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ADVANCE
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by **Rakesh Yadav**
Selected
Excise Inspector

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My Beloved Daughters
Unnati Yadav & Pragati Yadav

जीतने की इच्छा सभी में होती है, मगर जीतने के लिए तैयारी करने की इच्छा बहुत कम लोगों में होती है।

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Editor-in-chief
Karan Chaudhary

Preface

Nothing thrills a writer more than the success of his book. With this book, I hope to reach a much wider section of the student community and others, who relentlessly compete for various Government – jobs.

I am thankful to Almighty and my family (My parents, brother, wife, daughters and son), who extended their help in various invisible ways. I sincerely hope, the book **ADVANCE MATHS** will meet a good response. I would humbly appreciate suggestions, doubt, etc. concerned with this book at the following.

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Content

CHAPTER
01

UNIT DIGIT

Last Digit of number is called Unit Digit

1 2 3 4
$\downarrow$
unit digit

In This no. 4 is unit digit.

The unit digit of the Resultant value depends upon The unit digits of all participating numbers.

Ex.1: 23 + 34 + 46 + 78 = 181, unit digit of 181.

Sol. $\therefore$ unit digit = 1

It is clear that the unit digit of the Resultant value 181 depends upon the unit digits 3, 4, 6, 8

3 + 4 + 6 + 8 = 21

So, units digit = 1

Ex.2: What is the unit digit of ?

31 × 37 × 36 × 46 × 89

sol. 31 × 37 × 36 × 46 × 89

Unit digit = 1, 7, 6, 6, 9

multiply the unit digits = 1 × 7 × 6 × 6 × 9

$\Rightarrow$ 1 × 7 = 7

$\Rightarrow$ 7 × 6 = 42

$\Rightarrow$ 2 × 6 = 12

$\Rightarrow$ 2 × 9 = 18

unit digit = 8

Ex.3: What is the unit digit of?

31 × 33 × 37 × 39 × 43

Sol. 31 × 33 × 37 × 39 × 43

multiply the unit digits

= 1 × 3 × 7 × 9 × 3

unit digit = 7

Ex.4: What is the unit digit of ?

91 × 93 × 95 × 96 × 97 × 98

Sol. multiply the unit digit

1 × 3 × 5 × 6 × 7 × 8 = 0

Ex.5: Find the unit digit of 135 × 136 × 170

Sol. The unit digits = 5, 6, 0

multiply the units digit

= 5 × 6 × 0

= unit digit = 0

Ex.6: Find the unit digit at the product of all the odd prime numbers.

sol. The prime numbers are 3, 5, 7, 11, 13, 17, ... etc.

Now we know that if 5 is multiplied by any odd number it always gives the last digit 5. So the required unit digit will ne '5',

Ex.7: Find the unit digit of 584 × 328 × 547 × 613

Sol. The unit digits = 4, 8, 7, 3

multiplying the unit digits

= 4 × 8 × 7 × 3

= unit digit = 2

Ex.8: Find the unit digit of the product of all the even numbers

Sol. The even number are

2, 4, 6, 8, 10, 12, ... etc.

Now we know that if '0' is multiplied by any number it always gives the last digit 0. so the required unit digit will be 0.

Ex.9: Find the unit digit 4!

Sol. 4! = 4 × 3 × 2 × 1 = 2<u>4</u>

unit digit = 4

Ex.10: Find the unit digit 5!

Sol. 5! = 5 × 4 × 3 × 2 × 1 = 120

unit digit = 0

* Factorial 5 and more than 5 express gives unit digit 0.

Unit digit when 'N' is Raised to a power

unit digit of 0, 1, 5 and 6 has any power (odd or even) no change

Ex.11: $(3765)^{437}$

unit digit = $(5)^{437} = 5$

Ex.12: $(6736)^{32567}$

unit digit = $(6)^{32567} = 6$

Ex.13: $(32541)^{325}$

unit digit = $(1)^{325} = 1$

* **4 and 9** $\rightarrow$

4: even power $\rightarrow$ unit digit $\rightarrow$ 6; odd power $\rightarrow$ 4

9: even power $\rightarrow$ unit digit $\rightarrow$ 1; odd power $\rightarrow$ 9

Ex.14: Find the unit place $(67354)^{1237}$

Sol. $(67354)^{1237}$

unit digit = $(4)^{1237} = (4)^{\text{odd power}}$

So, unit digit = 4

Ex.15: Find the unit place $(3259)^{1214}$

Sol. $(3259)^{1214}$

unit digit = $(9)^{1214} = (9)^{\text{even power}}$

unit digit = 1

Ex.16: Find the unit place $(6734)^{312}$

Sol. $(6734)^{312}$

unit digit = $(4)^{312} = (4)^{\text{even}}$

unit digit = 6

Rule of (2, 3, 7 and 8) $\rightarrow$

unit digit when 'N' is raised to a power

If the value of the power is

Power $\longrightarrow$

unit digit $\downarrow$	1 or 4n+1	2 or 4n+2	3 or 4n+3	4 or 4n+4
2	2	4	8	6
3	3	9	7	1
7	7	9	3	1
8	8	4	2	6

here n $\rightarrow$ Natural No.

If those number which unit digit 2, 3, 7 and 8. $\rightarrow$

all unit digit have cyclicity 4

Ex.18: Find the unit place 3^{35}

Sol. $3^{35} = 3^{32} \times 3^{3}$

Break the power form of 4n

$(3^4)^8 \times 3^3 = (......1) \times (...7)$

unit place = 1 × 7 = 7

Ex.19: Find the unit place $(127)^{39}$

Sol. $(127)^{39}$

unit place = $(7)^{39}$

$= (7)^{36} \times (7)^3 = (7^4)^9 \times (7)^3$

$= (...1) \times (....3)$

unit place = 1 × 3 = 3

Ex.20: Find the unit place $(678)^{562}$

Sol. $(678)^{562}$

unit digit $= (8)^{562}$

$= (8)^{560} \times (8)^2$

$= (8^4)^{140} \times (8)^2$

$= (...6) \times (...4)$

unit digit = 6 × 4 = 2<u>4</u> = **4**

Ex.21: Find the unit place $(327)^{640}$

Sol. $(327)^{640}$

unit digit = $(7)^{640}$

640 is multiple of 4

then = $(7^4)^{160}$

unit digit = $(1)^{160}$ = **1**

Ex.22: Find the unit digit of $(2137)^{753}$

Sol. $(2137)^{753}$

unit digit = $(7)^{753}$

= $(7)^{752} \times 7^1$

= $(7^4)^{188} \times 7^1$

= $(....1) \times 7$

unit digit = 1×7 = **7**

Ex.23: Find the unit digit of $(13)^{2003}$

Sol. $(13)^{2003}$

unit digit = $(3)^{2003}$

= $3^{2000} \times 3^3$

= $(3^4)^{500} \times 3^3$

= $(....1)^{500} \times 27$

= $1 \times 27 = 27$

unit digit = **7**

Ex.24: Find the unit digit of $(22)^{23}$

Sol. $(22)^{23}$

unit digit = $(2)^{23}$

= $(2)^{20} \times 2^3 = (2^4)^5 \times 8$

= $(....6)^5 \times 8$

unit digit = $6 \times 8 = 48$ = **8**

Ex.25: Find the unit digit of $(37)^{105}$

Sol. $(37)^{105}$

unit digit = $(7)^{105}$

= $(7)^{104} \times 7^1$

= $(7^4)^{26} \times 7^1$

= $(...1)^{26} \times 7$

unit digit = 1×7 = **7**

Ex.26: Find the unit place

$(23)^{21} \times (24)^{22} \times (26)^{23} \times (27)^{24} \times (25)^{25}$

Sol. $(23)^{21} \times (24)^{22} \times (26)^{23} \times (27)^{24} \times (25)^{25}$

unit digit = $(3)^{21} \times (4)^{22} \times (6)^{23} \times (7)^{24} \times (5)^{25}$

Break the power multiple of 4

$= \underbrace{3^{20} \times 3^1}_{} \times \underbrace{4^{22}}_{\text{even power}} \times \underbrace{6^{23}}_{\text{same digit}} \times (7^4)^6 \times \underbrace{5^{25}}_{\text{same digit}}$

$= 3 \times 6 \times 6 \times 1 \times 5$

unit digit = 0

Note:- unit digit = even × 5 = '0'

Ex.27: Find the unit place

$(235)^{215} + (314)^{326} + (6736)^{213} + (3167)^{112}$

unit digit

$\underbrace{(5)^{215}}_{\text{Same}} + \underbrace{(4)^{326}}_{\text{even power}} + \underbrace{(6)^{213}}_{\text{Same}} + \underbrace{(7)^{112}}_{(7^4)^{28}}$

$= 5 + 6 + 6 + 1$

unit digit = 18 = 8

Ex.28: Find the unit place of

$$\frac{12^{55}}{3^{11}} + \frac{8^{48}}{16^{18}}$$

Sol. $\frac{12^{55}}{3^{11}} + \frac{8^{48}}{16^{18}}$

$= \frac{(3 \times 4)^{55}}{3^{11}} + \frac{(2^3)^{48}}{(2^4)^{18}}$

$= \frac{3^{55} \times 4^{55}}{3^{11}} + \frac{2^{144}}{2^{72}}$

$= 3^{44} \times 4^{55} + 2^{72}$

unit digt = $(.....1) \times (......4) + 6$

= 4 + 6 = 10,

unit digit = 0

EXERCISE

1. Find the unit digit of 584 × 389 × 476 × 786
(a) 7 (b) 3 (c) 4 (d) 6

2. Find the unit digit of
641 × 673 × 677 × 679 × 681
(a) 9 (b) 3 (c) 6 (d) 7

3. Find the unit digit of
$(5627)^{153} \times (671)^{230}$
(a) 7 (b) 9 (c) 3 (d) 1

4. Find the unit digit of
$(3625)^{333} \times (4268)^{645}$
(a) 6 (b) 3 (c) 4 (d) 0

5. Find the unit digit
$(3694)^{1793} \times (615)^{317} \times (841)^{941}$
(a) 5 (b) 3 (c) 4 (d) 0

6. Find the unit digit of
$(7^{95} - 3^{58})$
(a) 7 (b) 3 (c) 4 (d) 0

7. Find the unit place of
$(17)^{1999} + (11)^{1999} - (7)^{1999}$
(a) 0 (b) 1 (c) 2 (d) 7

8. Find the unit digit of
$3^6 \times 4^7 \times 6^3 \times 7^4 \times 8^2 \times 9^5$
(a) 6 (b) 9 (c) 0 (d) 2

9. Find the unit digit of 111! (factorial 111).
(a) 0 (b) 1 (c) 5 (d) 3

10. The last digit of the number obtained by multiplying the numbers 81 × 82 × 83 × 84 × 85 × 86 × 87 × 88 × 89 will be
(a) 0 (b) 9 (c) 7 (d) 2

11. Find the units digit of the expression $25^{6251} + 36^{528} + 22^{853}$
(a) 4 (b) 3 (d) 6 (d) 5

12. Find the units digit of the expression $55^{725} + 73^{5810} + 22^{853}$
(a) 4 (b) 0 (c) 6 (d) 5

13. Find the units digit of the expression $11^1 + 12^2 + 13^3 + 14^4 + 15^5 + 16^6$.
(a) 1 (b) 9 (d) 7 (d) 0

14. Find the last digit of the number $1^3 + 2^3 + 3^3 + 4^3 .... + 99^3$.
(a) 0 (b) 1 (d) 2 (d) 5

15. Unit digit in $(264)^{102}+(264)^{103}$ is:
(a) 0 (b) 4 (c) 6 (d) 8

16. Unit digit $[(251)^{98}+(21)^{29}-(106)^{100}+(705)^{35}-(16)^4+259]$ is
(a) 1 (b) 4 (c) 5 (d) 6

17. The unit digit in the expansion of $(2137)^{754}$ is
(a) 1 (b) 3 (c) 7 (d) 9

18. The digit in unit's place of the product $(2153)^{167}$ is :
(a) 1 (b) 3 (c) 7 (d) 9

19. The digit in unit's place of the product
$(2464)^{1793} \times (615)^{317} \times (131)^{491}$ is
(a) 0 (b) 2 (c) 3 (d) 5

20. What will be the unit digit in the product of 7^{105} ?
(a) 5 (b) 7 (c) 9 (d) 1

21. what is the number of unit place in $(329)^{78}$
(a) 1 (b) 7 (c) 9 (d) 3

22. unit digit of the number $(22)^{23}$ is:
(a) 4 (b) 6 (c) 6 (d) 8

23. The unit digit in the product $(122)^{173}$ is:
(a) 2 (b) 4 (c) 6 (d) 8

24. The unit digit in the sum of $(124)^{372}$ + $(124)^{373}$ is :
(a) 5 (b) 4 (c) 2 (d) 0

25. The last digit of $(1001)^{2008}$ + (1002) is:
(a) 0 (b) 3 (c) 4 (d) 6

26. Find the unit digit in the product: $(4387)^{245} \times (621)^{72}$.
(a) 1 (b) 2 (c) 5 (d) 7

27. The unit digit of the expression $25^{6251} + 36^{528} + 73^{54}$ is
(a) 6 (b) 5 (c) 4 (d) 0

28. The unit's digit in the product $7^{71} \times 6^{63} \times 3^{65}$ is
(a) 1 (b) 2 (c) 3 (d) 4

29. The last digit of 3^{40} is
(a) 1 (b) 3 (c) 7 (d) 9

30. The digit in unit's place of the number $(1570)^2 + (1571)^2 + (1572)^2 + (1573)^2$ is :
(a) 4 (b) 1 (c) 2 (d) 3

31. The unit digit in $3 \times 38 \times 537 \times 1256$ is
(a) 4 (b) 2 (c) 6 (d) 8

32. The unit digit in the product $(2467)^{153} \times (341)^{72}$ is
(a) 1 (b) 3 (c) 7 (d) 9

33. The unit digit in the product $(6732)^{170} \times (6733)^{172} \times (6734)^{174} \times (6736)^{176}$
(a) 1 (b) 3 (c) 4 (d) 5

34. Find the unit digit of the product of all the prime number between 1 and 99999
(a) 9 (b) 7 (c) 0 (d) N.O.T.

35. Find the unit digit of the product of all the elements of the set which consists all the prime numbers greater than 2 but less than 222.
(a) 4 (b) 5 (c) 0 (d) N.O.T.

36. Find the last digit of $222^{888} + 888^{222}$
(a) 2 (b) 6 (c) 0 (d) 8

37. Find the last digit of $32^{32^{32}}$
(a) 4 (b) 8 (c) 6 (d) 2

38. Find the last digit of the expression:
$1^2 + 2^2 + 3^2 + 4^2 + \ldots\ldots + 100^2$.
(a) 0 (b) 4 (c) 6 (d) 8

39. Find the unit digit of $1^1 + 2^2 + 3^3 + \ldots\ldots 10^{10}$.
(a) 9 (b) 7 (c) 0 (d) N.O.T.

40. Find the unit digit of $13^{24} \times 68^{57} + 24^{13} \times 57^{68} + 1234 + 5678$.
(a) 4 (b) 7 (c) 0 (d) 8

41. The unit digit of the expression
$$\frac{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10}{100}$$
(a) 7 (b) 9 (c) 8 (d) N.O.T.

42. Find the unit digit of the expression $888^{9235!} + 222^{9235!} + 666^{2359!} + 999^{9999!}$.
(a) 5 (b) 9 (c) 3
(d) None of these

43. The last digit of the following expression is: $(1!)^1 + (2!)^2 + (3!)^3 + (4!)^4 + \ldots + (10!)^{10}$
(a) 4 (b) 5 (c) 6 (d) 7

44. The last 5 digits of the following expression will be
$(1!)^5 + (2!)^4 + (3!)^3 + (4!)^2 + (5!)^1 + (10!)^5 + (100!)^4 + (1000!)^3 + (10000!)^2 + (100000!)$
(a) 45939 (b) 00929
(c) 20929
(d) Can't determined

45. The unit digit of the following expression $(1!)^{99} + (2!)^{98} + (3!)^{97} + (4!)^{96} + \ldots\ldots (99!)^1$ is:
(a) 1 (b) 3 (c) 7 (d) 6

46. The unit digit of $(12345k)^{72}$ is 6. The value of k is:
(a) 8 (b) 6 (c) 2
(d) all of these

47. The unit digit of the expression $(1!)^{1!} + (2!)^{2!} + (3!)^{3!} + \ldots.. + (100!)^{100!}$
(a) 0 (b) 1 (c) 2 (d) 7

48. The last digit of the expressioin $4 \times 9^2 \times 4^3 \times 9^4 \times 4^5 \times 9^6 \times \ldots\ldots \times 4^{99} \times 9^{100}$ is :
(a) 4 (b) 6 (c) 9 (d) 1

49. The last digit of the expression $4 + 9^2 + 4^3 + 9^4 + 4^5 + 9^6 + \ldots\ldots + 4^{99} + 9^{100}$ is:
(a) 0 (b) 3 (c) 5
(d) None of these

50. The unit digit of $2^{3^4} \times 3^{4^5} \times 4^{5^6} \times 5^{6^7} \times 6^{7^8} \times 7^{8^9}$ is:
(a) 0 (b) 5
(c) Can't be determined
(d) None of these

ANSWER KEY

1. (d)	6. (c)	11. (b)	16. (b)	21. (a)	26. (d)	31. (d)	36. (c)	41. (c)	46. (d)
2. (a)	7. (b)	12. (c)	17. (d)	22. (d)	27. (d)	32. (c)	37. (c)	42. (b)	47. (d)
3. (a)	8. (a)	13. (b)	18. (c)	23. (a)	28. (d)	33. (c)	38. (a)	43. (d)	48. (b)
4. (d)	9. (a)	14. (a)	19. (a)	24. (d)	29. (a)	34. (c)	39. (b)	44. (b)	49. (a)
5. (d)	10. (a)	15. (a)	20. (b)	25. (b)	30. (a)	35. (b)	40. (a)	45. (c)	50. (a)

SOLUTION

1. (d) $584 \times 389 \times 476 \times 786$
unit digit 4, 9, 6, 6
Multiplying the unit digit
$= 4 \times 9 \times 6 \times 6$
unit digit = 6

2. (a) $641 \times 673 \times 677 \times 679 \times 681$
unit digit = 1, 3, 7, 9, 1
Multiply the unit digit
$= 1 \times 3 \times 7 \times 9 \times 1$
$= 21 \times 9 = 189$
unit digit = 9

3. (a)$(5627)^{153} \times (671)^{230}$
unit digit $(7)^{153} \times (1)^{230}$
$= (7)^{152} \times 7^1 \times 1$
$= (7^4)^{38} \times 7 \times 1$
$= (\ldots.1)^{38} \times 7$
unit digit $= 1 \times 7 = 7$

4. (d) $(3625)^{333} \times (4268)^{645}$
unit digit $(5)^{333} \times (8)^{645}$
$= 5 \times (8)^{644} \times 8^1$
$= 5 \times (8^4)^{161} \times 8^1$
$= 5 \times (6)^{161} \times 8$
unit digit $= 5 \times 6 \times 8 = 240 = \mathbf{0}$

5. (d)$(3694)^{1793} \times (615)^{317} \times (841)^{941}$
unit digit $(4)^{1793} \times (5)^{317} \times (1)^{941}$
$4^{\text{odd power}} = 4$
$5^n = 5$
$4 \times 5 \times 1 = 20$
Hence, unit digit = 0

6. (c) $7^{95} - 3^{58}$

$= 7^{92} \times 7^3 - 3^{56} \times 3^2$

$= (7^4)^{23} \times 343 - (3^4)^{14} \times 9$

$= (.....1)^{23} \times 3 - (.....1)^{14} \times 9$

unit digit = (.....3) – (......9)

= 13 – 9 = **4**

7. (b) $(17)^{1999} + (11)^{1999} - (7)^{1999}$

unit digit = $(7)^{1999} + (1)^{1999} - (7)^{1999}$

$\because$ $(7)^{1999} - (7)^{1999}$ gives = 0

Then, unit digit = 1

8. (a) Unit digit = $3^6 \times 4^7 \times 6^3 \times 7^4 \times 8^2 \times 9^5$

The unit digit of $3^6 = 3^4 \times 3^2 = 9$

The unit digit of $4^7 = 4$

The unit digit of $6^3 = 6$

The unit digit of $7^4 = 1$

The unit digit of $8^2 = 4$

The unit digit of $9^5 = 9^4 \times 9^1 = 9$

multiply the unit digits = 9 × 4 × 6 × 1 × 4 × 9

unit digit = 6

9. (a) 111! = 1 × 2 × 3 × 4 × 5 × .... × 100 × 111

Since there is product of 5 and 2 hence it will give zero as the unit digit.

