 × 3 × 5 × 7 = 2520

Largest number of four digit = 9999

$$\begin{array}{l} 2520)9999(3 \\ \qquad 7560 \\ \qquad 2439 \end{array}$$

Required number = 9999 – 2439 – 4 = 7556

$$\text{where, } 4 = \begin{Bmatrix} 15 - 11 = 4 \text{ or} \\ 18 - 14 = 4 \text{ or} \\ 21 - 17 = 4 \text{ or} \\ 21 - 20 = 4 \end{Bmatrix}$$

30. (d); Required number = HCF of [(122–2),(243–3)]

i.e., HCF of (120, 240)

By Division Method

$$\begin{array}{l} 120)240(2 \\ \qquad 240 \\ \qquad \times \end{array}$$

∴ Required number = 120

Chapter 3

Surds and Indices

Index: $a \times a \times a \times a \times a$ .... m times

or $(a \times a \times a \times$ ....m times$) \times (a \times a \times a \times$ ....n times)

i.e. $a \times a \times a \times$ .... $(m + n)$ times

Important formulae : If $a > 0,\ a \neq 1$, m and n are integers then

(i) $a^m \times a^n = a^{m+n}$ (ii) $a^m \times a^n \times a^Q = a^{m+n+Q}$ (iii) $(a^m)^n = a^{mn}$ (iv) $\frac{a^m}{a^n} = a^{m-n}$

(v) $a^0 = 1$ (vi) $a^{-m} = \frac{1}{a^m}$ (vii) $a^{m^n} = a^{(m)^n}$ (viii) $(ab)^n = a^n b^n$

Surd : If 'a' is a rational number and n is a positive integer such the n^{th} root of 'a', i.e., $a^{1/n}$ $\sqrt[n]{a}$ is an irrational number, then $a^{1/n}$ is called a surd. In other words, an irrational root of a rational number is called a surd.

Example: $\sqrt{2},\ \sqrt[3]{4},\ \sqrt[4]{18},\ \sqrt[7]{4},\ \sqrt[3]{9}$ etc. are surds

i.e. we can say that every number expressed in a surd is an irrational number.

Types of surds:

(i) Pure Surd: $\sqrt{7},\ 3\sqrt{11},\ \sqrt[4]{125}$ are pure surds.

(ii) Mixed Surd: $3\sqrt{2},\ 7\sqrt[2]{11},\ \sqrt{32}$ are mixed surds.

(iii) Similar surds: $3\sqrt{3},\ \sqrt{3}$ and $6\sqrt{5},\ 7\sqrt{125} = 35\sqrt{5}$

order of surds: $\sqrt{7},\ \sqrt[3]{4},\ \sqrt[4]{8},\ \sqrt[5]{125}$ are respectively surds of order 2, 3, 4 and 5

Conjugate of surds : Two binomial surds which differ only in sign (+ or –) between the terms connecting them, are known as conjugate surds

Example : Conjugate of $5+\sqrt{7}$ is $5-\sqrt{7}$

Condition for two Surds to be equal : If a, b, c, d are all rational numbers and b and d are not perfect square then

$a+\sqrt{b} = c+\sqrt{d}$, i.e. a = c and b = d

Square root of surd of $a+\sqrt{b}$ form

$$\sqrt{\quad +\sqrt{b}} = \sqrt{\frac{a+\sqrt{a^2-b}}{2}} + \sqrt{\frac{a-\sqrt{a^2-b}}{2}}, \qquad \sqrt{\quad -\sqrt{b}} = \sqrt{\frac{\sqrt{\ ^2-b}}{2}} - \sqrt{\frac{a-\sqrt{a^2-b}}{2}}$$

Important formulae:

(i) $a^{-P} = \frac{1}{a^P}$ (ii) If $a^y = n$ then $a = (n)^{1/y}$ (iii) If $a^x = b^y$ then $a = (b)^{y/x}$

(iv) $x^n = a,\ x = \sqrt[n]{a}$ (v) $\sqrt[n]{a} = a^{1/n}$ (vi) $\left(\sqrt[n]{a}\right)^m = a^{\frac{m}{n}}$