Hence the unit digit of 111! is 0 (zero).

10. (a) 81 × 82 × 83 × 84 × 85 × 86 × 87 × 88 × 89

Unit digits = 1, 2, 3, 4, ...,9

Product of unit digits

= 1 × 2 × 3 ×...........×9

Because 5 multiply any even no. Then

we gets unit digit = 0

11. (b) $25^{6251} + 36^{528} + 22^{853}$

unit digit = $(5)^{6251} + (6)^{528} + (2)^{853}$

$\downarrow \quad \downarrow$

unit digit = $(....5) + (....6) + (2)^{852} \times 2^1$

$= (....5) + (....6) + (2^4)^{213} \times 2$

$= 5 + 6 + (6)^{213} \times 2$

Sum of unit digit = 5 + 6 + 6 × 2

= 5 + 6 + 12 = 23

Hence, unit digit = 3

12. (c) $55^{725} + 73^{5810} + 22^{853}$

unit digit = $(5)^{725} + (3)^{5810} + (2)^{853}$

$= (...5) + (3^4)^{1452} \times 3^2 + (2^4)^{213} \times 2^1$

$= 5 + (1)^{1452} \times 9 + (16)^{213} \times 2^1$

Sum of unit digit = 5 + 1 × 9 + 6 × 2 = 5 + 9 + 12 = 26

unit digit = 6

13. (b) $11^1 + 12^2 + 13^3 + 14^4 + 15^5 + 16^6$

unit digit = $(1)^1 + (2)^2 + (3)^3 + (4)^4 + (5)^5 + (6)^6$

Sum of unit digit = 1 + 4 + 7 + 6 + 5 + 6 = 29

unit digit = 9

14. (a) $1^3 + 2^3 + 3^3 + 4^3 .... + 99^3$

Sum of cube of natural no.

$$= \left(\frac{n(n+1)}{2}\right)^2 = \left(\frac{99(99+1)}{2}\right)^2$$

$$= \left(\frac{99 \times 100}{2}\right)^2 = (99 \times 50)^2$$

$= (4850)^2$

Unit digit = 0

15. (a) $(264)^{102} + (264)^{103}$

unit digit

$4^1 \rightarrow 4 \rightarrow 4$

$4^2 \rightarrow 16 \rightarrow 6$

$4^3 \rightarrow 64 \rightarrow 4$

Rule: When 4 has odd power, then unit digit is: 4

When 4 has even power, then unit digit is 6

$(264)^{102} + (264)^{103}$

$\downarrow \qquad \downarrow$

$(4)^{102} + (4)^{103}$

$\downarrow \qquad \downarrow$

(even power) (odd power)

unit digit 6 + 4 = 10 $\rightarrow$ **0**

Alternate :

$\Rightarrow (264)^{102} + (264)^{103}$

$\Rightarrow (264)^{102}(1 + 264)$

$\Rightarrow (264)^{102} \times 265$

Multiplication of 5 & 2 = 0

Hence, unit digit is 0.

16 (b) $[(251)^{98} + (21)^{29} - (106)^{100} + (705)^{35} - (16)^4 + (259)]$

unit place of 1,5 and 6 will remain same

$= [(1)^{98}+(1)^{29}-(6)^{100}+(5)^{35}-(6)^4+9]$

= [1 + 1 – 6 + 5 – 6 + 9]

$\Rightarrow$ 16 – 12 = 4

Hence, unit digit = 4

17. (d) $(2137)^{754}$

$= (7)^{754}$ will give unit digit

		unit digit
$7^1 = 7$	$\rightarrow$	7
$7^2 = 49$	$\rightarrow$	9
$7^3 = 343$	$\rightarrow$	3
$7^4 = 2401$	$\rightarrow$	1
$7^5 = 16807$	$\rightarrow$	7

4

& will repeat

754 divide by 4 = $\frac{754}{4}$

So, remainder is 2

$7^2 =$ **9**

Unit Place = 9

18. (c) $(2153)^{167}$

unit digit = 3^{167}

unit digit

$3^1 \rightarrow 3 \rightarrow 3$

$3^2 \rightarrow 9 \rightarrow 9$

$3^3 \rightarrow 27 \rightarrow 7$

$3^4 \rightarrow 81 \rightarrow 1$

This cycle will continue

$\Rightarrow$ divide the power of 3 by 4

$\frac{167}{4} \Rightarrow$ remainder is 3

$3^3 \Rightarrow 7$

Unit digit = 7

19. (a) $(2464)^{1793} \times (615)^{317} \times (131)^{491}$

$4^1 \rightarrow 4 \rightarrow 4$

$4^2 \rightarrow 16 \rightarrow 6$

$4^3 \rightarrow 64 \rightarrow 4$

So odd power of 4 will have 4 as unit digit and even power will have 6 as unit digit 5 and 1 have same unit digits respectively

$(2464)^{1793} \times (615)^{317} \times (131)^{491}$

$\downarrow$

odd power

$\downarrow$

unit digit $\rightarrow 4 \times 5 \times 1 = 20$

$\Rightarrow 20 \Rightarrow 0$ unit digit

20. (b) 7^{105}

$\Rightarrow \quad 7^1 \rightarrow 7 \rightarrow 7$

$\Rightarrow \quad 7^2 \rightarrow 49 \rightarrow 9$

$\Rightarrow \quad 7^3 \rightarrow 343 \rightarrow 3$

$\Rightarrow \quad 7^4 \rightarrow 2401 \rightarrow 1$

Divide power of 7 by 4

$\frac{105}{4} \rightarrow$ remainder = $1 \Rightarrow 7^1$ is left

unit digit = 7

21. (a) $(329)^{78}$

$\Rightarrow$ If power of 9 is odd, then unit digit number be 9. If power is even then unit digit number be 1.

Hence, unit digit = 1

22. (d) $(22)^{23}$

	Result	unit digit
2^1	2	2
2^2	4	4
2^3	8	8
2^4	16	6
2^5	32	2

Cycle completes

So divide power of 22 by 4

$\frac{23}{4}$ = remainder 3

$2^3 = 8$

unit digit = 8

23. (a) $(122)^{173}$

Unit digit

$2^1 \to 2 \to 2$
$2^2 \to 4 \to 4$
$2^3 \to 8 \to 8$
$2^4 \to 16 \to 6$ Cycle
$2^5 \to 32 \to 2$

$2^{173} = 2^{4\times43+1} = 2^{4\times43}\times2 = 16^{43}\times2$
$= 6^{43}\times2 = 6\times2 = 12$

unit digit = 2

24. (d) $(124)^{372}$ $(124)^{373}$

$\downarrow$ $\downarrow$

4^{372} 4^{373}

When 4 has odd power then unit digit is 4 when 4 has even power then unit digit is 6

$4^1 \to 4 \to 4$
$4^2 \to 16 \to 6$
$4^3 \to 64 \to 4$
$4^4 \to 256 \to 6$

4^{372} 4^{373}
$\downarrow$ $\downarrow$
$6 + 4 = 10$

last (unit) digit = 0

25. (b) $(1001)^{2008} + 1002$

$\downarrow$

Unit digit $\to 1^{2008} + 1002$

Unit digit will be 1 in case of 1 respective of power

$\Rightarrow$ 1+ 1002 = 1003

unit digit (last digit) = 3

26. (d)

unit place

$7^1 \to 7 \to 7$
$7^2 \to 49 \to 9$
$7^3 \to 343 \to 3$
$7^4 \to 2401 \to 1$ Cycle

$(4387)^{245} \times (621)^{72}$
$\downarrow$
$(7)^{245} \times (1)^{72}$
$\downarrow$
$(7)^{4\times61+1} \times 1$
$\downarrow$
$(7)^{4\times61}\times7 \times 1$
$\downarrow$
$(1)^{61}\times7 \times 1$

unit digit = 7

27. (d) 5 always gives unit digit 5 and 6 always gives unit digit 6

unit digit

$3^1 \to 3 \to 3$
$3^2 \to 9 \to 9$
$3^3 \to 27 \to 7$ Cycle
$3^4 \to 81 \to 1$

$25^{6251} + 36^{528} + 72^{54}$
$\downarrow$ $\downarrow$ $\downarrow$
$5^{6251} + 6^{528} + 3^{54}$
$\downarrow$ $\downarrow$ $\downarrow$ $54/4$ = r = 2

unit digit $\to 5 + 6 + 3^2$

$= 5 + 6 + 9 = 20 = 0$

Hence, unit digit = 0

28. (d) $7^{71} \times 6^{63} \times 3^{65}$

$\downarrow$ $\downarrow$ $\downarrow$

unit place 7^3 6^3 3^1

$\downarrow$ $\downarrow$ $\downarrow$

unit digit $\Rightarrow 3 \times 6 \times 3 = 54$

$\Rightarrow 4$

29. (a) 3^{40} :

Divide = $\frac{40}{4} \Rightarrow$ remainder = 0

Unit digit

$3^1 \to 3 \to 3$
$3^2 \to 9 \to 9$
$3^3 \to 27 \to 7$ Cycle
$3^4 \to 81 \to 1$

Hence, unit digit of 3^{40} of completing all cycle = 1

30. (a)

$(1570)^2 + (1571)^2 + (1572)^2 + (1573)^2$
$\downarrow$ $\downarrow$ $\downarrow$ $\downarrow$
unit digit $\to 0^2 + 1^2 + 2^2 + 3^2$
$\downarrow$ $\downarrow$ $\downarrow$ $\downarrow$
$0 + 1 + 4 + 9 = 14$

unit digit = 4

31. (d) $3 \times 38 \times 537 \times 1256$
$\times 24 \times 28 \times 48$

Note:- Always multiply only unit digit of first no. to second and product's unit digit no. with 3rd no. Again product of last's unit digit to fourth and so on.

Hence, unit digit = 8

32. (c) $(2467)^{153} \times (341)^{72}$

$\downarrow$ $\downarrow$

$(7)^{153} \times (1)^{72}$

$\downarrow$

[153/4= remainder = 1]

$\Rightarrow$ $7^1 \times 1 = 7$

		Result	Unit digit
7^1	=	7	7
7^2	=	49	9
7^3	=	343	3
7^4	=	2401	1

Hence, unit digit = 7

33. (c) $(6732)^{170} \times (6733)^{172} \times (6734)^{174} \times (6736)^{176}$

unit digit = $(2)^{170} \times (3)^{172} \times (4)^{174} \times (6)^{176}$
$= (2^4)^{42} \times 2^2 \times (3^4)^{43} \times (4)^{174} \times (6)^{176}$
$= (\ldots6) \times 4 \times (\ldots1) \times (\ldots6) \times (\ldots6)$

Multiplication of unit digit
$= 6 \times 4 \times 1 \times 6 \times 6 = 864$

Hence, unit digit = **4**

34. (c) The set of prime number S = {2, 3, 5, 7, 11, 13, ...}

Since there is one 5 and one 2 which gives 10 after multiplying mutually, it means the unit digit will be zero.

Hence, unit digit = 0

35. (b) The set of required prime number = The set of reqired prime number
= {3, 5, 7, 11, ...}

Since there is no any even number is the set so when 5 will multiply with any odd number, it will always give 5 as the last digit.

Hence the unit digit will be 5.

36. (c) The last digit of the expression will be same as the last digit of $2^{888} + 8^{222}$.

Now the last digit of 2^{888} is 6 and the last digit of the 8^{222} is 4.

$\therefore$ 6 + 4 = 10.

Hence, unit digit = 0

37. (c) Find the last digit of $2^{32^{32}}$

But $2^{32^{32}} = 2^{32 \times 32 \times 32 \,\ldots\ldots\, \times 32 \text{ times}}$

$\Rightarrow$ $2^{32^{32}} = 2^{4 \times 8 \times (32 \times 32 \,\ldots\ldots\, \times 31 \text{ times})}$

$\Rightarrow$ $2^{32^{32}} = 2^{4n}$

where n = 8 × (32 × 32 .... × 32 times)

Again $2^{4n} = (16)^n \Rightarrow$ unit digit is 6, for every $n \in N$

Hence, the required unit digit = 6

38. (a) Sum of square natural number = $\frac{n(n+1)(2n+1)}{6}$

Here, n = 100

$= \frac{100\times101\times201}{6} = 338350$

Then, Unit digit = 0

39. (b) Find the unit digit of $1^1 + 2^2 + 3^3 + \ldots\ldots 10^{10}$.

The unit digit of $1^1 = 1$

The unit digit of $2^2 = 4$

The unit digit of $3^3 = 7$

The unit digit of $4^4 = 6$

The unit digit of $5^5 = 5$

The unit digit of $6^6 = 6$

The unit digit of $7^7 = 3$

The unit digit of $8^8 = 6$

The unit digit of $9^9 = 9$

The unit digit of $10^{10} = 0$

Thus the unit digit of the given expression will be 7.

($\therefore$ 1 + 4 + 7 + 6 + 5 + 6 + 3 + 6 + 9 = 47)

40. (a) The unit of 3^{24} is 1

The unit digit of 8^{57} is 8

The unit digit of 4^{13} is 4

The unit digit of 7^{68} is 1

So the resultant value of the unit digits

= 1 × 8 + 4 × 1 + 4 + 8

= 8 + 4 + 4 + 8 = 24

Thus the unit digit of the whole expression is 4.

41. (c) Since in the numerator of the product of the expression there will be 2 zeros at the and thase two zeros will be cancelled by 2 zeros of the denominator. Hence finally we get a non-zero unit digit in the expression.

Now, $\dfrac{1\times2\times3\times4\times5\times6\times7\times8\times9\times10}{100}$

$= \dfrac{1\times2^1\times3^1\times2^2\times5^1\times2^1\times 3^1\times7^1\times2^3\times3^2\times2^1\times5^1}{5^2\times2^2}$

$= \dfrac{1\times2^8\times3^4\times5^2\times7^1}{5^2\times2^2}$

$= 1 \times 2^6 \times 3^4 \times 7$

Therefore, the unit digit of the given expression will be same as that of $1 \times 2^6 \times 3^4 \times 7$.

Now, The unit digit of $1 \times 2^6 \times 3^4 \times 7$ is 8.

($\therefore$ the product of unit digits of 1, 2^6, 3^4, 7 si 1 × 4 × 1 × 7 = 28)

Hence, the unit digit of $\dfrac{10!}{100}$ is 8.

42. (b) First of all we find the unit digit individually of all the four terms,

So, the unit digit of $888^{9235!}$ is equal to the unit digit of $8^{9235!}$

Now, the unit digit of $8^{9235!}$ equal to the unit of 8^4 (since 9235! is divisible by 4), which is 6.

unit digit of $(888)^{9235!} = (8)^{4n} = 6$

unit digit of $(222)^{9235!} = (2)^{4n} = 6$

unit digit of $(666)^{2359!} = (6)^{\text{anypower}} = 6$

unit digit of $(999)^{9999!} = (9)^{\text{even power}} = 1$

Thus the unit digit of the expression is 9. ($\therefore$ 6 + 6 + 6 + 1 = 19)

43. (d) The unit digit of the given expression will be equal to the unit digit of the sum of the unit digits of every term of the expression.

Now, The unit digit of $(1!)^2 = 1$

The unit digit of $(2!)^2 = 4$

The unit digit of $(3!)^3 = 6$

The unit digit of $(4!)^4 = 6$

The unit digit of $(5!)^5 = 0$

The unit digit of $(6!)^6 = 0$

Thus the last digit of the $(7!)^7$, $(8!)^8$, $(9!)^9$, $(10!)^{10}$ will be zero.

So, the unit digit of the given expression = 7

($\therefore$ 1+4+6 + 6 + 0 + 0 + 0 + 0 + 0 = 17)

44. (b) The last digit of $(1!)^5 = 1$

The last digit of $(2!)^4 = 16$

The last digit of $(3!)^3 = 216$

The last digit of $(4!)^2 = 576$

The last digit of $(10!)^5 = 00000$

The last digit of $(100!)^4 = 00000$

$(1000!)^3 = 00000$

$(10000!)^2 = 00000$

$(100000!)^1 = 00000$

Thus the last 5 digits of the given expression = 00929

[$\therefore$ 1 + 16 + 216 + 576 + 120 + 00000 + 00000 + 00000 + 00000 + 00000 + = 00929]

45. (c) $(1!)^{99} + (2!)^{98} + (3!)^{97} + (4!)^{96} + \ldots\ldots (99!)^1$

unti digit $(1!)^{99} = (1!)^{99} = 1$

unit digit $(2!)^{98} = 1 \times 2 = (2)^{98} = 4$

unit digit $(3!)^{97} = 1 \times 2 \times 3 = (6)^{97} = 6$

unit digit=$(4!)^{96}$ =1 × 2 × 3 × 4=$(4)^{96}$ = 6

unit digit=$(5!)^{95}$=1×2×3×4× 5 = $(0)^{95}$ = 0

....

....

same unit digit $(99!)^1$ = (1×2....99) = $(0)^1$ = 0

Then, Sum of unit digit = 1 + 4 + 6 + 6 + 0 + 0 +.... 0 = 17

unit digit = **7**

46. (d) unit digit $(12345k)^{72} = 6$

if we put the value of k = 2, 6, 8

Then we get unit digit = 6

47. (d) $(1!)^{1!} + (2!)^{2!} + (3!)^{3!} + \ldots\ldots + (100!)^{100!}$

unit digit $(1!)^{1!} = 1^1 = 1$

unit digit $(2!)^{2!} = (2)^2 = 4$

unit digit $(3!)^{3!} = (6)^6 = 6$

unit digit $(4!)^{4!} = (4)^{24} = 6$

unit digit $(5!)^{5!} = (0)^{120} = 0$

....

....

unit digit $(100!)^{100!}$

$\Rightarrow$ $= (0)^{1\times2\times3\times\ldots\ldots\times100} = 0$

Sum of unit digit = 1 + 4 + 6 + 6 + 0 + 0 + 0 = 17

unit digit = 7

48. (b) $4 \times 9^2 \times 4^3 \times 9^4 \times 4^5 \times 9^6 \times \ldots\ldots \times 4^{99} \times 9^{100}$

unit digit $4^1 = 4$

unit digit $9^2 = 1$

unit digit $4^3 = 4$

unit digit $9^4 = 1$

unit digit $4^5 = 4$

....

....

unit digit $4^{99} = 4$

unit digit $9^{100} = 1$

Then multiply the unit digit

$\underline{4} \times 1 \times \underline{4} \times 1 \times \underline{4} \times 1 \ldots\ldots \underline{4} \times 1$

Pair of 4 × 1 (4) is equal 50

we can say this expression = 4^{50}

Then, unit place = 6

49. (a) $4 + 9^2 + 4^3 + 9^4 + 4^5 + 9^6 + \ldots\ldots + 4^{99} + 9^{100}$

unit digit $4^1 = 4$

unit digit $9^2 = 1$

unit digit $4^3 = 4$

unit digit $9^4 = 1$

unit digit $4^5 = 4$

....

....

unit digit $4^{99} = 4$

unit digit $9^{100} = 1$

Then,

Sum of unit place

4 + 1 + 4 + 1 + 4+ 1.......4 + 1

Pair of 4 + 1 (5) is equal to 50

We can say this expression

= 50 × 5 = 250

unit digit = 0

50. (a) $2^{3^4} \times 3^{4^5} \times 4^{5^6} \times 5^{6^7} \times 6^{7^8} \times 7^{8^9}$

We know unit digit of

$2^{3^4} = 2^{81} = 2$

unit digit of $5^{6^7} = 5$

Then we know that when even number is multplied by 5 then we get unit place = '0'

So, last digit = 0

CHAPTER

02

NUMBER OF ZEROES

Number of zeroes in an Expression zero:- zero will be formed by 2 and 5

Ex. $10 = 2 \times 5$

$100 = 2^2 \times 5^2$

$1000 = 2^3 \times 5^3$

$\Rightarrow$ We can say that for 'n' number of zeroes at end of the product. We need exactly 'n' combinations of 5 and 2

Ex.1 Find the number of zeroes at the end of the product:-
$5 \times 7 \times 9 \times 2 \times 11$

Sol. $5 \times 7 \times 9 \times 2 \times 11$

In this product we see

Number of 2's = 1

Number of 5's = 1

Number of pair 2's and 5's = 1

$\therefore$ Number of zero = 1

Ex.2 Find the number of zeroes at the end of the product:- $12 \times 27 \times 63 \times 113 \times 1250 \times 24 \times 650$

Sol. $\underline{12} \times 27 \times 63 \times 113 \times \underline{1250} \times \underline{24} \times \underline{650}$

Break the numbers form of 2 and 5 multiple

In this series 27, 63 & 113 are not multiple of 2 & 5.

$\therefore$ The multiple of 2 & 5 are 12, 1250, 24 & 650

$\Rightarrow$ $12 = 2 \times 2 \times 3 = 2^2 \times 3$

$\Rightarrow$ $1250 = 2 \times 5 \times 5 \times 5 \times 5 = 2^1 \times 5^4$

$\Rightarrow$ $24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$

$\Rightarrow$ $650 = 2 \times 5 \times 5 \times 13 = 2^1 \times 5^2 \times 13$

$\underline{2^2} \times 3 \times 27 \times 63 \times 113 \times \underline{2^1} \times \underline{5^4} \times \underline{2^3} \times 3 \times \underline{2^1} \times \underline{5^2} \times 13$

Number of 2's = 7

Number of 5's = 6

$\Rightarrow$ Number of pair 2's and 5's = 6 there are 7 two's and 6 fives. Hence we shall be able to form only 6 pairs of 2 and 5, Hence there will be 6 zeroes at the end of the product of numbers.

Ex.3 Find the number of zeroes at the end of the product:-
$1 \times 3 \times 5 \times 7 \times 9 \ldots\ldots.. 97 \times 99$

Sol. $1 \times 3 \times 5 \times 7 \times 9 \ldots\ldots.. 97 \times 99$

In this series the number of zero and the end of the product is "0". Because there is no even number present in this series while it is necessary to be 2 and 5 for the Zero

The highest power of k that can exactly divided n! we divide n by k, n by k^2, n by k^3 and so on till we get $\left[\frac{n}{k^x}\right]$ equal to 1 an then add up as.

$$\left[\frac{n}{k}\right]+\left[\frac{n}{k^2}\right]+\left[\frac{n}{k^3}\right]+\left[\frac{n}{k^4}\right]+\ldots+\left[\frac{n}{k^x}\right]$$

Ex.4 Find the largest power of 5 contained in 124!

Sol. $\left[\frac{124}{5}\right]+\left[\frac{124}{5^2}\right] = 24 + 4 = 28$

[We cannot do it further since 124 is not divisible by 5^3]

Hence, there are 28 times 5 alternate as a factor in 124!

Alternate:-

Divide successive quotients till we get 0 as the last quotient

$$\begin{array}{r|l} 5 & 124 \\ \hline 5 & 24 \rightarrow \\ \hline & 4 \rightarrow \end{array} \Big\} \text{28 (add up all the quotients)}$$

Ex.5 Find the largest power of 3 that can divide 270!

Sol. $\left[\frac{270}{3}\right]+\left[\frac{270}{3^2}\right]+\left[\frac{270}{3^3}\right]+\left[\frac{270}{3^4}\right]$

$+\left[\frac{270}{3^5}\right] = 90 + 30 + 10 + 3 + 1 = 134$

Hence, there are 134 times 3 involved as a factor in 270!

Alternate:-

Divide successive quotients till we get 0 as the last quotient

$$\begin{array}{r|l} 3 & 270 \\ \hline 3 & 90 \rightarrow \\ \hline 3 & 30 \rightarrow \\ \hline 3 & 10 \rightarrow \\ \hline 3 & 3 \rightarrow \\ \hline 3 & 1 \rightarrow \\ \hline & 0 \end{array} \Bigg\} \text{134 (add up all the quotients)}$$

* Alternate method is easier than first.

Ex.6 Find the largest power of 2 that can contained in:-
$1 \times 2 \times 3 \times 4 \ldots\ldots.. 22$?

Sol. $1 \times 2 \times 3 \times 4 \ldots\ldots.. 22$

$$\begin{array}{r|l} 2 & 22 \\ \hline 2 & 11 \rightarrow \\ \hline 2 & 5 \rightarrow \\ \hline 2 & 2 \rightarrow \\ \hline 2 & 1 \rightarrow \\ \hline & 0 \end{array} \Bigg\} 19$$

Number of 2's = 11 + 5 + 2 + 1 = 19

Hence, there are 19 times 2 involved as a factor in 22!

Ex.7 Find the largest power of 5 that can contained in
$1 \times 2 \times 3 \times 4 \ldots\ldots.. 41 \times 42$

Sol.
$$\begin{array}{r|l} 5 & 42 \\ \hline 5 & 8 \rightarrow \\ \hline 5 & 1 \rightarrow \\ \hline & 0 \end{array} \Big\} 9$$

Hence, there are 9 times 5 involved as a factor in 42!

Ex.8 Find the largest power of 7 that can exactly divide 777!

Sol.
$$\begin{array}{r|l} 7 & 777 \\ \hline 7 & 111 \rightarrow \\ \hline 7 & 15 \rightarrow \\ \hline 7 & 2 \rightarrow \\ \hline & 0 \end{array} \Bigg\} \text{128 (add up all the quotients)}$$

Thus the highest power of 7 is 128 by which 777! can be divided.

Ex.9 Find the number of zeroes at the end of the product 10!

Sol. $10! = 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times \underline{10}$

2×5

* In this type expression it is clear that in any factorial value, the number of 5's will always be lesser then the number of 2's In this condition. all we need to do is to count the number of 5's

Number of 5's = 2

Number of 2's = 5

But pair of 2's and 5's are =2

Then,

Number of zero's = 2

Alternate:-

Highest power of 5's in 10!

$\frac{10}{5} = 2$

Number of 5's = 2

Then number of zeroes = 2

Ex.10 Find the number of zeroes at the end of the product 100!

Sol.
$$\begin{array}{r|l} 5 & 100 \\ 5 & 20 \rightarrow \\ 5 & 4 \rightarrow \\ & 0 \end{array} \Big\} \text{ 24 (add up all the quotients)}$$

Number of 5's = 20 + 4 = 24

then number of zeroes = 24

Ex.11 Find the number of zeroes at the end of the product 1000!.

Sol. 1 × 2 × 3 × .... 999 × 1000
$$\begin{array}{r|l} 5 & 1000 \\ 5 & 200 \rightarrow \\ 5 & 40 \rightarrow \\ 5 & 8 \rightarrow \\ 5 & 1 \rightarrow \\ & 0 \end{array} \Bigg\} \text{ 249}$$

The highest power of Number 5's = 200 + 40 + 8 + 1= 249

then number of zeroes at the end of the product = 249

Ex.12 Find the number of zeroes at the end of the product

1 × 3 × 5 × 7 .... 73 × 1024

Sol. 1 × 3 × 5 × 7 .... 73 × 1024

Number of 5's from 1 to 73
$$\begin{array}{r|l} 5 & 73 \\ 5 & 14 \rightarrow \\ 5 & 2 \rightarrow \\ & 0 \end{array} \Big\} \text{ 16 (add up all the quotients)}$$

Total number of 5's = 14 + 2 = 16

we know that

1024 = 2^{10}

number of 2 = 10

number of pairs (2 and 5) = 10

then number of zeroes 10

Ex.13 Find the number of zeroes at the end of the product

12 × 13 × 14 84

Sol. In this types expression first for us complete the sereis

$\underline{1} \times \underline{2} \times \underline{3}$ .... $\underline{11}$ × 12 × 13 .... 84 - $\underline{1} \times \underline{2} \times \underline{3}$...... $\underline{11}$

Number of zero (1 to 84)
$$\begin{array}{r|l} 5 & 84 \\ 5 & 16 \rightarrow \\ 5 & 3 \rightarrow \\ & 0 \end{array} \Big\} \text{ 19}$$

Number of zero (1 to 11)
$$\begin{array}{r|l} 5 & 11 \\ 5 & 2 \\ & 0 \end{array} \Big\} \text{ 2}$$

Number of zeroes = 19 – 2 = 17

Ex.14 Find the number of zeroes at the end of the product

512 × 513 1120

Sol. $\underline{1} \times \underline{2} \times \underline{3}$ .... $\underline{511}$ × 512 × 513 ....1120 – $\underline{1} \times \underline{2} \times \underline{3}$ .... $\underline{511}$
$$\begin{array}{r|l} 5 & 1120 \\ 5 & 224 \rightarrow \\ 5 & 44 \rightarrow \\ 5 & 8 \rightarrow \\ 5 & 1 \rightarrow \\ & 0 \end{array} \Bigg\} \text{ 277}$$
$$\begin{array}{r|l} 5 & 511 \\ 5 & 102 \rightarrow \\ 5 & 20 \rightarrow \\ 5 & 4 \rightarrow \\ & 0 \end{array} \Bigg\} \text{ 126}$$

Number of zeroes = 277 - 126 = 151

Ex.15 Find the number of zeroes at the end of the product

$1^5 \times 2^5 \times 3^5$ .... 32^5

Sol. In this type every second terms has power of 2's. It means power of 2's more than that of 5 So count the power of 5's

power of 5's = total power of 5's

$5^5 = (1\times 5)^5 = 1^5 \times 5^5 = 5$

$10^5 = (2 \times 5)^5 = 2^5 \times 5^5 = 5$

$15^5 = (3 \times 5)^5 = 3^5 \times 5^5 = 5$

$20^5 = (4 \times 5)^5 = 4^5 \times 5^5 = 5$

$25^5 = (5 \times 5)^5 = 5^5 \times 5^5 = 10$

$30^5 = (6 \times 5)^5 = 6^5 \times 5^5 = 5$

Number of 5's power = 35

then number of zeroes at the end of the product = 35

Ex.16 Find the number of zeroes at the end of the product

$1^1 \times 2^2 \times 3^3 \times 4^4$........ 28^{28}

Sol. count the numbe of 5's

power of 5's = total power of 5's

$5^5 = (1\times 5)^5 = 1^5 \times 5^5 = 5$

$10^{10} = (2 \times 5)^{10} = 2^{10} \times 5^{10} = 10$

$15^{15} = (3 \times 5)^{15} = 3^{15} \times 5^{15} = 15$

$20^{20} = (4 \times 5)^{20} = 4^{20} \times 5^{20} = 20$

$25^{25} = (5 \times 5)^{25} = 5^{25} \times 5^{25} = 50$

Number of 5's power

= 5 + 10 + 15 + 20 + 50

= 100

Then number of zeroes at the end of product = 100

Ex.17 Find the number of zeroes at the end of the product

$a = 1^3$, $b = 2^4$, $c = 3^5$, ..., $z = 26^{28}$

a × b × c × d .... × z

Sol. Count the number of 5's

power of 5's = total power of 5's

$5^7 = (1\times 5)^7 = 1^7 \times 5^7 = 7$

$10^{12} = (2 \times 5)^{12} = 2^{12} \times 5^{12} = 12$

$15^{17} = (3 \times 5)^{17} = 3^{17} \times 5^{17} = 17$

$20^{22} = (4 \times 5)^{22} = 4^{22} \times 5^{22} = 22$

$25^{27} = (5 \times 5)^{27} = 5^{27} \times 5^{27} = 54$

Number of 5's power

= 7 + 12 + 17 + 22 + 54 = 112

Then number of zeroes at the end of product = 112

Ex.18 Find the number of zeroes at the end of the product

$1^1 \times 2^2 \times 3^3 \times 4^4$........ 100^{100}

Sol. Count the power of 5's

$5^5 = 5$

$10^{10} = 10$

$15^{15} = 15$

.   .

.   .

.   .

.   .

$100^{100} = 100$

5 + 10 + 15 .... + 100

it is an a.p.series

we use a.p. formula

number of term = $\boxed{\frac{l-a}{d}+1}$

l = last term of a.p.

a = first term of a.p.

d = common difference

number of term = $\frac{100-5}{5}+1$

= 20

sum of n^{th} term of a.p.

$= \frac{n}{2}\left[2a+(n-1)d\right]$

$= \frac{20}{2}\left[2\times 5+(20-1)\times 5\right]$

$= 10\left[10+19\times 5\right]$

$= 10\left[105\right] = 1050$

but $25^{25} = (5 \times 5)^{25} = 5^{25} \times 5^{25}$

$50^{50} = (2 \times 5 \times 5)^{50} = 2^{50} \times 5^{50} \times 5^{50}$

$75^{75} = (3 \times 5 \times 5)^{75} = 3^{75} \times 5^{75} \times 5^{75}$

$100^{100} = (4 \times 5 \times 5)^{100} = 4^{100} \times 5^{100} \times 5^{100}$

then number of 5's power

= 25 + 50 + 75 + 100

= 250

then number of total zeroes at the end of product

= 1050 + 250 = 1300

Ex.19 Find the number of zeroes at the end of the product

10 × 20 × 30 80

Sol. $10^1 \times 1 \times 10^1 \times 2 \times 10^1 \times 3....10^1 \times 8 = 10^8$ [1 × 2 × 3 .... 8]

from 1 to 8, number of 0's = 1
$\therefore$ Only one pair (2 & 5)
then total number of 0's
= 1 + 8 = 9

Ex.20 Find the number of zeroes at the end of the product
10 × 20 × 30 1000

Sol. $10^1 \times 1 \times 10^1 \times 2 \times 10^1 \times 3 \ldots . 10^1 \times 100$
$= 10^{100} [1\times2\times3\times\ldots\ldots100]$
from 1 to 100 number of 0's

$$\begin{array}{r|l} 5 & 100 \\ 5 & 20 \rightarrow \\ 5 & 4 \rightarrow \\ & 0 \end{array} \Big\} \text{ 24 (add up all the quotients)}$$

number of 0's = 20 + 4 = 24
and 10^{100}, here number of zero = 100
total number of 0's
= 24 + 100 = 124
then number of zeroes = 124

EXERCISE

1. Find the number of zeroes at the end of the product 47!
(a) 8 (b) 9 (c) 10 (d) 11

2. Find the number of zeroes at the end of the product 125!
(a) 25 (b) 30 (c) 31 (d) 28

3. Find the number of zeroes at the end of the product 378!
(a) 93 (b) 90 (c) 75 (d) 81

4. Find the number of zeroes at the end of the product 680!
(a) 163 (b) 169 (c) 170 (d) 165

5. Find the number of zeroes at the end of the product 1000!
(a) 200 (b) 249 (c) 248 (d) 250

6. Find the number of zeroes at the end of the product 500!
(a) 100 (b) 124 (c) 120 (d) 125

7. Find the number of zeroes at the end of the product 1132!
(a) 280 (b) 271 (c) 281 (d) 272

8. Find the number of zeroes at the end of the product 1098!
(a) 280 (b) 270 (c) 271 (d) 262

9. Find the number of zeroes at the end of the product 2346!
(a) 580 (b) 583 (c) 575 (d) 580

10. Find the number of zeroes at the end of the product 2700!
(a) 673 (b) 670 (c) 669 (d) 675

11. Find the number of zeroes at the end of the product 10 × 15 × 44 × 28 × 70
(a) 2 (b) 3 (c) 4 (d) 5

12. Find the number of zeroes at the end of the product 12×5×15×24×13×30×75
(a) 4 (b) 5 (c) 2 (d) 3

13. Find the number of zeroes at the end of the product 2×4×6×.......48×50
(a) 6 (b) 12 (c) 7 (d) 5

14. Find the number of zeroes at the end of the product 1×3×5×7×9×11........99×101
(a) 24 (b) 5 (c) 2 (d) 0

15. Find the number of zeroes at the end of the product 21×22×23.......59×60
(a) 14 (b) 4 (c) 10 (d) 12

16. Find the number of zeroes at the end of the product
35 × 36 × 37 × ......89 × 90
(a) 21 (b) 7 (c) 14 (d) 20

17. Find the number of zeroes at the end of the product
41 × 42 ....109 × 110
(a) 26 (b) 9 (c) 17 (d) 25

18. Find the number of zeroes at the end of the product
140!×5×15×22×11×44×135
(a) 34 (b) 35 (c) 36 (d) 37

19. Find the number of zeroes at the end of the product
25! × 32! × 45!
(a) 10 (b) 23 (c) 22 (d) 7

20. Find the number of zeroes at the end of the product
1140! × 358! × 171!
(a) 282 (b) 325 (c) 411 (d) 370

21. Find the number of zeroes at the end of the product $1 \times 2^2 \times 3^3 \times 4^4 \times 5^5 \ldots\ldots 49^{49}$
(a) 225 (b) 250 (c) 240 (d) 245

22. Find the number of zeroes at the end of the product
$100^1 \times 99^2 \times 98^3 \times 97^4 \ldots . 1^{100}$
(a) 970 (b) 1124 (c) 875 (d) 975

23. Find the number of zeroes at the end of the product
$1^{1!} \times 2^{2!} \times 3^{3!} \times 4^{4!} \ldots\ldots 10^{10!}$
(a) 51 (b) 10 (c) 5! + 10!
(d) N.O.T

24. Find the number of zeroes at the end of the product
$2^2 \times 5^4 \times 4^2 \times 10^8 \times 6^{10} \times 15^{12} \times 8^{14} \times 20^{16} \times 10^{18} \times 25^{20}$
(a) 98 (b) 90 (c) 94 (d) 100

25. Find the number of zeroes at the end of the product
3200+1000+40000+32000+15000
(a) 15 (b) 13 (c) 2 (d) 3

26. Find the number of zeroes at the end of the product
3200×1000×40000×32000×15000
(a) 15 (b) 2 (c) 14 (d) 16

27. Find the number of zeroes at the end of the product
20×40×7600×600×300×1000
(a) 11 (b) 10 (c) 2 (d) 3

28. Find the number of zeroes at the end of the product 100! + 200!
(a) 24 (b) 25 (c) 49
(d) N.O.T

29. Find the number of zeroes at the end of the product
$1^1 \times 2^2 \times 3^3 \times 4^4 \ldots\ldots 10^{10}$
(a) 10 (b) 15 (c) 5
(d) N.O.T