(vii) $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$

Types of Questions

1. The greatest among $\sqrt{7}-\sqrt{5}, \sqrt{5}-\sqrt{3},$ $\sqrt{9}-\sqrt{7}, \sqrt{11}-\sqrt{9}$

Sol. On rationalising

$$\frac{\left(\sqrt{7}-\sqrt{5}\right)\times\left(\sqrt{7}+\sqrt{5}\right)}{\sqrt{7}+\sqrt{5}}, \frac{\left(\sqrt{5}-\sqrt{3}\right)\times\left(\sqrt{5}+\sqrt{3}\right)}{\sqrt{5}+\sqrt{3}},$$

$$\frac{\left(\sqrt{9}-\sqrt{7}\right)\times\left(\sqrt{9}+\sqrt{7}\right)}{\sqrt{9}+\sqrt{7}}, \frac{\left(\sqrt{11}-\sqrt{9}\right)\times\left(\sqrt{11}+\sqrt{9}\right)}{\left(\sqrt{11}+\sqrt{9}\right)}$$

$$= \frac{2}{\sqrt{7}+\sqrt{5}}, \frac{2}{\sqrt{5}+\sqrt{3}}, \frac{2}{\sqrt{9}+\sqrt{7}}, \frac{2}{\sqrt{11}+\sqrt{9}}$$

smallest one = $\frac{2}{\sqrt{11}+\sqrt{9}} = \sqrt{11}-\sqrt{9}$

Greatest one = $\frac{2}{\sqrt{5}+\sqrt{3}} = \sqrt{5}-\sqrt{3}$

2. Greatest among the following numbers $\sqrt[3]{9}, \sqrt{3}, \sqrt[4]{16}, \sqrt[6]{80}$

Sol. LCM 3, 2, 4, and 6 = 12

$9^{12/3}$, $3^{12/2}$, $16^{12/4}$, and $80^{12/6}$

9^4, 3^6, 16^3, 80^2

6561, 729, 4096, 6400

i.e. largest one = $\sqrt[3]{9}$.

3. By how much does $\sqrt{12}+\sqrt{18}$ exceed $\sqrt{3}+\sqrt{2}$?

Sol. Required value will be the differnce between $\sqrt{12}+\sqrt{18}$ and $\left(\sqrt{3}+\sqrt{2}\right)$

$= \left(\sqrt{12}+\sqrt{18}\right)-\left(\sqrt{3}+\sqrt{2}\right)$

$= \left(2\sqrt{3}+3\sqrt{2}\right)-\left(\sqrt{3}+\sqrt{2}\right)$

$= \sqrt{3}+2\sqrt{2}$

4. Is $2^x = 3^y = 6^{-z}$ then $\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$ is equal to

Sol. Let $2^x = 3^y = 6^{-z} = k$

i.e. $2 = k^{1/x}$, $3 = k^{1/y}$, $6 = k^{-1/z,}$

$2 \times 3 = 6$

$k^{1/x} \times k^{1/y} = k^{-1/z}$, $k^{1/x+1/y} = k^{-1/z}$

$\frac{1}{x}+\frac{1}{y} = -\frac{1}{z}$, $\frac{1}{x}+\frac{1}{y}+\frac{1}{z} = 0$

5. Find the value of $3+\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{3}+3}+\frac{1}{\sqrt{3}-3}$

Sol. $3+\frac{1\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}}+\frac{1}{(\sqrt{3}+3)}\times\frac{3-\sqrt{3}}{3-\sqrt{3}}+\frac{1}{\left(\sqrt{3}-3\right)}\times\frac{3+\sqrt{3}}{3+\sqrt{3}}$

$= 3+\frac{\sqrt{3}}{3}+\frac{3-\sqrt{3}}{6}-\frac{3+\sqrt{3}}{6} = \frac{18}{6} = 3$

6. The value of $\sqrt{5+2\sqrt{6}}-\frac{1}{\sqrt{5+2\sqrt{6}}}$

Sol. Expression $\sqrt{5+2\sqrt{6}}-\frac{1}{\sqrt{5+2\sqrt{6}}}$

$= \frac{\left(\sqrt{5+2\sqrt{6}}\right)^2-1}{\sqrt{5+2\sqrt{6}}} = \frac{5+2\sqrt{6}-1}{\sqrt{5+2\sqrt{6}}}$

$= \frac{4+2\sqrt{6}}{\sqrt{3}+\sqrt{2}}\times\left(\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}\right)$

$= 4\sqrt{3}+6\sqrt{2}-4\sqrt{2}-4\sqrt{3} = 2\sqrt{2}$

7. If a = 64 and b = 289, then the value of $\left(\sqrt{\sqrt{a}+\sqrt{b}}-\sqrt{\sqrt{\ \ }\ \ \sqrt{a}}\right)^{1/2}$

Sol. $\left(\sqrt{\sqrt{\ \ }+\sqrt{289}}-\sqrt{\ \sqrt{\ \ }+\sqrt{289}}\right)^{1/2}$