30. Find the number of zeroes at the end of the product
100! × 200!
(a) 49 (b) 24 (c) 73
(d) N.O.T

31. Find the number of zeroes at the end of the product 5×10×15×20×25×30×35×40×45×50
(a) 8 (b) 12 (c) 10 (d) 14

32. Find the number of zeroes at the end of the product
2 × 4 × 6 × 8 × 10 200
(a) 49 (b) 24 (c) 25 (d) 50

33. Find the number of zeroes at the end of the product
1 × 3 × 5 × 7 ....99 × 64
(a) 23 (b) 6 (c) 0 (d) 5

34. Find The No. zero at the end of the product of $2^{222} \times 5^{555}$
(a) 222 (b) 555 (c) 777 (d) 333

35. Find the number of zeroes at the end of the product
10 + 100 + 1000 + ...100000000
(a) 8 (b) 28 (c) 0 (d) 1

36. Find the number of zeroes at the end of the product
$10^1 \times 10^2 \times 10^3 \times 10^4 \ldots\ldots 10^{10}$
(a) 10 (b) 55 (c) 50 (d) 45

37. Find the number of zeroes at the end of the product
$2^1\times5^2\times2^3\times5^4\times2^5\times5^6\times2^7\times5^8\times2^9\times5^{10}$
(a) 30 (b) 25
(c) 55 (d) 50

38. Find the number of zeroes at the end of the product
$(3^{123} - 3^{122} - 3^{121})\ (2^{121} - 2^{120} - 2^{119})$
(a) 1 (b) 0 (c) 119 (d) 120

39. Find the number of zeroes at the end of the product
$(8^{123} - 8^{122} - 8^{121})\ (3^{223} - 3^{222} - 3^{221})$
(a) 1 (b) 2 (c) 0 (d) 3

40 Find the number of zeroes at the end of the product
5× 10 × 15 75
(a) 11 (b) 15 (c) 10 (d) 18

ANSWER KEY

1. (c)	5. (b)	9. (b)	13. (a)	17. (c)	21. (b)	25. (c)	29. (b)	33. (b)	37. (b)
2. (c)	6. (b)	10. (a)	14. (d)	18. (d)	22. (b)	26. (d)	30. (c)	34. (a)	38. (a)
3. (a)	7. (c)	11. (b)	15. (c)	19. (b)	23. (c)	27. (a)	31. (a)	35. (d)	39. (b)
4. (b)	8. (c)	12. (b)	16. (c)	20. (c)	24. (a)	28. (a)	32. (b)	36. (b)	40. (a)

SOLUTION

1. (c)
$$\begin{array}{r|l} 5 & 47 \\ \hline 5 & 9 \\ \hline 5 & 1 \\ \hline & 0 \end{array}$$
No. of zeroes = 9 + 1 = 10

2. (c)
$$\begin{array}{r|l} 5 & 125 \\ \hline 5 & 25 \rightarrow \\ \hline 5 & 5 \rightarrow \\ \hline 5 & 1 \rightarrow \\ \hline & 0 \end{array} \Bigg\} 31$$
No of zeroes
= 25 + 5 + 1 = 31

3. (a)
$$\begin{array}{r|l} 5 & 378 \\ \hline 5 & 75 \rightarrow \\ \hline 5 & 15 \rightarrow \\ \hline 5 & 3 \rightarrow \\ \hline & 0 \end{array} \Bigg\} 93$$
No. of zeroes = 75 + 15 + 3 = 93

4. (b)
$$\begin{array}{r|l} 5 & 680 \\ \hline 5 & 136 \rightarrow \\ \hline 5 & 27 \\ \hline 5 & 5 \\ \hline 5 & 1 \rightarrow \\ \hline & 0 \end{array} \Bigg\} 169$$
No. of zeroes = 136 + 27 + 5 + 1
= 169

5. (b)
$$\begin{array}{r|l} 5 & 1000 \\ \hline 5 & 200 \rightarrow \\ \hline 5 & 40 \rightarrow \\ \hline 5 & 8 \rightarrow \\ \hline 5 & 1 \rightarrow \\ \hline & 0 \end{array} \Bigg\} 249$$
No. of zeroes = 200 + 40 + 8+ 1
= 249

6. (b)
$$\begin{array}{r|l} 5 & 500 \\ \hline 5 & 100 \rightarrow \\ \hline 5 & 20 \rightarrow \\ \hline 5 & 4 \rightarrow \\ \hline & 0 \end{array} \Bigg\} 124$$
No. of zeroes = 100 + 20 + 4
= 124

7. (c)
$$\begin{array}{r|l} 5 & 1132 \\ \hline 5 & 226 \rightarrow \\ \hline 5 & 45 \rightarrow \\ \hline 5 & 9 \rightarrow \\ \hline 5 & 1 \rightarrow \\ \hline & 0 \end{array} \Bigg\} 281$$
No. of zeroes = 226 + 45 + 9 + 1
= 281

8. (c)
$$\begin{array}{r|l} 5 & 1098 \\ \hline 5 & 219 \rightarrow \\ \hline 5 & 43 \rightarrow \\ \hline 5 & 8 \rightarrow \\ \hline & 1 \rightarrow \\ \hline & 0 \end{array} \Bigg\} 271$$
No. of zeroes = 219 + 43 + 8 + 1
= 271

9.(b)
$$\begin{array}{r|l} 5 & 2346 \\ \hline 5 & 469 \rightarrow \\ \hline 5 & 93 \rightarrow \\ \hline 5 & 18 \rightarrow \\ \hline 5 & 3 \rightarrow \\ \hline & 0 \end{array} \Bigg\} 583$$
No. of zeroes = 469 + 93 + 18 + 3
= 583

10.(a)
$$\begin{array}{r|l} 5 & 2700 \\ \hline 5 & 540 \rightarrow \\ \hline 5 & 108 \rightarrow \\ \hline 5 & 21 \rightarrow \\ \hline 5 & 4 \rightarrow \\ \hline & 0 \end{array} \Bigg\} 673$$
No. of zeroes = 540 + 108 + 21 + 4
= 673

11.(b) 10 × 15 × 44 × 28 × 70
$\underline{2} \times \underline{5} \times 3 \times \underline{5} \times \underline{2} \times \underline{2} \times 11 \times \underline{2}$
$\times \underline{2} \times 7 \times \underline{2} \times 5 \times 7$
In this expression
No of 2's = 6
No. of 5's = 3
Pair of 2's and 5's = 3
So, No of zeroes = 3

12.(b) 12×5×15×24×13×30×75
$\underline{2} \times \underline{2} \times 3 \times \underline{5} \times 3 \times \underline{5} \times \underline{2} \times \underline{2}$
$\times \underline{2} \times 3 \times 13 \times \underline{2} \times 3 \times \underline{5} \times \underline{5} \times \underline{5} \times 3$
No. of 2's $\rightarrow$ 6
No. of 5's $\rightarrow$ 5
Pair of 2's and 5's = 5
No. of zeroes = 5

13.(a) 2×4×6×.......48×50
$\Rightarrow$ 2×1×2×2×2×3 .... 2×24×2×25
$\Rightarrow$ 2^{25} (1×2×3×4×.......25)
There are many 2's In This series we count 5's
$$\begin{array}{r|l} 5 & 25 \\ \hline 5 & 5 \rightarrow \\ \hline 5 & 1 \rightarrow \\ \hline & 0 \end{array} \Big\} 6$$
No. of 5's = 5 + 1 = 6
Then No. of zeroes = 6

14. (d) 1×3×5×7×9×11........99×101
There is no 'zero' in this expression because there is no even present here.

15.(c) 21×22×23.......59×60
1×2×3.....19×20×21×22×23......59×60
\- 1×2×3......20

$$\begin{array}{r|l} 5 & 60 \\ 5 & 12 \rightarrow \\ 5 & 2 \rightarrow \\ & 0 \end{array} \Big\} 14 \qquad \begin{array}{r|l} 5 & 20 \\ 5 & 4 \rightarrow \\ & 0 \end{array} \Big\} 4$$

No. of zeroes 1 to 60 = 12 + 2 = 14

No. of zeroes 1 to 20 = 4

No. of zeroes 21 to 60 = 14 - 4 = 10

16. (c) 35 × 36 × 37 × ......89 × 90

$\underbrace{1\times2\times3\times4.....34\times35\times36......89\times90}$

$-\underbrace{1\times2\times3\times.........\times33\times34}$

No. of zeroes 1 to 90 = 18 + 3 = 21

$$\begin{array}{r|l} 5 & 90 \\ 5 & 18 \rightarrow \\ 5 & 3 \rightarrow \\ & 0 \end{array} \Big\} 21$$

No. of Zeroes 1 to 34

$$\begin{array}{r|l} 5 & 34 \\ 5 & 6 \rightarrow \\ 5 & 1 \rightarrow \\ & 0 \end{array} \Big\} 7$$

No. of Zeroes = 6 + 1 = 7

No. of zeroes 35 to 90 = 21 – 7 = 14

17.(c) 41 × 42 ....109 × 110

$\underbrace{1\times2\times3\times4.....40\times41\times42......109\times110}$

$-\underbrace{1\times2\times3.....40}$

No. of zeroes 1 to 110 = 22 + 4 = 26

No. of zeroes 1 to 40 =8 + 1 = 9

$$\begin{array}{r|l} 5 & 110 \\ 5 & 22 \rightarrow \\ 5 & 4 \rightarrow \\ & 0 \end{array} \Big\} 26 \qquad \begin{array}{r|l} 5 & 40 \\ 5 & 8 \rightarrow \\ 5 & 1 \rightarrow \\ & 0 \end{array} \Big\} 9$$

No. of zeroes 41 to 110 = 26–9 = 17

18.(d) 140! would have 28 + 5 + 1 = 34

$$\begin{array}{r|l} 5 & 140 \\ 5 & 28 \rightarrow \\ 5 & 5 \rightarrow \\ 5 & 1 \rightarrow \end{array} \Big\} 34$$

Now Remaining part

5 × 15 × 22 × 11 × 44 × 135

$5\times 3 \times \underline{5} \times \underline{2} \times 11 \times 11 \times \underline{2} \times \underline{2} \times 11 \times \underline{5} \times 27$

No. of 2's = 3

No. of 5's = 3

Pair of (2 & 5) = 3

Remaining part of the Expression would have 3 zeroes

Total No. of zeroes = 34 + 3 = 37

19.(b)

$$\begin{array}{r|l} 5 & 25 \\ 5 & 5 \rightarrow \\ 5 & 1 \rightarrow \\ & 0 \end{array} \Big\} 6 \qquad \begin{array}{r|l} 5 & 32 \\ 5 & 6 \rightarrow \\ 5 & 1 \rightarrow \\ & 0 \end{array} \Big\} 7$$

$$\begin{array}{r|l} 5 & 45 \\ 5 & 9 \rightarrow \\ 5 & 1 \rightarrow \\ & 0 \end{array} \Big\} 10$$

No. of zeroes In 25! = 5 + 1 = 6

No of zeroes In 32! = 6 + 1 = 7

No. of zero In 45! 9 + 1 = 10

Total No. of zero = 6 + 7 + 10= 23

20.(c)

$$\begin{array}{r|l} 5 & 1140 \\ 5 & 228 \rightarrow \\ 5 & 45 \rightarrow \\ 5 & 9 \rightarrow \\ 5 & 1 \rightarrow \\ & 0 \end{array} \Bigg\} 283 \qquad \begin{array}{r|l} 5 & 358 \\ 5 & 71 \rightarrow \\ 5 & 14 \rightarrow \\ 5 & 2 \rightarrow \\ & 0 \end{array} \Bigg\} 87$$

$$\begin{array}{r|l} 5 & 171 \\ 5 & 34 \rightarrow \\ 5 & 6 \rightarrow \\ 5 & 1 \rightarrow \\ & 0 \end{array} \Bigg\} 41$$

No. of zeroes in 1140!

= 228 + 45 + 9 + 1 = 283

No. of zeroes in 358!

= 71 + 14 + 2 = 87

No. of zeroes in 171!

= 34 + 6 + 1 = 41

Total No. of zeroes= 283+87+41 = 411

21.(b) The Fives will be less than the two's Hence, we need to count only the Fives

Thus,

$5^5 = (5\times 1)^5 = 5$

$10^{10} = (5\times 2)^{10} = 10$

$15^{15} = (3\times 5)^{15} = 15$

$20^{20} = (4\times 5)^{20} = 20$

$25^{25} = (5\times 5)^{25} = 50$

$30^{30} = (5\times 6)^{30} = 30$

$35^{35} = (5\times 7)^{35} = 35$

$40^{40} = (5\times 8)^{40} = 40$

$45^{45} = (5\times 9)^{45} = 45$

5+10+15+20+50+30+35+40+45

No. of Fives = 250

Then,

Number of zeroes = 250

22.(b) The Five will be less than the two's Then count the number of five

$100^1\times 95^6\times 90^{11}.........10^{91}\times 5^{96}$

(1+6+11......91+96) using sum of A.P.

a = 1, d = 5

No. of term = $\frac{96-1}{5}+1 = 20$

$S_n = \frac{20}{2}[2\times1+19\times5]$

= 10 [2 + 95] = 970

But

$100^1=(5\times5\times4)^1 = 5^1\times 5^1$

$75^{26} = (5\times5\times3)^{26} = 5^{26}\times 5^{26}$

$50^{51} = (5\times5\times2)^{51} = 5^{51}\times 5^{51}$

$25^{76} = (5\times5)^{76} = 5^{76}\times 5^{76}$

Then no of zeroes

$= \underline{1}+\underline{26}+\underline{51}+\underline{76} = 154$

Total number of zeroes

= 154 + 970 = 1124

23 (c) Count the No. of 5's

$5^{5!}$ and $10^{10!}$

No. of 5's = 5! + 10!

Then ,

No. of zeroes = 5! + 10!

24.(a) Count the No. of 5's

Then

$5^4\times 10^8\times 15^{12}\times 20^{16}\times 25^{20}$

= 4 + 8 + 12 + 16 + 40

= 80

So, No. of zero = 80

25.(c)

$$\begin{array}{r} 3200 \\ 1000 \\ 40000 \\ 32000 \\ +\ 15000 \\ \hline 91200 \end{array}$$

No. or zero = 2

26.(d) 3200 × 1000 × 40000 × 32000 × 15000

No. of zero's 2 + 3 + 4 + 3 + 3 = 15

But 1500 = 3 × 5 × 100

Here 5 is present

When 5 is multiply by even number, then unit digit '0' is get.

Then,

No. of Total zero = 15 + 1 =16

27.(a) 20 × 40 × 7600 × 600 × 300 × 1000

No. of zeroes = 1 + 1 + 2 + 2 + 2 + 3 = 11

28.(a) 100! + 200!

No. of zeroes In 100! = 20 + 4 = 24

No. of zeroes In 200! = 40 + 8 + 1 = 49

When you add the two Number (One with 24 zeroes and the other with 49 zeroes at It's end)

The Total No. of zeroes = 24

29.(b) $1^1\times 2^2\times 3^3\times 4^4.......10^{10}$

Count the Number of 5's

5^5 no of fives = 5

10^{10} No. of Fives = 10

No. of zeroes = 5 + 10 = 15

30.(c) 100!   200!

$$\begin{array}{r|l} 5 & 100 \\ 5 & 20 \rightarrow \\ 5 & 4 \rightarrow \\ & 0 \end{array} \Big\} 24 \qquad \begin{array}{r|l} 5 & 200 \\ 5 & 40 \rightarrow \\ 5 & 8 \rightarrow \\ 5 & 1 \rightarrow \\ & 0 \end{array} \Big\} 49$$

No. of zeroes In 100! = 24
No. of zeroes In 200! = 49
When you multiply two numbers (One with 24 zeroes and the other with 49 zeroes at It's end). The Resultant Total No. of zeroes = 24 + 49 = 73

31.(a) $5 \times 10 \times 15 \times 20 \times 25 \times .....\times 50$
$5\times1\times5\times2\times5\times3\times5\times4$ ......5×10
5^{10} $(1\times2\times3\times4$ ....$10)$
The two will be less than the fives hence we need to count only the two's
1 to 10 no of 2's

$$\begin{array}{r|l} 2 & 10 \\ 2 & 5 \rightarrow \\ 2 & 2 \rightarrow \\ 2 & 1 \rightarrow \\ & 0 \end{array} \Big\} 8$$

No. of 2's = 5 + 2 + 1 = 8
Then No. of zeroes = 8

32.(b) $2 \times 4 \times 6 \times 8 \times 10$ 200
$= 2\times1\times2\times2\times2\times2\times3$ .... 2×100
$= 2^{100}$ $(1\times2\times3\times$.........$100)$
We count No of 5

$$\begin{array}{r|l} 5 & 100 \\ 5 & 20 \rightarrow \\ 5 & 4 \rightarrow \\ & 0 \end{array} \Big\} 24$$

No of 5's = 24
Then No. of zeroes = 24

33.(b) $1\times3\times5\times7$.......99×2^6
Here No. of 5 is more than no.of 2 then count the number of 2
No. of 2's = 6
Now No. of zero = 6

34.(a) $2^{222} \times 5^{555}$
No. of 2's = 222
No. of 5's = 555
No. of 2's are less than Number of 5's
Pair (2's & 5's) = 222
No. of zero = 222

35.(d) 10+100+1000+ 100000000

$$\begin{array}{r} 10 \\ 100 \\ 1000 \\ .... \\ .... \\ 100000000 \\ \hline 111111110 \end{array}$$

This there is only one zero at the end of result

36.(b) $10^1 \times 10^2 \times 10^3 \times 10^4$ ....10^{10}
$10^{(1+2+3........10)} = 10^{55}$

$$\boxed{\because\ 1+2+3....10 = \frac{10(10+1)}{2} = 55}$$

No. of zero = 55

37.(b) $2^1 \times 5^2 \times 2^3 \times 5^4 \times 2^5 \times 5^6 \times 2^7 \times 5^8 \times 2^9 \times 5^{10}$
$\Rightarrow 2^{(1+3+5+7+9)} \times 5^{(2+4+6+8+10)}$
$\Rightarrow 2^{25} \times 5^{30}$
Number of 2's are less than the Number of 5's
= Pair of (2×5) = 25
No of zero = 25

38.(a) $(3^{123} - 3^{122} - 3^{121})$ $(2^{121} - 2^{120} - 2^{119})$
$\Rightarrow 3^{121}$ $(3^2 - 3^1 - 3^0)$ 2^{119} $(2^2 - 2^1 - 2^0)$
$\Rightarrow (3^{121})(2^{119})$ (9-3-1) (4 - 2 - 1)
$\Rightarrow (3^{121})(2^{119})$ (5) (1)
$= 5^1 \times 2^{119} \times 3^{121}$
No. of 5's = 1
No. of 2's = 119
Pair of (2 & 5) = 1
No. of zero = 1

39.(b) $(8^{123} - 8^{122} - 8^{121})$ $(3^{223} - 3^{222} - 3^{221})$
$\rightarrow 8^{121}$ $(8^2 - 8^1 - 1)$ 3^{221} $(3^2 - 3^1 - 1)$
$\rightarrow 8^{121}$ (64 - 9) 3^{221} (9 - 4)
$\rightarrow 8^{121} \times 55 \times 3^{221} \times 5$
$= 8^{121} \times 3^{221} \times 5 \times 11 \times 5$
$= (2^3)^{121} \times 3^{221} \times 5^2 \times 11$
$= 11 \times 5^2 \times 2^{363} \times 3^{221}$
No. of 2's = 363
No. of 5's = 2
Pair of (2 & 5) = 2
No . of zero = 2

40.(a) $5^1 \times 1 \times 5^1 \times 2 \times 5^1 \times 3$ .... $5^1 \times 15$
$= 5^{15}$ $(1 \times 2 \times 3$ .... $15)$
each term multiple of 5 So power of 5's more than 2 then count the number of 2 from 1 to 15.

$$\begin{array}{r|l} 2 & 15 \\ 2 & 7 \rightarrow \\ 2 & 3 \rightarrow \\ 2 & 1 \rightarrow \\ & 0 \end{array} \Big\} 11$$

number of (2 and 5) pairs = 11
then number of zeroes = 11

CHAPTER

03

FACTOR

Factor→ A number which divides a given number exactly is called factor (or divisor) of that given number and the given number is called a multiple of that number.

Ex. 1, 2, 4, and 8 are factors of 8 because 8 is perfectly divisible of 1, 2, 4 and 8

Factors and Multiple

Ex. Factors of 35 = 1, 5, 7, 35

Ex. Factors of 24 = 1, 2, 3, 4, 6, 8, 12, 24

Same,

Multiple of 2 = 2, 4, 6, 8, 10, ...

Multiple of 7 = 7, 14, 21, 28, 35 ....

* 1 is a factor of every number
* every number is a factor of itself
* every number, except 1 has at least 2 factor
* every number has infinite number of its multiples
* every number is a multiple of itself

Number of Factors

Let N be the composite number and a, b, c,.. be its prime factors and p, q, r be the indices (or powers) of a, b, c, i.e, if N can be expressed as $N = a^p.b^q.c^r$ then total number of factors of N = (p + 1) × (q + 1) × (r + 1)

If a is even prime factor, b and c are odd prime factors

The number of even factors

= (P) × (q + 1) × (r + 1)

The number of odd factors

= (1) × (q + 1) × (r + 1)

Ex.1 Find the total number of factors of 8.

Sol. 8 = 1, 2, 4 and 8 are Perfectly divisible

So number of factors = 4

* This method is easy for smaller number but for larger number its a probelm So use for alternate method

Alternate

$8 = 2 \times 2 \times 2 = 2^3$

Number of Total factors

= 3 + 1 = 4

Ex.2 Find the total number of factors of 240

Sol.

$$\begin{array}{r|l} 2 & 240 \\ 2 & 120 \\ 2 & 60 \\ 2 & 30 \\ 3 & 15 \\ 5 & 5 \\ & 1 \end{array}$$

$240 = 2 \times 2 \times 2 \times 2 \times 3 \times 5$

$= 2^4 \times 3^1 \times 5^1$

Total Factors

= (4 + 1) × (1 + 1) × (1 + 1)

= 5 × 2 × 2 = 20

Ex.3 Find the total number of factors of 500.

Sol.

$$\begin{array}{r|l} 2 & 500 \\ 2 & 250 \\ 5 & 125 \\ 5 & 25 \\ 5 & 5 \\ & 1 \end{array}$$

500 = 2 × 2 × 5 × 5 × 5

$= 2^2 \times 5^3$

No. of factors = (2 + 1) × (3 + 1)

= 3 × 4 = 12

Number of even Factor

Ex4. Find the number of even factors of 24.

Sol. Factor of 24 = 1, 2, 3, 4, 6, 8, 12, 24

Even Factor of 24 = 2, 4, 6, 8, 12, 24,

So,

Total number of even Factor of 24 = 6

Alternate

$$\begin{array}{r|l} 2 & 24 \\ 2 & 2 \\ 2 & 6 \\ 3 & 3 \\ & 1 \end{array}$$

$24 = 2^3 \times 3^1$

Number of even factor = 3×(1+1)

= 3 × 2 = 6

Ex.5 Find the number of even factor of 60.

Sol.

$$\begin{array}{r|l} 2 & 60 \\ 2 & 30 \\ 3 & 15 \\ 5 & 5 \\ & 1 \end{array}$$

60 = 2 × 2 × 3 × 5

$= 2^2 \times 3^1 \times 5^1$

No. of even factor = 2 × (1+1) × (1+1)

= 2 × 2 × 2 = 8

No. of odd factor

Ex.6 Find the number of odd factors of 40.

Sol. 40 = 1, 2, 4, 5, 8, 10, 20, 40

Odd factors = 1, 5

Number of odd factors = 2

Alternate

$$\begin{array}{r|l} 2 & 40 \\ 2 & 20 \\ 2 & 10 \\ 5 & 5 \\ & 1 \end{array}$$

40 = 2 × 2 × 2 × 5

$= 2^3 \times 5^1$

No. of odd Factors= 1 × (1 + 1)

= 1 × 2

= 2

Ex.7 Find the number of factors, number of even factors and number of odd factors of 180

Sol. 180 = 2 × 2 × 3 × 3 × 5

$= 2^2 \times 3^2 \times 5^1$

Total Number of factors

= (2 + 1) × (2 + 1) × (1 + 1)

= 3 × 3 × 2 = 18

Number of even factors
$= 2\,(2 + 1) \times (1 + 1)$
$= 2 \times 3 \times 2 = 12$
Number of odd factors
$= 1 \times (2 + 1) \times (1 + 1)$
$= 1 \times 3 \times 2 = 6$

Ex.8 Find the number of factors, number of even factors and number of odd factors of 360.

Sol.
$$\begin{array}{r|l} 2 & 360 \\ 2 & 180 \\ 2 & 90 \\ 3 & 45 \\ 3 & 15 \\ 5 & 5 \\ & 1 \end{array}$$
$360 = 2 \times 2 \times 2 \times 3 \times 3 \times 5$
$= 2^3 \times 3^2 \times 5^1$
Total number of factors
$= (3 + 1) \times (2 + 1) \times (1 + 1)$
$= 4 \times 3 \times 2 = 24$
Number of even factors
$= 3 \times (2 + 1) \times (1 + 1)$
$= 3 \times 3 \times 2 = 18$
Number of odd factors
$= 1 \times (2 + 1) \times (1 + 1)$
$= 3 \times 2 = 6$

Ex.9 Find the number of factors, number of even factors and number of odd factors of 100

Sol.
$$\begin{array}{r|l} 2 & 100 \\ 2 & 50 \\ 5 & 25 \\ 5 & 5 \\ & 1 \end{array}$$
$100 = 2 \times 2 \times 5 \times 5 = 2^2 \times 5^2$
Total no. of Factor
$= (2 + 1) \times (2 + 1) = 3 \times 3 = 9$
No. of even factor $= 2 \times (2 + 1)$
$= 2 \times 3 = 6$
No. of odd factor $= 1 \times (2 + 1)$
$= 1 \times 3 = 3$

Sum of factors

Let N be the composite number and a, b, c,... be its prime factors and p, q, r be the indices (or powers) of a, b, c, i.e. if N can be expressed as $N = a^p.b^q.c^r$

then the sum of all the divisors (or factors) of N
$= (a^o + a^1 + a^2 \ldots. a^p) \times (b^o + b^1 + b^2 \ldots.b^q) \times (c^o + c^1 + c^2 \ldots\ldots c^r)$

If a is even prime factor and b and c odd prime factors then

- Sum of even factors $= (a^1 + a^2 \ldots a^p) \times (b^o + b^1 + b^2 \ldots. b^q) \times (c^o + c^1 + c^2 \ldots.. c^r)$
- Sum of odd actor $= (a^o) \times (b^o + b^1 + b^2 \ldots. b^q) \times (c^o + c^1 + c^2 \ldots\ldots c^r)$

Ex10. Find the sum of all factors of 8.

Sol. factors of 8 = 1, 2, 4, 8
Sum of factors = 1 + 2 + 4 + 8
= 15

This method is easy for smaller number but for larger number its a probelm So use for alternate method

Alternate
$8 = 2^3$
sum of all factors $= (2^0 + 2^1 + 2^2 + 2^3)$
$= 1 + 2 + 4 + 8 = 15$

$\boxed{\mathbf{(a^o = 1,\ where\ a = real\ number)}}$

Ex.11 find the sum of all factors, sum of even factors and sum of odd factors of 24.

Sol. factors of 24 = 1, 2, 3, 4, 6, 8, 12, 24
sum of factors = 1 + 2 + 3 + 4 + 6 + 8 + 12 + 24 = 60
sum of even factors
= 2 + 4 + 6 + 8 + 12 + 24 = 56
Sum of odd factors = 1 + 3 = 4

Alternate
$$\begin{array}{r|l} 2 & 24 \\ 2 & 12 \\ 2 & 6 \\ 3 & 3 \\ & 1 \end{array} \quad 24 = 2^3 \times 3^1$$
sum of all factors
$= (2^0 + 2^1 + 2^2 + 2^3) \times (3^0 + 3^1)$
$= (1 + 2 + 4 + 8) \times (1 + 3)$
$= 15 \times 4 = 60$
Sum of even factors
$= (2^1 + 2^2 + 2^3) \times (3^0 + 3^1)$
$= (2 + 4 + 8) \times (1 + 3)$
$= 14 \times 4 = 56$
Sum of odd factos
$= (2^0) \times (3^0 + 3^1) = 1 \times 4 = 4$

Ex12. find the sum of all factors, sum of even factors and sum of odd factors of 360.

Sol.
$$\begin{array}{r|l} 2 & 360 \\ 2 & 180 \\ 2 & 90 \\ 3 & 45 \\ 3 & 15 \\ 5 & 5 \\ & 1 \end{array} \quad 360 = 2^3 \times 3^2 \times 5^1$$
sum of all factors
$= (2^0 + 2^1 + 2^2 + 2^3) \times (3^0 + 3^1 + 3^2) \times (5^0 + 5^1)$
$= 15 \times 13 \times 6 = 1170$
Sum of even factors
$=(2^1 + 2^2 + 2^3) \times (3^0 + 3^1 + 3^2) \times (5^0 \times 5^1)$
$= 14 \times 13 \times 6 = 1092$
sum of odd factors
$= 2^0 \times (3^0 + 3^1 + 3^2) \times (5^0 \times 5^1)$
$= 1 \times 13 \times 6 = 78$

Ex13. find the sum of all factors, sum of even factors and sum of odd factors of 100.

$$\begin{array}{r|l} 2 & 100 \\ 2 & 50 \\ 5 & 25 \\ 5 & 5 \\ & 1 \end{array} \quad 100 = 2^2 \times 5^2$$
Sum of all factors
$= (2^0 + 2^1 + 2^2) \times (5^0 + 5^1 + 5^2)$
$= 7 \times 31 = 217$
Sum of even factors
$= (2^1 + 2^2) \times (5^0 + 5^1 + 5^2)$
$= 6 \times 31 = 186$
Sum of odd factors
$= (2^0) \times (5^0 + 5^1 + 5^2)$
$= 1 \times 31 = 31$

Prime Factorisation

Prime Factorisation : If a natural number is expressed as the product of prime numbers (factors) then the factorisation of the number is called its prime factorisation.

(i) **72**
$$\begin{array}{r|l} 2 & 72 \\ 2 & 36 \\ 2 & 18 \\ 3 & 9 \\ 3 & 3 \\ & 1 \end{array}$$
$72 = 2 \times 2 \times 2 \times 3 \times 3$
$72 = 2^3 \times 3^2$
number of prime factors = 3 + 2 = 5

(ii) **540**
$$\begin{array}{r|l} 2 & 540 \\ 2 & 270 \\ 3 & 135 \\ 3 & 45 \\ 3 & 15 \\ 5 & 5 \\ & 1 \end{array}$$
$540 = 2 \times 2 \times 3 \times 3 \times 3 \times 5$
$= 2^2 \times 3^3 \times 5^1$

No. of prime factor = 2 + 3 + 1
= 6

(iii) find the number of prime factor
$2^3 \times 5^7 \times 21^4 \times 10^8$

Sol. $2^3 \times 5^7 \times 21^4 \times 10^8$
$2^3 \times 5^7 \times (3 \times 7)^4 \times (2 \times 5)^8$
$2^3 \times 5^7 \times 3^4 \times 7^4 \times 2^8 \times 5^8$
$2^{11} \times 3^4 \times 5^{15} \times 7^4$
Total No. of prime factors
= 11 +4 + 15 + 4 = 34

Ex14. The Number of prime Factors In the expression $6^4 \times 8^6 \times 10^8 \times 12^{10}$ is
(a) 48 (b) 64
(b) 72 (d) 80

Sol. $6^4 \times 8^6 \times 10^8 \times 12^{10}$
$\Rightarrow (2\times3)^4 \times (2\times2\times2)^6 \times (2\times5)^8 \times (2\times2\times3)^{10}$
$\Rightarrow 2^4\times3^4 \times (2^3)^6 \times 2^8 \times 5^8 \times (2^2 \times 3)^{10}$
$\Rightarrow 2^4 \times 3^4 \times 2^{18} \times 2^8 \times 5^8 \times 2^{20} \times 3^{10}$
$\Rightarrow 2^{4+18+8+20} \times 3^{4+10} \times 5^8$
$= 2^{50} \times 3^{14} \times 5^8$
Total No. of prime factor
= 50 + 14 + 8 **= 72**

EXERCISE

1 Find the number of Factors of 1728
(a) 28 (b) 29 (c) 30 (d) 31

2. Find the Number of Factor of 1420
(a) 12 (b) 13 (c) 14 (d) 15

3. Find the Number of Divisors of 10800
(a) 30 (b) 60 (c) 120 (d) 180

4. Find the No. of Prime Factor of 240.
(a) 4 (b) 5 (c) 6 (d) 8

5. Find the No. of prime factor. $(30)^{26} \times (25)^{51} \times (12)^{23}$
(a) 249 (b) 250 (c) 255 (d) 260

6. Find the No. of Prime Factor $(30)^{15} \times (22)^{11} \times (15)^{24}$
(a) 110 (b) 115 (c) 120 (d) 125

7. Find the No. of Prime Factor 180
(a) 4 (b) 5 (c) 6 (d) 7

8. Find the No. of Prime Factor of 536
(a) 4 (b) 5 (c) 6 (d) 3

9. Find the No. of prime Factor of 1044
(a) 4 (b) 5 (c) 10 (d) 9

10. Find The No. of prime factor of $(56)^{20} \times (36)^{31} \times (42)^{13} \times (13)^{21}$
(a) 240 (b) 242 (c) 264 (d) 248

11. Find the total Number of Prime Factors of
$2^{17} \times 6^{31} \times 7^5 \times 10^{11} \times 11^{10} \times 21^{12}$
(a) 142 (b) 144 (c) 140 (d) 146

12. Find the prime Factors 210
(a) 3 (b) 4 (c) 5 (d) 6

13. Find the sum of odd factors of544
(a) 16 (b) 18 (c) 20 (d) 22

14. For the Number 2450 find
(i) Number of all factors
(ii) Number of even factors
(iii) Number of odd factors
(a) 18,9,9 (b) 18,10,8
(c) 18,8,10 (d) 18,12,6

15. For the Number 760
(i) The sum and Number of all factors
(ii) The Sum and Number of even factors
(iii) The Sum and Number of odd factors

16. For The Number 96
(i) Sum and number of all factors
(ii) The sum and Number of even factors
(iii) The sum and Number of odd factors

17. For the Number 270
(i) The sum & Number of all Factor
(ii) The sum & Number of even factor
(iii) The sum & Number of odd Factor

ANSWER KEY

1. (a)	3. (b)	5. (a)	7. (b)	9. (b)	11. (c)	13. (b)
2. (a)	4. (c)	6. (b)	8. (a)	10. (c)	12. (b)	14. (a)

SOLUTION

1. **(a)**

$$\begin{array}{r|l} 2 & 1728 \\ 2 & 864 \\ 2 & 432 \\ 2 & 216 \\ 2 & 108 \\ 2 & 54 \\ 3 & 27 \\ 3 & 9 \\ 3 & 3 \\ & 1 \end{array}$$

1728 = 2×2×2×2×2×2×3×3×3
$= 2^6 \times 3^3$
No. of factors = (6 + 1) × (3 + 1)
= 7 × 4 = 28

2 **(a)**

$$\begin{array}{r|l} 2 & 1420 \\ 2 & 710 \\ 5 & 355 \\ 71 & 71 \\ & 1 \end{array}$$

1420 = 2 × 2 × 5 × 71
$= 2^2 \times 5^1 \times 71^1$
No. of factors
= (2 + 1) × (1+1) × (1+1)
= 3 × 2 × 2 = 12

3. **(b)**

$$\begin{array}{r|l} 2 & 10800 \\ 2 & 5400 \\ 2 & 2700 \\ 2 & 1350 \\ 3 & 675 \\ 3 & 225 \\ 3 & 75 \\ 5 & 25 \\ 5 & 5 \\ & 1 \end{array}$$

10800 = 2×2×2×2×3×3×3×5×5
$= 2^4 \times 2^3 \times 5^2 = 2^4 \times 3^3 \times 5^2$

No. of factors = (4 + 1) (3 + 1) (2 + 1)
= 5 × 4 × 3 = 60

4. **(c)**
$$\begin{array}{r|l} 2 & 240 \\ 2 & 120 \\ 2 & 60 \\ 2 & 30 \\ 3 & 15 \\ 5 & 5 \\ & 1 \end{array}$$

$240 = 2 \times 2 \times 2 \times 2 \times 3 \times 5$
$= 2^4 \times 3^1 \times 5^1$

No. of prime factor = 4+1+1=6

5. **(a)** $(30)^{26} \times (25)^{51} \times (12)^{23}$

Break The form of prime factor

$\Rightarrow$ $(2^1 \times 3^1 \times 5^1)^{26} \times (5 \times 5)^{51} \times (2 \times 2 \times 3)^{23}$

$\Rightarrow$ $\underline{2^{26}} \times \underline{3^{26}} \times 5^{26} \times 5^{102} \times \underline{2^{46}} \times \underline{3^{23}}$

$\Rightarrow$ $2^{26+46} \times 3^{26+23} \times 5^{26+102}$

$\Rightarrow$ $2^{72} \times 3^{49} \times 5^{128}$

No. of prime factors

$\Rightarrow$ 72 + 49 + 128 = 249

6. **(b)** $(30)^{15} \times (22)^{11} \times (15)^{24}$

$\Rightarrow$ $(2 \times 3 \times 5)^{15} \times (2 \times 11)^{11} \times (3 \times 5)^{24}$

$\Rightarrow$ $2^{15} \times 3^{15} \times 5^{15} \times 2^{11} \times 11^{11} \times 3^{24} \times 5^{24}$

$\Rightarrow$ $2^{15+11} \times 3^{15+24} \times 5^{15+24} + 11^{11}$

$\Rightarrow$ $2^{26} \times 3^{39} \times 5^{39} \times 11^{11}$

No. of Prime factor

26 + 39 + 39 + 11 = 115

7. **(b)**
$$\begin{array}{r|l} 2 & 180 \\ 2 & 90 \\ 3 & 45 \\ 3 & 15 \\ 5 & 5 \\ & 1 \end{array}$$

$180 = 2 \times 2 \times 3 \times 3 \times 5$
$= 2^2 \times 3^2 \times 5^1$

No. of prime Factor = 2+2+1 = 5

8. **(a)**
$$\begin{array}{r|l} 2 & 536 \\ 2 & 268 \\ 2 & 134 \\ 67 & 67 \\ & 1 \end{array}$$

$536 = 2 \times 2 \times 2 \times 67 = 2^3 \times 67^1$

No. of prime factor = 3+1 = 4

9. **(b)**
$$\begin{array}{r|l} 2 & 1044 \\ 2 & 522 \\ 3 & 261 \\ 3 & 87 \\ 29 & 29 \\ & 1 \end{array}$$