$\left(\sqrt{8+17}-\sqrt{-8+17}\right)^{1/2} = (5-3)^{1/2} = \sqrt{2}$

Foundation

Questions

1. If $3^x - 3^{x-1} = 18$, then x^x is equal to
(a) 3 (b) 8
(c) 27 (d) 216

2. If $a^{2x+2} = 1$, where a is a positive real number other than 1, then x = ?
(a) – 2 (b) – 1
(c) 0 (d) 1

3. $\frac{\left[(12)^{-2}\right]^2}{\left[(12)^2\right]^{-2}} = ?$
(a) 12 (b) 4.8
(c) $\frac{12}{144}$ (d) 1

4. Value of ? in expression
$7^{8.9} \div (343)^{1.7} \times (49)^{4.8} = 7^?$ is
(a) 13.4 (b) 12.8
(c) 11.4 (d) 9.6

5. If $\{(2^4)^{1/2}\}^? = 256$, find the value of '?'.
(a) 1 (b) 2
(c) 4 (d) 8

6. $(16)^9 \div (16)^4 \times 16^3 = (16)^?$
(a) 6.75 (b) 8
(c) 10 (d) 12

7. $(42 \times 229) \div (9261)^{1/3} = ?$
(a) 448 (b) 452
(c) 456 (d) 458

8. Evaluate $(0.00032)^{2/5}$.
(a) $\frac{1}{625}$ (b) $\frac{1}{225}$
(c) $\frac{1}{125}$ (d) $\frac{1}{25}$

9. If $\frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}} = a + b\sqrt{35}$, then the value of $(a - b)$ is:
(a) 5 (b) 6
(c) 8 (d) None of these

10. If P = 124, then $\sqrt[3]{P(P^2+3P+3)+1} = ?$
(a) 5 (b) 7
(c) 123 (d) 125

11. If $\left(\frac{p}{q}\right)^{n-1} = \left(\frac{q}{p}\right)^{n-3}$, then the value of n is:
(a) $\frac{1}{2}$ (b) $\frac{7}{2}$
(c) 1 (d) 2

12. Value of ? in $\sqrt[3]{512} \div \sqrt[4]{16} + \sqrt{576} = ?$ is:
(a) 24 (b) 31
(c) 28 (d) 18

13. Value of $3 + \frac{1}{\sqrt{3}} + \frac{1}{3+\sqrt{3}} + \frac{1}{\sqrt{3}-3} - 3$ is:
(a) $3+\sqrt{3}$ (b) 3
(c) 1 (d) 0

14. $16^{5/4} = ?$
(a) 64 (b) 31
(c) 32 (d) 33

15. $\left(\frac{32}{243}\right)^{-3/5} = ?$
(a) $\frac{27}{8}$ (b) $\frac{27}{7}$
(c) $\frac{27}{6}$ (d) $\frac{27}{2}$

16. Find the value of $(243)^{0.16} \times (243)^{0.04}$
(a) 0.16 (b) $\frac{1}{3}$
(c) 3 (d) 0.04

17. $17^{3.5} \times 17^{7.3} \div 17^{4.2} = 17^?$
(a) 8.4 (b) 8
(c) 6.6 (d) 6.4

18. If $289 = 17^{\frac{x}{5}}$, then x = ?
(a) 16 (b) 8
(c) 10 (d) $\frac{2}{5}$

19. $\left[\left\{\left(-\frac{1}{2}\right)^2\right\}^{-2}\right]^{-1} = ?$
(a) $\frac{1}{16}$ (b) 16
(c) $-\frac{1}{16}$ (d) – 16

20. Find the value of $(10)^{200} \div (10)^{196}$.
(a) 10000 (b) 1000
(c) 100 (d) 100000

21. If $\left(\frac{1}{5}\right)^{3a} = 0.008$, find the value of $(0.25)^a$.
(a) 20.5 (b) 22.5
(c) 0.25 (d) 6.25

22. Value of $\sqrt{\quad + 2\sqrt{6}} - \frac{1}{\sqrt{5-2\sqrt{6}}}$ is:
(a) $2\sqrt{2}$ (b) 0
(c) $2\sqrt{3}$ (d) $\sqrt{5}-1$