$1044 = 2 \times 2 \times 3 \times 3 \times 29$
$= 2^2 \times 3^2 \times 29^1$

No. of prime factor
= 2 + 2 + 1 = 5

10. **(c)** $(56)^{20} \times (36)^{31} \times (42)^{13} \times (13)^{21}$

$\Rightarrow$ $(2 \times \underline{2} \times \underline{2} \times \underline{7})^{20} \times (\underline{2}^2 \times \underline{3}^2)^{31} \times (\underline{2} \times \underline{3} \times \underline{7})^{13} \times (13)^{21}$

$\Rightarrow$ $(2^3 \times 7)^{20} \times (2^{62}) \times (3)^{62} \times (\underline{2} \times \underline{3} \times \underline{7})^{13} \times (13)^{21}$

$\Rightarrow$ $2^{60} \times 7^{20} \times 2^{62} \times 3^{62} \times 2^{13} \times 3^{13} \times 7^{13} \times 13^{21}$

$\Rightarrow$ $2^{60+62+13} \times 3^{62+13} \times 7^{20+13} \times 13^{21}$

$\Rightarrow$ $2^{135} \times 3^{75} \times 7^{33} \times 13^{21}$

Number of prime factors
= 135 + 75 + 33 + 21 = 264

11. **(c)** $2^{17} \times 6^{31} \times 7^5 \times 10^{11} \times 11^{10} \times 21^{12}$

$\Rightarrow$ $2^{17} \times (2 \times 3)^{31} \times 7^5 \times (2 \times 5)^{11} \times 11^{10} \times (3 \times 7)^{12}$

$\Rightarrow$ $2^{17} \times 2^{31} \times 3^{31} \times 7^5 \times 2^{11} \times 5^{11} \times 11^{10} \times 3^{12} \times 7^{12}$

$\Rightarrow$ $2^{17+31+11} \times 3^{31+12} \times 5^{11} \times 7^{5+12} \times 11^{10}$

$\Rightarrow$ $2^{59} \times 3^{43} \times 5^{11} \times 7^{17} \times 11^{10}$

Total No. of Prime Factors
= 59 + 43 + 11 + 17 + 10
= 140

12. **(b)**
$$\begin{array}{r|l} 2 & 210 \\ 3 & 105 \\ 5 & 35 \\ 7 & 7 \\ & 1 \end{array}$$

$210 = 2^1 \times 3^1 \times 5^1 \times 7^1$
= 1 + 1 + 1 + 1 = 4

13. **(b)** $544 = 2 \times 2 \times 2 \times 2 \times 2 \times 17$
$= 2^5 \times 17^1$

Sum of odd factors
$= (2^0) \times (17^0 + 17^1)$
= 1 × (1 + 17)
= 1 × 18 = **18**

14. (a)
$$\begin{array}{r|l} 2 & 2450 \\ 5 & 1225 \\ 5 & 245 \\ 7 & 49 \\ 7 & 7 \\ & 1 \end{array}$$

2450 = 2 × 5 × 5 × 7 × 7
$2450 = 2^1 \times 5^2 \times 7^2$

Number of Factor = (1 + 1) (2 + 1) (2 + 1)
= 2 × 3 × 3 = 18

Number of even Factor = 1 × (2 + 1) × (2 + 1)
= 1 × 3 × 3 = 9

Number of odd factor = 1(2+1)×(2+1)
= 3 × 3 = 9

15.
$$\begin{array}{r|l} 2 & 760 \\ 2 & 380 \\ 2 & 190 \\ 5 & 95 \\ 19 & 19 \\ & 1 \end{array}$$

$760 = 2 \times 2 \times 2 \times 5 \times 19$
$= 2^3 \times 5^1 \times 19^1$

(i) Numbe of factor
= (3+1)×(1+1)×(1+1)
= 4×2×2=16

Sum of factor
$= (2^0 + 2^1 + 2^2 + 2^3) \times (5^0 + 5^1) \times (19^0 + 19^1)$
= (1+2+4+8) × (1+5) × (1+19)
= 15×6×20 = 1800

(ii) Number of even factor
= 3 × (1 + 1) × (1 + 1)
= 3 × 2 × 2 = 12

Sum of even factor
$= (2^1 + 2^2 + 2^3) \times (5^0 + 5^1) \times (19^0 + 19^1)$
= 14 × 6 × 20 = 1680

(iii) Number of odd factors
= 1 × (1+1) × (1+1)
= 1 × 2 × 2 = 4

Sum of odd factors
$= (2^0) \times (5^0 + 5^1) \times (19^0 + 19^1)$
= 1 × 6 × 20 = 120

16. 96 = 2×2×2×2×2×3
$= 2^5 \times 3^1$

(i) Number of all factor
= (5+1) × (1+1) = 6 × 2 = 12

Sum of all factor
$= (2^0 + 2^1 + 2^2 + 2^3 + 2^4 + 2^5) \times (3^0 + 3^1)$
= (1+2+4+8+16+32)×(1+3)
= 63 × 4 = 252

(ii) Number of even factor
= 5 × (1+1) = 5 × 2 = 10

Sum of even factor
$= (2^1 + 2^2 + 2^3 + 2^4 + 2^5) \times (3^0 + 3^1)$
= (2+4+8+16+32) × (1+3)
= 62 × 4 = 248

(iii) Number of odd factor
= 1× (1 + 1) = 1 × 2 = 2

Sum of odd factor
$= (2^0) \times (3^0 + 3^1)$
= 1 × 4 = 4

17.Sol.

$$\begin{array}{r|l} 2 & 270 \\ 3 & 135 \\ 3 & 45 \\ 3 & 15 \\ 5 & 5 \\ & 1 \end{array}$$

$270 = 2\times3\times3\times3\times5$

$= 2^1\times3^3\times5^1$

(i) Number of all factor

$= (1+1)\times(3+1)\times(1+1)$

$= 2\times4\times2=16$

Sum of all factor

$= (2^0+2^1)\times(3^0+3^1+3^2+3^3)\times(5^0+5^1)$

$= 3 \times 40 \times 6 = 720$

(ii) Number of even factor

$= 1 \times (3+1) \times (1+1)$

$= 1 \times 4 \times 2 = 8$

Sum of even factor

$= (2^1)\times(3^0+3^1+3^2+3^3) \times (5^0 + 5^1)$

$= 2 \times 40 \times 6 = 480$

Number of odd factors

$= 1\times(3+1)\times(1+1)$

$= 4\times2 = 8$

Sum of odd factors

$=2^0\times(3^0+3^1+3^2+3^3)\times(5^0+5^1)$

$= 1\times40\times6 = 240$

CHAPTER

04

DIVISIBILITY

Rule of Divisibility

* **Divisibility by 2** ⟶ If Last digit of the number is divisible by 2

Divisibility by 4 ⟶ If Last two digits of the number are divisibile by 4

Divisibility by 8 ⟶ If Last three digits of the number are divisible by 8

Divisibility by 16 ⟶ If Last four digits of the number are divisible by 16

Divisibility by 32 ⟶ If Last five digits of the number are divisible by 32

* **Divisibility of 3** ⟶ All such numbers the Sum of whose digits are divisible by 3

Divisibility of 9 ⟶ All such numbers the Sum of whose digits are divisible by 9

* **Divisibility by 6** ⟶ A number is divisible by 6 If it is simultaneously divisible by 2 and 3

* **Divisibility by 5** ⟶ If Last digit (0 and 5) is divisible by 5

Divisibility by 25 ⟶ If Last two digits of the number are divisibile by 25

Divisibility by 125 ⟶ If Last three digits of the number are divisible by 125

* **Divisibility by 7** ⟶ Double the last digit and subtract it from the remaining leading truncated number. If the result is divisible by 7, then so was the original number.

* **Divisibility by 11** ⟶ The difference of the sum of the digits in the odd places and the sum of digits in the even places is '0' or multiple of 11 is divisible

* **Divisibility by 3, 7, 11, 13, 21, 37 and 1001** ⟶

(i) If any number is made by repeating a digit 6 times the number will be divisible by 3, 7, 11, 13, 21, 37 and 1001 etc.

(ii) A six digit number if formed by repeating a three digit number; for example, 256, 256 or 678, 678 etc. Any number of this form is always exactly divisible by 7, 11, 13, 1001 etc.

Some important points

(a) If a is divisible by b then ac is also divisible by b.

(b) If a is divisible by b and b is divisible by c then a is divisible by c.

(c) If n is divisible by d and m is divisible by d then (m + n) and (m – n) are both divisible by d. This has an important implication. Suppose 48 and 528 are both divisible by 8. Then (528 + 48) as well as (528 – 48) are divisible by 8

Ex.1: Check to see if 203 is divisible by 7

Sol.

$$\begin{array}{r|l} 20 & 3 \\ -6 & \times 2 \\ \hline 14 & \end{array}$$

Step I. Double the last digit = 3 × 2 = 6

Step II. Substract that from the rest of the Number = 20 – 6 = 14

Step III. Check to see if the difference is divisible by 7. 14 is divisible by 7 therefore 203 is also divisible by 7

Ex.2: Check to see if 68734 is divisible by 7

Sol.

$$\begin{array}{r|l} 6873 & 4 \\ -8 & \times 2 \\ \hline 686 & 5 \\ -10 & \times 2 \\ \hline 67 & 6 \\ -12 & \times 2 \\ \hline 55 & \end{array}$$

55 is not divisible by 7 So, 68734 is not divisible by 7

Ex.3: Check to see if 24983 is divisible by 7

Sol.

$$\begin{array}{r|l} 2498 & 3 \\ -6 & \times 2 \\ \hline 249 & 2 \\ -4 & \times 2 \\ \hline 24 & 5 \\ -10 & \times 2 \\ \hline 14 & \end{array}$$

14 is divisible by 7, therefore 24983 is also divisible by 7

Ex.4: Check to see if 65432577 is divisible by 7

Sol. When any number is made of more than five digits then we check divisiblity by 7 another rule

Step I. First for we make pair of 3 digits from right side (last)

<u>65</u> <u>432</u> <u>577</u>

Step II. Add alternate pairs = 65 + 577 = 642

Step III. Substract from remaning (3^{rd}) pair = 642 –432 = 210

If difference is divisible by 7 therefore number is also divisible by 7

Here difference = 210

210 is divisible by 7. Therefore 65432577 will be divisible by 7.

Note:- We can use First rule of divisibility by 7 but when a number has more than 5 digits this rule is easy for solve problem.

Ex.5: Check to see if 23756789765 is divisible by 7

Sol. <u>23</u> <u>756</u> <u>789</u> <u>765</u>

Step I. Add alternate pair

765 + 756 = 1521

23 + 789 = 812

Step II. Substract pairs

1521 – 812 = 709

709 is not divisible by 7 therefore 23756789765 is not divisible by 7

Ex.6: If 5432*7 is divisible by 9, then the digit in place of * is

(a) 0 (b) 1 (c) 6 (d) 9

Sol. (c) $\frac{5+4+3+2+x+7}{9} = \frac{21+x}{9}$

Put the value of 'x'. So the number is completely divisible by 9. Put $x = 6$

$= \frac{21+6}{9} = \frac{27}{9} =$ '0' remainder

Property: A number is completely divisible by 9 it the sum of the digits of the number is completely divisible by 9 and give no remainder.

Ex.7: When 335 is added to 5A7, the result is 8B2. 8B2 is divisible by 3. What is the largest posisble value of A?

(a) 8 (b) 2 (c) 1 (d) 4

Sol.
$$\begin{array}{r} 5\ A\ 7 \\ 3\ 3\ 5 \\ \hline 8\ B\ 2 \end{array}$$

$\Rightarrow$ A $\rightarrow$ 1, 2, 3, 4, 5 &

B $\rightarrow$ 5, 6, 7, 8, 9

8B2 is exactly $\therefore$ 8 + B + 2 = multiple of 3

$\therefore$ B = 5 or 8 $\Rightarrow$ A = 1 or 4

Ex.8: If * is a digit such that 5824* is divisible by 11, then * equals :

(a) 2 (b) 3 (c) 5 (d) 6

Sol. **(c)** 5 8 2 4 *

$\Rightarrow$ 5 +2 + * = 8 + 4

7 + * = 12

* = 12 - 7 = 5

Property: A number will be exactly divisible by 11 when the difference of the sum of odd place digits and even place digits is zero or divisible by 11.

Ex.9: Both the end digits of a 99 digit number N are 2 N is divisible by 11 then all the middle digits are:

(a) 1 (b) 2 (c) 3 (d) 4

Sol. (d) A number is divisible by 11 if the difference of the sum of digits at odd and even places by either zero or multiple of 11.

If the middle digit be 4, then 24442 or 244442 etc are divisible by 11.

Alternate:-

2............................2

This number has 99 digits. First (1st) and last (99th) term is 2 (given) middle terms (2nd to 98th) is assume 4.

3rd to 98th term = (4 .... 4)

differece between odd and even place of terms (3rd to 98th) = 0

Remaining Terms

1st, 2nd and 99th (last)

Here,

1st term = 2

2nd terms = 4

Last terms = 2

Difference of odd and even place of the remaning term

(2 + 2) – 4 = 0

So, If the middle digit be 4, then 24442 or 244442 etc are divisible by 11.

Ex.10: Both the end digits of a 100 digit number N are 2. N is divisible by 11 then all the middle digits are:

(a) Only 4 (b) Only 2

(c) Only 3 (d) Any digit

Sol. (d) A number is divisible by 11 if the difference of the sum of digits at odd and even places by either zero or multiple of 11.

If the middle digit be any digit then 211112 or 23333332 etc are divisible by 11.

Alternate:-

2............................2

This number has 100 digits. First (1st) and last (100th) term is 2 (given) middle terms (2nd to 99th) is assume any digit.

2rd to 99th term = (n .... n)

differece between odd and even places (2rd to 99th) = 0

Remaining Terms

1st, and 100th (last)

Here,

1st term = 2

Last terms = 2

Difference of odd and even place of the remaning terms

(2 – 2) = 0

So, If the middle digit be any digit, then (2..........2) is divisible by 11.

Ex.11: If the number 243x 51 is divisible by 9 then the value of the digit marked as x would be:

(a) 3 (b) 1 (c) 2 (d) 4

Sol. **(a)** 243x 51 is divisible by 9

divisiblty of 9 = sum of digit divisible by 9

$= 2 + 4 + 3 + x + 5 + 1 = \frac{15+x}{9}$

x would be $3 = \frac{18}{9}$

So, $x = 3$

Ex.12. $2^{71} + 2^{72} + 2^{73} + 2^{74}$) is divisible by

(a) 9 (b) 10 (c) 11 (d) 13

Sol.(b) Expression

$= 2^{71}\,(1 + 2 + 4 + 8)$

$= 2^{71} \times 15 = 2^{71} \times 3 \times 5$

Which is exactly divisible by 10.

Ex.13. A 4 digit number is formed by repeating a 2-digit number such of 2525, 3232, etc.Any number of this form is always exactly divisible by :

(a) 7 (b) 11 (c) 13

(d) Smallest 3-digit prime number

Sol.(d) Let the unit digit be x and ten's digit be y.

$\therefore$ Number

=1000y + 100x + 10y + x

= 1010y + 101x = 101 (10y + x)

Clearly, this number is divisible by 101, which is the smallest three-digit prime number.

Ex. 14: A six digit number is formed by repeating a three digit number; for example, 256, 256 or 678, 678 etc. Any number of this form is always exactly divisible by:

(a) 7 only (b) 11 only

(c) 13 only (d) 1001

Sol. (d) The number (x y z x y z) can be written, after given corresponding weightage of the places at which the digits occur, as 100000 x + 10000y + 1000z + 100+ 10y + z

= 100100x + 10010y + 1001z

= 1001 (100x + 10y + z)

Since 1001 is a factor, the number is divisible by 1001.

As the number is divisible by 1001, it will also be divisible by all three namely, 7, 11 and 13 and not by only one of these because all three are factors of 1001.

Ex.15: Which of the following number will always divide a six–digit number of the form *xyxyxy* (where $1 \le x \le 9$, $1 \le y \le 9$)?

(a) 1010 (b) 10101
(c) 11011 (d) 11010

Sol. (b) Number = *xy xy xy*
= $xy \times 10000 + xy \times 100 + xy$
= $xy\,(10000 + 100 + 1)$
= $xy \times 10101$

Ex.16. 47 is added to the product of 71 and an unknown number. The new number is divisible by 7 giving the quotient 98. The unknown number is a multiple of
(a) 2 (b) 5 (c) 7 (d) 3

Sol. **(d)** Let the unknown number be x.
$\therefore$ $71 \times x + 47 = 98 \times 7$
$\Rightarrow$ $71x = 686 - 47 = 639$
$\Rightarrow$ $x = \frac{639}{71} = 9 = 3\times3$

Ex.17. When an integer K is divided by 3, the remainder is 1, and when K + 1 is divided by 5, there remainder is 0. Of the following, a possible value of K is
(a) 62 (b) 63 (c) 64 (d) 65

Sol. **(c)** Take option (d)
When 64 is divided by 3, remainder = 1
When 65 is divided by 5, remainder = 0

Ex.18. If n is a whole number greater than 1, then $n^2\,(n^2-1)$ is always divisible by:
(a) 16 (b) 12 (c) 10 (d) 8

Sol.(b) $n^2\,(n^2-1) = n^2\,(n+1)\,(n-1)$
Now, we put values n = 2,3.....
When n = 2
$\therefore$ $n^2\,(n^2-1) = 4\times3\times1 = 12$, which is a multiple of 12
When n= 3.
$n^2\,(n^2-1) = 9\times4\times2 = 72$,
Which is also a multiple of 12.
etc.

Ex.19. If n is even, $(6^n - 1)$ is divisible by
(a) 37 (b) 35 (c) 30 (d) 6

Sol.(b) When n = 2.
$6^n - 1 = 6^2 - 1 = 36 - 1 = 35$
When, n = an even number, $a^n - b^n$ is always divisible by $(a^2 - b^2)$.

Relation among divisor, dividend quotient and remainder :

$\rightarrow$ 27 is divided by 6 then $\frac{27}{6}$

dividend
divisor 6) 27 (4 → **Quotient**
24
3 → **Remainder**

$6 \times 4 + 3 = 27$
divisor × quotient + remainder = dividend

Ex.20: In a problem involving division, the divisor is eight times the quotient and four times the remainder. If the remainder is 12, then the dividend is:
(a) 300 (b) 288 (c) 512 (d) 524

Sol. (a) Remainder = 12
Divisor = 4 × 12 = 48
Quotient = $\frac{48}{8} = 6$
Dividend = divisor × quotient + remainder = 48 × 6 + 12
= 288 + 12 = 300

Ex.21: The divisor is 25 times the quotient and 5 times the remainder. If the quotient is 16, the dividend is :
(a) 6400 (b) 6480
(c) 400 (d) 480

Sol. (b) Dividend= divisor × quotient + Remainder
According to Question
Divisor = 16×25 = 5×R
$\Rightarrow$ $R = \frac{1}{5}\times16\times25$
Dividend
$= [(16\times25)\times16] + \frac{1}{5}\times16\times25$
= [16 × 25 × 16] + 80 = 6480

Ex.22: In a division problem, the divisor is 4 times the quotient and 3 times the remainder. If remainder is 4, the dividend is:
(a) 36 (b) 40 (c) 12 (d) 30

Sol. (b)
Divisor) Dividend (Quotient
–
Remainder

According to the question
12) Dividend (3
–
4

Dividend is (Divisor × Quotient) + Remainder
= (12 × 3) + 4
= 40

Ex.23: In a division sum, the divisor is 10 times the quotient and 5 times the remainder. If the remainder is 46, the dividend is:

Sol. Remainder = 46
Divisor = 5 × 46 = 230
Quotient = $\frac{230}{10} = 23$
Dividend = divisor × quotient + remainder
= 230 × 23 + 46
= 5290 + 46 = 5336

Ex.24: On dividing 397246 by a certain number, the quotient is 865 and the remainder is 211. Find the divisor.

Sol. Divisor = $\frac{397246-211}{865} = 459$

Ex.25: A number when divided by 14 leaves a remainder of 8, but when the same number is divided by 7, it will leave the remainder:
(a) 3 (b) 2 (c) 1
(d) Can't be determined

Sol. 14) N (x
8 → Remainder
N = $14x + 8$
According to the question,
$\frac{N}{7} = \frac{\overset{0}{14x}+\overset{1}{8}}{7}$
Remainder = $\frac{8}{7} = 1$

Ex.26: If a number is divided by 102 and leaves remainder 91. If this number is divided by 17 the remainder

Sol.
dividend
divisor 102) M (P → **Quotient**
91 → **Remainder**

We know that

$\boxed{\text{divisor×quotent+ remainder =divided}}$

M = 102 × P + 91
Now = $M = \frac{17\times6\times P+91}{17}$
$M = \frac{17\times6P}{17} + \frac{91}{17}$

$$R = \frac{91}{17} \Rightarrow R = 6$$

Note: If we have to find remainder of those term which divide previous term we will take remainder of it and divide by this term and we have to get.

$$\frac{91}{17} \quad \mathbf{R = 6}$$

Ex.27: If a number is divided by 84 and leaves remainder 37. If this number is divided by 12.

Sol. Then the remainder 84 is divisible by 12

So, remainder $= \frac{37}{12} = \mathbf{1}$

Ex.28: A number when divided by 899 gives a remainder 63. If the same number is divided by 29, the remainder will be:

(a) 10 (b) 5 (c) 4 (d) 2

Sol. (b) $\frac{\text{Remainder}}{29} = \frac{63}{29}$

$\Rightarrow$ remainder = 5

Ex.29: A number when divided by 296 gives a remainder 75. When the same number is divided by 37 the remainder will be

(a) 1 (b) 2 (c) 8 (d) 11

Sol. (a) $\frac{\text{Remainder}}{37} = \frac{75}{37}$

remainddder = 1

Ex.30: A number being divided by 52 gives remainder 45. If the number is divided by 13, the remainder will be:

(a) 5 (b) 6 (c) 12 (d) 7

Sol. (b) since 13 is factor of 52. So divide its remainder by 13

Remainder $= \frac{45}{13} = 6$

Ex.31: If A number is divided by 225 a remainder at 70. But when a square of the number is divided by 15. What is the remainder?

Sol. $225\overline{)\ N\ (}Q$

$\underline{\quad\quad}$
70

N = 225 Q + 70

Square of number = N^2

= $(225\ Q + 70)^2$

Then = $\frac{(\overset{\overset{0}{\uparrow}}{225\ Q} + \overset{\overset{10}{\uparrow}}{70})}{15}$

$= \frac{(10)^2}{15} = \frac{100}{15}$

$\boxed{\text{Remainder} = 10}$

Alternate:-

$$\frac{(\text{Remainder})^2}{15} = \frac{(70)^2}{15}$$

$$= \frac{(\overset{\overset{10}{\uparrow}}{70})^2}{15} = \frac{(10)^2}{15} = \frac{100}{15}$$

Remainder = 10

Ex.32: If a number is divided 36 and leaves remainder 23. If cube of this number is divided by 12. Then what is the remainder.

Sol. $36\overline{)\ N\ (}Q$

$\underline{\quad\quad}$
23

N = 36 Q + 23

cube of number

= $N^3 = (36\ Q + 23)^3$

Now, $\frac{(\overset{\overset{0}{\uparrow}}{36\ Q} + \overset{\overset{-1}{\uparrow}}{23})^3}{12}$

$= \frac{(-1)^3}{12} = \frac{-1}{12}$

Remainder = 12 – 1 = 11

Alternate:-

Remainder = $\frac{(\overset{\overset{-1}{\uparrow}}{23})^3}{12}$

$= \frac{(-1)^3}{12} = \frac{-1}{12}$

Remainder = 12 – 1 = 11

Ex.33: Two number when divided by 17. Leave remainder 13 and 11 respectively if the sum of those two numbers is divided by 17 the remainder will be

Sol. N_1 (First Number) = $17x+13$

N_2 (Second no.) = $17y + 11$

$$\frac{(N_1+N_2)}{17} = \frac{17(x+y)}{17} + \frac{13+11}{17}$$

Remainder $= \frac{24}{17} = 7$

Ex.34: When a number is divided certain divisor, remainder is 35 but another no. is divided by the same divisor remainder is 27. If the sum of both number is divided by the same certain divisor remainder is 20. Find the certain divisor

Sol. $N_1 = Dx + 35$ .... (i)

$N_2 = Dy + 27$ ......(ii)

Here N_1 = First no.

N_2 = Second no.

D = certain divisor

x & y = Quotient

(i) + (ii)

According to the question

$$\frac{N_1+N_2}{D} = \frac{D(x+y)+62}{D}$$

Here divisor is same

Then Remainder = $D\overline{)\ 62\ (}1$

$\underline{-\ 42}$

20

Remainder = 20

Quotient = 1

Dividend = 62

Divisor = 62 – 20 × 1 = 42

$\boxed{\text{Divisor} = 42}$

Alternate:

$$\begin{array}{|ccc|} \frac{N_1}{D} & \frac{N_2}{D}, & \frac{N_1+N_2}{D} \\ \downarrow & \downarrow & \downarrow \\ R_1 & R_2 & R_3 \\ & \boxed{D = R_1+R_2-R_3} & \end{array}$$

Then divisor = 35 + 27 – 20

= **42**

Successive Division : If the quotient in a division is further used as a dividend for the next divisor and again the latest obtained divisor is used as a dividend for another divisor and so on, then it is called then " successive division" i.e, if we divide 150 by 4, we get 37 as quotient and 2 as a remainder then if 37 it divided by another divisior say 5 then we get 7 as a quotient and 2 remainder and again if we divide 7 by another divisior say 3 we get 2 as quotient and 1 as a remainder i.e, we can represent it as following

4	150		
5	37 →	2	Remainder
3	7 →	2	
	2 →	1	

Now you can see that the quotient obtained in the first division behaves as a dividend for another divisor 5. Once again the quotient 7 is treated as a dividend for the next divisor 3. Thus it is clear from the above discussion as

Dividend	Divisior	Quotient	Remainder
150	4	37	2
37	5	7	2
7	3	2	1

So the 150 is successively divided by 4, 5, and 3 the corresponding remainders are 2, 2 and 1.

Ex.35: The least possible number when successively divided by 2, 5, 4, 3 gives respective remainders of 1, 1, 3, 1 is :

(a) 372 (b) 275 (c) 273 (d) 193

Sol. The problem can be expressed as

2	A		
5	B →	1	Remainder
4	C →	1	
3	D →	3	
	E →	1	

So it can be solved as

(((((E × 3) + 1)4 + 3) 5 + 1)2 + 1) =A (where A is the required number)

So for the least possible number E = 1 (the least positive integer)

then A= (((((1 × 3) + 1) × 4 + 3) 5 + 1)2 + 1)

[Since at E = 0, we get a two digit number]

So it can be solved as

2	**193**	1
5	**96**	1
4	**19**	3
3	**4**	1
	1	

D = 1 × 3 + 1 = 4

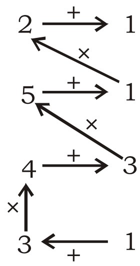
C = 4 × 4 + 3 = 19

B = 19 × 5 + 1 = 96

A = 96 × 2 + 1 = 193

So $\boxed{\text{Number= 193}}$

Alternate :



Step 1. (1 + 3) × 4 = 16

Step 2. (16 + 3) × 5 = 95

Step 3. (95 + 1) × 2 = 192

Step 4. (192 + 1) = **193**

Or

Number = (((1 + 3) × 4 + 3) × 5 + 1) × 2 + 1

= ((16 + 3) × 5 + 1) × 2 + 1

= 96 × 2 + 1

$\boxed{\text{Number = 193}}$

Ex.36: A number when divided successively by 4 and 5 leaves remainders 1 and 4 respectively. When it is successively divided by 5 and 4 the respective remainder will be

Sol. The least number × in this case will be determind as follows

4	X	
5	y –1	
	1 – 4	

y = 5 × 1 + 4 = 9

X = 4 × y + 1 = 4 × 9 + 1 = 37

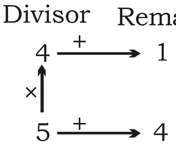
Now

5	37
4	7 –2
	1 – 3

Here, the respective remainder are 2, 3

Alternate:-

Successive Divisor Remainder



Number = (4 + 5) × 4 + 1 = 36 + 1 = 37

5	37
4	7 – 2
	1 – 3

Remainder = 2, 3

Alternate II.

4	37	1
5	9	4
	1	

1 × 5 + 4 = 9

9 × 4 + 1 = 37

Number = 37

Now, divided by 5 and 4 successively

5) 37 (7
35
2 → Remainder

4) 7 (1
4
3 → Remainder

Remainder = 2, 3

Ex.37: Find the smallest no. which one successive divided 5, 3 and 7 give remainder 2, 1 and 2 respectively

Sol.

5	**142**	2
3	**28**	1
7	**9**	2
	1	

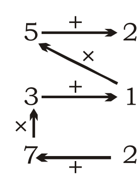
1 × 7 + 2 = 9

9 × 3 + 1 = 28

28 × 5 + 2 = 142

Number = 142

Alternate:-



$\left[\left((2+7)\times 3+1\right)\times 5\right]+2$

= (28 × 5) + 2

Number = 142

Ex.38: A least number when successively divided by 2, 3, 5 it leaves the respective remainder 1, 2 and 3. What will be the remainder if this number will be divided by 7 ?

Sol.

2	**53**	1
3	**26**	2
5	**8**	3
	1	

Step. I 5 × 1 + 3 = 8

Step. II 8 × 3 + 2 = 26

Step. III 26 × 2 + 1 = 53

So the least number = 53

According to the question,
53 is divided by 7 then remainder = **4**

Ex.39: Find the smallest no. which when successive divided by 4, 5 and 6 give remainder 2, 1 and 1. Also find sequence of remainder if the sequence of divisor is reverse.

Sol.

4	146	2
5	36	1
6	7	1
	1	

6 × 1 + 1 = 7
7 × 5 + 1 = 36
36 × 4 + 2 = 146
Number = 146
According to the question,
Now divisor is 6, 5 and 4
Then successive remainder

6	146
5	24 – 2
4	4 – 4
	1 – 0

Remainder = 2, 4 and 0

Ex.40: A number when divided successively by 6, 7 and 8, it leaves the respective remainders of 3, 5 and 4, what will be the last remainder when such a least possible number is divided successively by 8, 7 and 6.

Sol.

6	**537**	3
7	**89**	5
8	**12**	4
	1	

Step. I 1 × 8 + 4 = 12
Step. II 12 × 7 + 5 = 89
Step. III 89 × 6 + 3 = 537
least number = 537
Now we divide 537 successively by 8, 7 and 6.

8	**537**	
7	**67 —1**	**Remainder**
6	**9 —4**	
	1 —3	

So, 3 is the last Remainder.

Ex.41: A number when divided by 3 leaves a remainder1. When the quotient is divided by 2. It leaves a remainder1. What will be the remainder when the number is divided by 6?

Sol.

$$\begin{tikzcd} 3 \arrow[r, "+"] & 1 \\ 2 \arrow[u, "\times"] \arrow[r, "+"] & 1 \end{tikzcd}$$

Number = $((1+2)\times 3)+1$
= 9 + 1 = 10
According to question,

Remainder = $\frac{10}{6}$ = 4

Ex.42: A number divided by 13 leaves a remainder 1 and if the quotient is divided by 5. We got a remainder of 3. What will be the remainder if the number is divided by 65?

Sol.

$$\begin{tikzcd} 13 \arrow[r, "+"] & 1 \\ 5 \arrow[u, "\times"] \arrow[r, "+"'] & 3 \end{tikzcd}$$

Number = $[(3+5)\times 13]+1$
= 8 × 13 + 1 = 105
According to the question,

Remainder = $\frac{105}{65}$ = 40

BINOMIAL THEOREMREM

* **Statement of the theorem:-**
According to the theorem, it is possible to expand any power of x +y into a sum of the form

$$\boxed{(x+y)^n = {}^nc_0\, x^n y^0 + {}^nc_1\, x^{n-1} y^1 + {}^nc_2\, x^{n-2} y^2 + \ldots.. + {}^nc_{n-1}\, x^1 y^{n-1} + {}^nc_n\, x^0 y^n}$$

Where each $\left({}^nc_k\right)$ is a specific positive integer known as **binomial coefficient.** (When an exponent is zero, the corresponding power expression is taken to be 1 and this multiplicative factor is often omitted from the term. Hence one often sees the right side written as $\left({}^nc_0\right) x^n + \ldots.$) This formula is also referred to as the **binomial formula or the binomial identity.** Using **summation notation,** it can be written as

$$(x+y)^n = \sum_{k=0}^{n} \left({}^nc_k\right) x^{n-k} y^k = \sum_{k=0}^{n} \left({}^nc_k\right) x^k y^{n-k}$$

The final expression follows from the previous one by the symmetry of x and y in the first expression, and by comparison it follows that the sequence of binomial coefficients in the formula is symmetrical. A simple variant of the binomial formula is obtained by **substituting** 1 for y, so that it involves only a single **variable,** In this form, the formula reads

$(1+x)^n = {}^nc_0\, x^0 + {}^nc_1\, x^1 + {}^nc_2\, x^2 + \ldots.. + {}^nc_{n-1}\, x^{n-1}$
${}^nc_n\, x^n$

or equivalently

$$(1+x)^n = \sum_{k=0}^{n} \left({}^nc_k\right) x^k.$$

Ex. (i) $(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$,
(ii) $(x+y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + {}^{y4}y$

Some important points

1. The powers of x start at n and decrease by 1 in each term until they reach 0 (with {{{1}}} often unwritten);
2. The powers of y start at 0 and increase by 1 until they reach n;
3. The n^{th} row of pascal's Triangle will be the coefficients of the expanded binomial when the terms are arranged in this way;
4. The number of terms in the expansion before like terms are combined is the sum of the coefficients and is equal to 2^n, and
5. there will be **(n + 1) terms** in the expression after

combining like terms in the expansion.

The binomial theorem can be applied to the powers of any binomal. for example.

$(x + 2)^3$

$= x^3 + 3x^3 + 3x^2(2) + 3x(2)^2 + 2^3$

$= x^3 + 6x^2 + 12x + 8.$

* For a binomial involving subtraction, the theorem can be applied by using the form $(x - y)^n = (x + (-y))^n$. This has the effect of changing the sign of every other term in expansion:

$(x - y)^3 = (x + (-y))^3$

$= x^3 + 3x^2(-y) + 3x(-y)^2 + (-y)^3$

$= x^3 - 3x^2y + 3xy^2 - y^3$

* $(a^n + b^n)$ is always divisible by $(a + b)$ when $n \to$ odd power

HINT

$\boxed{a^3+b^3 = \underline{(a+b)}\,(a^2+ab+b^2)}$

Ex.43: Which of the following number will not completly divide the $(29)^{37} + (17)^{37}$?

(a) 2 (b) 11 (c) 23 (d) 46

Sol. (b) $(29^{37} + 17^{37})$, $(29 + 17) = 46$ Completely divisible by $46 = 1, 2, 23, 46$

This will be completely divisible by all the factors of 46 So 11 will not divide the given number.

Ex.44: Which of the following will not completely divide $(3^{41} + 7^{82})$?

(a) 4 (b) 52 (c) 17 (d) 26

Sol. (c) $3^{41} + 7^{82}$

$\Rightarrow$ (Equalising the power)

$\Rightarrow 3^{41} + (7^2)^{41}$

$\Rightarrow 3^{41} + 49^{41}$, $3 + 49 =$ Completely Divisible by 52

$52 = 1, 2, 4, 13, 26, 52$

So, 17 is not the factor of 52 hence this number will be completely divisible by 17

Ex.45. $(49)^{15} - 1$ is exactly divisible by:

(a) 50 (b) 51 (c) 29 (d) 8

Sol.(d) $x^n - a^n$ is exactly divisible by $(x - a)$ if n is odd.

$\therefore (49)^{15} - (1)^{15}$ is exactly divisible by $49 - 1 = 48$, that is a multiple of 8.

Ex.46: Which of the following completely divide $(29^{47} + 23^{47} + 17^{47})$

(a) 21 (b) 22 (c) 23 (d) 24

Sol. (c) $\dfrac{29^{47}+17^{47}+23^{47}}{23}$

$29^{47} + 17^{47}$ will be completely divisible by 46 or its factor (2 and 23) and 23^{47} is completely divisible by 23 so 23 will completely divide this number

* $(a^n - b^n)$ is always divisible by $(a - b)$ where $n \to$ odd power

Hint

$\boxed{(a^3+b^3) = \underline{(a-b)}\,(a^2+ab+b^2)}$

* $(a^n - b^n)$ is always completely divisible by $(a - b)$, $(a + b)$ where $n \to$ (even power)

Hint

$\boxed{(a^2 - b^2) = (a-b)\,(a+b)}$

Ex.47 Which of the following will not divide $23^{10} - 1024$ completly.

(a) 3 (b) 5 (c) 7 (d) 4

Sol. 1024 is the value of 2^{10} and

$23^{10} - 2^{10} \to (23 - 2)$ and $(23 + 2)$ is completely divisible

$(23 - 2) = 21 = 1, 3, 7, 21$

$(23 + 2) = 25 = 1, 5, 25$

Hence this number is not divisibe by 4.

(a^n+b^n) $n \to$ odd (a^n+b^n) is perfectly divisble by $(a+b)$	(a^n-b^n) $n \to$ odd (a^n-b^n) is perfectly divisble by $(a-b)$	(a^n-b^n) $n \to$ even (a^n-b^n) is perfectly divisble by $(a+b)$, $(a-b)$	(a^n+b^n) $n \to$ even It can't be determined
$\dfrac{(a^3+b^3)}{(a+b)\,(a^2-ab+b^2)}$	$\dfrac{(a^3-b^3)}{(a-b)\,(a^2+b^2+ab)}$	$\dfrac{(a^2-b^2)}{(a+b)\,(a-b)}$	$a^2+b^2=$ .....