23. $\frac{1}{1+x^{b-a}+x^{c-a}} + \frac{1}{1+x^{a-b}+x^{c-b}} + \frac{1}{1+x^{b-c}+x^{a-c}} = ?$
(a) x^{a-b-c} (b) 1
(c) 0 (d) 3

24. $[P^{(b-c)}]^{(b+c)} \cdot [P^{(c-a)}]^{(c+a)} \cdot [P^{(a-b)}]^{(a+b)} = ?$
(a) 0 (b) P^{abc}
(c) 1 (d) P^{a+b+c}

25. If $\sqrt{5+\sqrt[3]{x}} = 3$, then the value of x is:
(a) 64 (b) 125
(c) 9 (d) 27

26. $\left[\left(\sqrt[5]{x^{\frac{-3}{5}}}\right)^{-\frac{5}{3}}\right]^5 = ?$
(a) x^5 (b) x^{-5}
(c) x (d) $\frac{1}{x}$

27. $\left(\sqrt{\quad\sqrt{\quad\sqrt{\quad+...}}}\right) \div 2^2 = ?$
(a) 0 (b) 1
(c) 2 (d) 8

28. If $a^x = b$, $b^y = c$ and $xyz = 1$, then what is the value of c^z?
(a) a (b) b
(c) ab (d) $\frac{a}{b}$

29. If $16 \times 8^{n+2} = 2^m$, then m is equal to:
(a) n + 8 (b) 2n + 10
(c) 3n + 2 (d) 3n + 10

30. If $x = \frac{\sqrt{3}+1}{\sqrt{3}-1}$ and $y = \frac{\sqrt{3}-1}{\sqrt{3}+1}$, then $x^2 + y^2$ is equal to:
(a) 14 (b) 13
(c) 15 (d) 10

Moderate

1. $\frac{16 \times 2^{n+1} - 4 \times 2^n}{16 \times 2^{n+2} - 2 \times 2^{\quad +}} = ?$
(a) 1 (b) 2
(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

2. $\frac{2^{2^3} \div (2^2)^3 \times 2^{-2}}{4^{2^3} \div (4^2)^3 \times 4^{-2}} = ?$
(a) 2 (b) $\frac{1}{2}$
(c) 1 (d) – 2

3. $\frac{(0.6)^0 - (0.1)^{-1}}{\left(\frac{3}{2^3}\right)^{-1}\left(\frac{3}{2}\right)^3 + \left(-\frac{1}{3}\right)^{-1}} + \left(\frac{8}{27}\right)^{-\frac{1}{3}} = ?$
(a) 1 (b) 0
(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

4. If $\frac{(81)^{4x} \times (27)^x \times 9^7}{(729)^{x+2}} = 3^9$, then find the value of x.
(a) $\frac{6}{13}$ (b) $\frac{5}{13}$
(c) $\frac{8}{13}$ (d) $\frac{7}{13}$

5. If $2^a + 3^b = 17$ and $2^{a+2} - 3^{b+1} = 5$, the values of a and b are :
(a) a = 2, b = 3 (b) a = 4, b = 6
(c) a = 3, b = 2 (d) a = – 3, b = – 2

6. $\left[\frac{3\sqrt{2}}{\sqrt{6}+\sqrt{3}} - \frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}} + \frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}}\right]$ is simplified to
(a) 0 (b) 1
(c) $\sqrt{3}$ (d) $\sqrt{6}$

7. Find the value of $\frac{2(\sqrt{2}+\sqrt{6})}{3\sqrt{2+3}}$.
(a) $\frac{1}{3}$ (b) $\frac{2}{3}$
(c) $-\frac{2}{3}$ (d) $\frac{4}{3}$

8. If $x=\frac{\sqrt{5}-2}{\sqrt{5}+2}$, then x^4+x^{-4} is
(a) a surd
(b) a rational number but not an integer
(c) an integer
(d) an irrational number but not a surd

9. The simplified value of $\sqrt{\frac{19+8\sqrt{3}}{7-4\sqrt{3}}}$ is :
(a) $11-6\sqrt{3}$ (b) $11+6\sqrt{3}$
(c) $10+5\sqrt{3}$ (d) $10-5\sqrt{3}$

10. The simplified value of $\left(28+10\sqrt{3}\right)^{\frac{1}{2}}-\left(7-4\sqrt{3}\right)^{-\frac{1}{2}}$ is:
(a) 1 (b) 2
(c) 3 (d) 4

11. The simplified value of $\sqrt{6-4\sqrt{3}+\sqrt{16-8\sqrt{3}}}$ is :
(a) $\sqrt{3}$ (b) $\sqrt{3}+1$
(c) $\sqrt{3}-1$ (d) $\sqrt{3}+\sqrt{2}$

12. If $x=\sqrt{\frac{\quad+2\sqrt{6}}{5-2\sqrt{6}}}$, then the value of $x^2(x-10)^2$ is
(a) 0 (b) 1
(c) – 1 (d) 2

13. The simplified value of $\frac{\sqrt{\quad-\sqrt{7}}}{\sqrt{\quad+3\sqrt{7}}-2\sqrt{2}}$ is:
(a) 1 (b) 2
(c) – 1 (d) – 2

14. $\frac{(625)^{6.25}\times(25)^{2.6}}{(625)^{6.75}\times(5)^{1.2}}=?$
(a) 5 (b) 10
(c) 15 (d) 25

15. The simplified value of $\frac{2+\sqrt{3}}{2-\sqrt{3}}$ is :
(a) $7+4\sqrt{3}$ (b) $7-4\sqrt{3}$
(c) $7+2\sqrt{3}$ (d) $7-2\sqrt{3}$

16. The simplified value of $\frac{(0.3)^{\frac{1}{3}}\left(\frac{1}{27}\right)^{\frac{1}{4}}(9)^{\frac{1}{6}}(0.81)^{\frac{2}{3}}}{(0.9)^{\frac{2}{3}}(3)^{-\frac{1}{2}}(243)^{-\frac{1}{4}}}$ is :
(a) 2.2 (b) 2.7
(c) 2.4 (d) 2.6

17. $\frac{2^{n+4}-2\times 2^n}{2\times 2^{n+3}}+2^{-3}=?$
(a) 1 (b) 2
(c) $\frac{1}{2}$ (d) $\frac{1}{3}$

18. If $10^{0.48}=x$, $10^{0.7}=y$ and $x^z=y^2$, then z = ?
(a) $1\frac{11}{12}$ (b) $2\frac{11}{12}$
(c) $3\frac{11}{12}$ (d) $-2\frac{11}{12}$

19. The simplified value of $(27)^{-\frac{2}{3}}+\left[\left(2^{-\frac{2}{3}}\right)^{-\frac{5}{3}}\right]^{-\frac{9}{10}}$ is :
(a) $\frac{11}{18}$ (b) $-\frac{11}{18}$
(c) $\frac{17}{18}$ (d) $-\frac{17}{18}$

20. Find the value of $\sqrt{8+3\sqrt{7}}-\sqrt{7+3\sqrt{5}}$.
(a) $\frac{\sqrt{14}+\sqrt{10}}{2}$ (b) $\frac{\sqrt{14}-\sqrt{10}}{2}$
(c) $\frac{\sqrt{14}+\sqrt{10}}{4}$ (d) $\frac{\sqrt{14}-\sqrt{10}}{4}$

21. Find the value of $\sqrt{38+5\sqrt{3}}+\sqrt{\quad-\sqrt{5}}$.
(a) $\frac{5\sqrt{6}-\sqrt{10}}{2}$ (b) $\frac{5\sqrt{6}+\sqrt{10}}{2}$
(c) $\frac{5\sqrt{6}-\sqrt{10}}{4}$ (d) $\frac{5\sqrt{6}+\sqrt{10}}{4}$

22. Find the value of $\sqrt{4-\sqrt{7}}+\sqrt{8+\quad\sqrt{\quad}}$
(a) $\sqrt{2}\left(\sqrt{7}+1\right)$ (b) $\sqrt{2}+1$
(c) 1 (d) $\sqrt{2}+2$

23. If $\sqrt{5}=2.236$ and $\sqrt{10}=3.162$, then the value of
$\frac{15}{\sqrt{10}+\sqrt{20}+\sqrt{40}-\sqrt{5}-\sqrt{80}}$ is:
(a) 4.398 (b) 5.398
(c) 4.938 (d) 5.938

24. $\frac{\left(2^{2n}-3.2^{2n-2}\right)\left(3^n-2.3^{n-2}\right)}{3^{n-4}\left(4^{n+3}-2^{2n}\right)}=?$
(a) $\frac{1}{2}$ (b) $\frac{1}{4}$
(c) $-\frac{1}{2}$ (d) $\frac{1}{6}$