EXERCISE

1. In a division sum, the divisor is 10 times the quotient and 5 times the remainder. If the remainder is 48, the dividend is:
(a) 808 (b) 5008
(c) 5808 (d) 8508

2. The divisor is 321, the quotient 11 and the remainder 260. Find the dividend.
(a) 3719 (b) 3971
(c) 3791 (d) 3179

3. In a division sum, the divisor is 5 times the remainder and the quotient is 6 times the remainder which is 73. What is the dividend ?
(a) 169943 (b) 159963
(c) 159943 (d) 159953

4. The sum of 20 odd natural number is equal to :
(a) 210 (b) 300 (c) 400 (d) 240

5. When a number is divided by 56, the remainder obtained is 29. What will be the remainder when the number is divided by 8 ?
(a) 4 (b) 5 (c) 3 (d) 7

6. A number when divided successively by 4 and 5 leave the remainder 1 and 4 respectively. When it is successively divided by 5 and 4 the respective remainders will be:
(a) 4,1 (b) 3,2 (c) 2,3 (d) 1,2

7. $4^{61}+4^{62}+4^{63}+4^{64}$ is divisible by :
(a) 3 (b) 10 (c) 11 (d) 13

8. $(3^{25} + 3^{26} + 3^{27} + 3^{28})$ is divisible by :
(a) 11 (b) 16 (c) 25 (d) 30

9. The least number, which must be added to 6709 to make it exactly divisible by 9, is
(a) 5 (b) 4 (c) 7 (d) 2

10. If 78*3945 is divisible by 11 where * is a digit, then * is equal to :
(a) 1 (b) 0 (c) 3 (d) 5

11. When a number is divided by 357 the remainder is 39. If same number is divided by 17, the remainder will be :
(a) 0 (b) 3 (c) 5 (d) 11

12. A number when divided by 6 leaves remainder 3. When the square of the same number is divided by 6, the remainder is :
(a) 0 (b) 1 (c) 2 (d) 3

13. When a number is divided by 893, the remainder is 193. What will be remainder when it is divided by 47 ?
(a) 3 (b) 5 (c) 25 (d) 33

14. A number divided by 13 leaves a remainder 1 and if the quotient, thus obtained, is divided by 5, we get a remainder of 3. What will be the remainder if the number is divided by 65 ?
(a) 28 (b) 16 (c) 18 (d) 40

15. Which of the following number is NOT divisible by 18 ?
(a) 54036 (b) 50436
(c) 34056 (d) 65043

16. If n is an integer, then (n^3 - n) is always divisible by :
(a) 4 (b) 5 (c) 6 (d) 7

17. A 4 digit number is formed by repeating a 2 digit number such as 2525, 3232, etc. Any number of this form is always exactly divisible by:
(a) 7 only (b) 11 only
(c) 13 only (d) Smallest 3 digit prime number

18. If two numbers are each divided by the same divisor, the remainders are respectively 3 and 4. If the sum of the two numbers be divided by the same divisor, the remainder is 2. The divisor is :
(a) 9 (b) 7 (c) 5 (d) 3

19. A number when divided by 5 leaves remainder 3. What is the remainder when the square of the same number is divided by 5 ?
(a) 1 (b) 2 (c) 3 (d) 4

20. If the number 4 8 3 2 7 * 8 is divisible by 11, then the missing digit (*) is
(a) 5 (b) 3 (c) 2 (d) 1

21. A number, when divided by 136, leaves remainder 36. If the same number is divided by 17, the remainder will be
(a) 9 (b) 7 (c) 3 (d) 2

22. Two numbers, when divided by 17, leaves remainder 13 and 11 respectively. If the sum of those two numbers is divided by 17, the remainder will be :
(a) 13 (b) 11 (c) 7 (d) 4

23. A number, when divided by 221, leaves a remainder 64. What is the remainder if the same number is divided by 13?
(a) 0 (b) 1 (c) 11 (d) 12

24. When a number is divided by 387, the remainder obtained is 48. If the same number is divided by 43, the remainder obtained will be ?
(a) 0 (b) 3 (c) 5 (d) 35

25. When two number are separately divided by 33, the remainders are 21 and 28 respectively. If the sum of the two number is divided by 33, the remainder will be ?
(a) 10 (b) 12 (c) 14 (d) 16

26. ($2^{71} + 2^{72} + 2^{73} + 2^{74}$) is divisible by :
(a) 9 (b) 10 (c) 11 (d) 13

27. When 'n' is divisible by 5 the remainder is 2. What is the remainder when n^2 is divided by 5 ?
(a) 2 (b) 3 (c) 1 (d) 4

28. A number when divided by 49 leaves 32 as remainder. The number when divided by 7 will have the remainder as:
(a) 4 (b) 3 (c) 2 (d) 5

29. When a number is divided by 36, the remainder is 19. What will be the remainder when the number is divided by 12 ?
(a) 7 (b) 5 (c) 3 (d) 0

30. In a division sum, the divisor is 10 times the quotient and 5 times the remainder. If the remainder is 46, then the dividend is :
(a) 4236 (b) 4306
(c) 4336 (d) 5336

31. When a number is divided by 24, the remainder is 16. The remainder when the same number is divided by 12 is
(a) 3 (b) 4 (c) 6 (d) 8

32. The expression $8^n - 4^n$, where n is a natural number is always divisible by
(a) 15 (b) 18 (c) 36 (d) 48

33. ($4^{61}+4^{62}+4^{63}$) is divisible by
(a) 3 (b) 11 (c) 13 (d) 17

34. When an integer K is divided by 3, the remainder is 1, and when K + 1 is divided by 5, the remainder is 0. Of the following, a possible value of K is:
(a) 62 (b) 63 (c) 64 (d) 65

35. A number when divided by 91 gives a remainder 17. When the same number is divided by 13, the remainder will be :
(a) 0 (b) 4 (c) 6 (d) 3

36. A number when divided by 280 leaves 115 as remainder. When the same number is divided by 35, the remainder is:
(a) 15 (b) 10 (c) 20 (d) 17

37. A certain number when divided by 175 leaves a remainder 132. When the same number is divided by 25, the remainder is:
(a) 6 (b) 7 (c) 8 (d) 9

38. Which one of the following will completely divide by $5^{71} + 5^{72} + 5^{73}$
(a) 150 (b) 160 (c) 155 (d) 30

39. Which of the following numbers will always divide a six-digit number of the form $xyxyxy$ (where $1 \leq x \leq$, $1 \leq y < 9$)?
(a) 1010 (b) 10101
(c) 11011 (d) 11010

40. A positive integer when divided by 425 gives remainder 45. When the same number is divided by 17, the remainder will be
(a) 5 (b) 2 (c) 11 (d) 13

41. A number x when divided by 289 leaves 18 as the remainder. The same number when divided by 17 leaves y as a remainder. The value of y is
(a) 5 (b) 2 (c) 3 (d) 1

42. When n is divided by 6, the remainder is 4. When 2n is divided by 6, the remainder is:
(a) 2 (b) 0 (c) 4 (d) 1

43. In a division sum, the divisor is 3 times the quotient and 6 times the remainder. If the remainder is 2, then the dividend is :
(a) 50 (b) 48 (c) 36 (d) 28

44. In a divison sum, the divisor is 12 times the quotient and 5 times the remainder. If the remainder is 36, then the dividend is :

(a) 2706 (b) 2726

(c) 2736 (d) 2262

45. For any integral value of n, $3^{2n} + 9n + 5$ when divide by 3 will leave the remainder

(a) 1 (b) 2

(c) 0 (d) 5

46. The quotient when 10^{100} is divided by 5^{75} is :

(a) 10^{25} (b) 2^{75}

(c) $2^{75}\times10^{25}$ (d) $2^{25}\times10^{75}$

47. The remainder obtained when $23^3 + 31^3$ is divided by 54

(a) 0 (b) 1

(c) 3 (d) C.N.D

48. $(19^5 + 21^5)$ is divisible by

(a) Only 10

(b) Only 20

(c) Both 10 & 20

(d) Neither 10 nor 20

49. If $(17)^{41} + (29)^{41}$ is divided by 23. Find the remainder

(a) 1 (b) 6 (c) 0 (d) 12

50. If $(3)^{41} + (7)^{82}$ always divisible by

(a) 10 (b) 49 (c) 52 (d) 44

51. If $m^n - n^m = (m + n)$; $(m, n) \in$ prime numbers, then what can be said about m and n:

(a) m, n are only even integers

(b) m, n are only odd integers

(b) m is even and n is odd

(d) none of these

ANSWER KEY

1. (c)	6. (c)	11. (c)	16. (c)	21. (d)	26. (b)	31. (b)	36. (b)	41. (d)	46. (c)
2. (c)	7. (b)	12. (d)	17. (d)	22. (c)	27. (d)	32. (d)	37. (b)	42. (a)	47. (a)
3. (c)	8. (d)	13. (b)	18. (c)	23. (d)	28. (a)	33. (a)	38. (c)	43. (a)	48. (c)
4. (c)	9. (a)	14. (d)	19. (d)	24. (c)	29. (a)	34. (c)	39. (b)	44. (c)	49. (c)
5. (b)	10. (d)	15. (d)	20. (d)	25. (d)	30. (d)	35. (b)	40. (c)	45. (b)	50. (c)
									51. (c)

SOLUTION

1. (c) Remainder = 48

Divisor = 48 × 5 = 240

Quotient $= \frac{240}{10} = 24$

Dividend = 240 × 24 + 48

= 5760 + 48

= 5808

2. (c) Dividend = Divisor × Quotient + Remainder

= 321 × 11 + 260

= 3531 + 260 = 3791

3. (c) Remainder = 73

Quotient = 6×73 = 438

Divisor = 5×73 = 365

Dividend = 365 × 438 + 73

= 159943

4. (c) 1, 3, 5, 7 ....20th term

a = 1, d = 2, n = 20

sum $= \frac{n}{2}[2a + (n - 1)d]$

$= \frac{20}{2}[2 \times 1+(20-1)2]$

= 10[2×1 + 19 ×2] = 400

Alternate :

The sum of first n odd natural numbers = $n^2 = 20^2 = 400$

5. (b) $\frac{\text{Remainder}}{8} = \frac{29}{8}$

Remainder = 5

6. (c) Number is divided successively

		Remainder
4	37	1
5	9	4
	1	

5×1+4=9

9×4+1= 37

Number is 37

5	37	2
4	7	3
	1	

Remainder is 2, 3

7. (b) $4^{61} + 4^{62} + 4^{63} + 4^{64}$

$= 4^{61}(4^0 + 4^1 + 4^2 + 4^3)$

$= 4^{61}(1 + 4 + 16 + 64)$

$= 4^{61} \times 85$

$= 4^{60} \times 4 \times 85$

$= 4^{60} \times 340$

$= 4^{60} \times 34 \times 10$

Now, check with option

Only, check with option

Only 10 can divide this.

8. (d) $(3^{25} + 3^{26} + 3^{27} + 3^{28})$

$= 3^{35}(3^0 + 3^1 + 3^2 + 3^3)$

$= 3^{25}(1 + 3 + 9 + 27)$

$= 3^{25} \times 40 = 3^{24} \times 120$

Now, check with option

Only, check with option

Only 30 can divide this.

9. (a) 6709

⇒ 6+7+0+9=22

[9 - (divisibility property)

Sum of digits must be divisible by 9]

So 22 + 5 = 27 is divisible by 9

5 is answer

10. (d) 78 * 3945

Odd place : 7 + * + 9 + 5 = 21 + *

Even place : 8 + 3 + 4 = 15

(21+ *) –(15)= either 11 or 0

(21 + *) – 15 = 11

21 + * = 26

* = 5

11. (c) $\frac{\text{Remainder of number}}{17} = \frac{39}{17}$

⇒ remainder = 5

12. (d) Shorcut Method

Let number is: 9 (Gives remainder 3 when divided by 6)

Now $\frac{9^2}{6} = \frac{81}{6}$ ⇒ Remainder = 3

13. (b) $\frac{\text{Remainder of no.}}{47} = \frac{193}{47}$

$\Rightarrow$ remainder = 5

14. (d)

13	105	1
5	8	3
	1	

$5 \times 1 + 3 = 8$

$13 \times 8 + 1 = 105$

remainder = $105 \div 65$

Remainder = 40

15. (d) A number will be divisible by 18 if it is divisible by 2 and 9

Clearly we can see 65043 is not divisible by 2. Because unit digit of 65043 is 3 so this will not be divsible by 18

16. (c) $(n^3 - n)$ and n is any integer.

put n = 2 so, $2^3 - 2 = 6$

It will be always divisble by 6 (Put n = 2,3,4...)

17. (d) Smallest 3 digit prime number is '101'

xyxy is always divisible by 101

Hence, 101 Will be the divisor.

18. (c) **Shortcut Method**

divisor = Remainder 1 + Remainder 2 – Remainder 3

$= 3 + 4 - 2 = 7 - 2 = 5$

19. (d) Let no. be 8

$\Rightarrow \frac{8^2}{5} = \frac{64}{5}$

= 4 remainder

Alternate:-

$$\text{Remainder} = \frac{(\text{Remainder})^2}{5}$$

$$= \frac{(3)^2}{5} = \frac{9}{5} = 4$$

20. (d) 4 8 3 2 7 * 8

odd place $\Rightarrow 4 + 3 + 7 + 8 = 22$

Even place $\Rightarrow 8 + 2 + * = 10 + *$

Difference should be either zero or 11,22,33 ....etc.

$\Rightarrow 22 - (10 + *) = 11$

$22 - 10 - * = 11$

$12 - * = 11$

$* = 1$

21. (d) $\frac{\text{Remainder of no.}}{17} = \frac{36}{17}$

$\Rightarrow$ remainder = 2

22. (c) (dividend = divisor × quotient + remainder)

First no. = (17 × n) +13

Let 'n' = 1

$\Rightarrow$ (17 × 1) + 13

$\Rightarrow$ 30

Second no. = (17 × n) + 11

= (17 × 1) + 11 = 28

According to question

$\frac{30+28}{17} = \frac{58}{17} \Rightarrow$ remainder = 7

Alternate:-

Divisor = Remainder 1 + Remainder 2 – Remainder 3

17 = 13 + 11 – Remainder 3

Remainder 3 = 24 – 17 = 7

23. (d) $\frac{\text{Remainder of no.}}{13} = \frac{64}{13}$

$\Rightarrow$ remainder = 12

24. (c) $\frac{\text{Remainder of no.}}{43} = \frac{48}{43}$

$\Rightarrow$ remainder = 5

25. (d) first no. = (33 × n) + 21

Let no = 1

= (33 × 1) + 21 = 54

Second no. = (33 × n) + 28

= (33 × 1) + 28 = 61

According to question

$\frac{54+61}{33} \Rightarrow \frac{115}{33}$

$\Rightarrow$ 16 Remainder

Alternate:-

Divisor = Remainder 1 + Remainder 2 – Remainder 3

33 = 21 + 28 – Remainder 3

Remainder 3 = 16

26. (b) $(2^{71} + 2^{72} + 2^{73} + 2^{74})$

$= 2^{71}(2^0 + 2^1 + 2^2 + 2^3)$

$= 2^{71}(1 + 2 + 4 + 8)$

$= 2^{71} \times 15 = 2^{70} \times 30$

It is divisible by 10

27. (d) $\frac{n}{5} \Rightarrow$ remainder 2

If we put n = 7 Then it satisfies above situation

So n = 7

$\frac{n^2}{5} = \frac{7^2}{5} = \frac{49}{5} \Rightarrow$ remainder= 4

28. (a) $\frac{\text{remainder of no.}}{7} = \frac{32}{7}$

$\Rightarrow$ Remainder = 4

29. (a) $\frac{\text{remainder of no.}}{12} = \frac{19}{12}$

$\Rightarrow$ remainder = 7

30. (d)

Quotient	:	Divisor	:	Remainder
1	:	10		
		5	:	1
1	:	10	:	2
$\downarrow \times 23$		$\downarrow \times 23$		$\downarrow \times 23$
23	:	230	:	46

Dividend = (Divisor × Quotient) + Remainder

= (230 × 23) + 46= 5336

31. (b) $\frac{\text{Remainder of no.}}{12} = \frac{16}{12}$

= 4 is remainder

32. (d) $8^n - 4^n$

n = 1,2,3.............(n is a natural number)

Put, n = 2,

expression = $8^2 - 4^2 = 64 - 16 = 48$

$\therefore$ $8^n - 4^n$ is divisible by 48

48 is completely divisible by 4 so f^n is divisible 4

33. (a) $(4^{61} + 4^{62} + 4^{63})$

$= 4^{61}(4^0 + 4^1 + 4^2)$

$= 4^{61}(1 + 4 + 16) = 4^{61} \times 21$

Now check the options

Only 3 divides it. So '3' is answer

34. (c) Always do these types of question by options to save time

Pick up the option and follow the question instruction

take option (c)

64 $\Rightarrow$ Divide 3 it gives remainder 1

Now add 1 to 64

$\frac{65}{5} \Rightarrow$ remainder '0' it satisfies

So, k = 64 this is answer

35. (b) $\frac{\text{Remainder of no.}}{13} = \frac{17}{13}$

remainder = 4

36. (b) $\frac{\text{Remainder of no.}}{35} = \frac{115}{35}$

Remainder = 10

37. (b) $\frac{\text{Remainder of no.}}{25} = \frac{132}{25}$

remainder = 7

38. (c) $5^{71} + 5^{72} + 5^{73}$

$= 5^{71}\ (5^0 + 5^1 + 5^2)$

$= 5^{71}\ (1 + 5 + 25)$

$= 5^{71} \times 31 = 5^{70} \times 155$

Check with option,

So 155 is answer

39. (b) Number = *xyxyxy*

$= xy \times 10000 + xy \times 100 + xy$

$= xy\ (10000 + 100 + 1)$

$= xy(10101)$

Hence, option (B) will divide answer

Alternate:

You can assume (121212, 343434..........) any number divisible by option, So that number is divisible by exactly that's the answer

40. (c) $\frac{\text{Remainder of no.}}{17} = \frac{45}{17}$

$\Rightarrow$ remainder = 11

41. (d) $\frac{\text{Remainder of no.}}{17} = \frac{18}{17}$

$\Rightarrow$ remainder = 1

42. (a) $\frac{n}{6}$ = remainder 4

If n = 10 $\Rightarrow \frac{10}{6}$

$\Rightarrow$ remainder = 4 (matched) n = 10

$2n = 2 \times 10 \Rightarrow \frac{20}{6}$

$\Rightarrow$ remainder = 2

Note : Always put value in these type of questions.

43. (a)

Remainder	Divisor	Quotient
	3	1
1	6	
1	6	2
$\downarrow \times 2$	$\downarrow \times 2$	$\downarrow \times 2$
Actual $\rightarrow$ 2	12	4

Dividend = (Divisor × Quotient) + remainder

$= (12 \times 4) + 2 = 50$

44. (c)

Remainder	Divisor	Quotient
	12	1
1	5	
12	60	5
$\downarrow \times 3$	$\downarrow \times 3$	$\downarrow \times 3$
36	180	15

Dividend = (divisor × quotient) + Remainder

$= (180 \times 15) + 36$

$= 2736$

45. (b) $3^{2n} + 9n + 5$

Put n = 1

$\Rightarrow 3^{2\times1} + 9\times 1 + 5$

$\Rightarrow 9 + 9 + 5 \Rightarrow 23$

$\Rightarrow \frac{23}{3} \Rightarrow$ remainder = 2

Note: value of n can be 1,2,3,4, ...

46. (c) $10^{100} \div 5^{75}$

$\frac{2^{100} \times 5^{100}}{5^{75}} = 2^{100} \times 5^{25} = 2^{25}\ .2^{75}\ .5^{25}$

$= 2^{75} \times 10^{25}$

47. (a) We know that $(a^n + b^n)$ is always divisible $(a + b)$ then. where n $\rightarrow$ odd power

$(23^3 + 31^3)$ is Always divisible by $(23 + 31) = 54$

So remainder is '**0**'

48. (c) $(a^n + b^n)$, is always divisible by $(a + b)$

When n $\rightarrow$ odd power

$(19 + 21) = 40$

Factor of 40 (1, 2, 4, 5, 10, 20, 40) is divisible by $(19^5 + 21^5)$ then options 10 & 20 is divisible

49. (c) $(a^n + b^n)$, is always divisible $(a + b)$

When n is odd power

Then,

$(17^{41} + 29^{41})$ is always divisible by $(17 + 29) = 46$

factor of 46 (1, 2, 23, 46)

So, $(17^{41} + 29^{41})$ is perfectly divisible by 23

hence, Remainder '0'

50. (c) $3^{41} + 7^{82}$

Equalising the power

$3^{41} + (7^2)^{41} = 3^{41} + 49^{41}$

$3^{41} + 49^{41}$ is always divisible $(3 + 49) = 52$

So 52 is divisible by $(3^{41} + 7^{82})$

51. (c) $m^n - n^m = m + n$

Consider m = 2 and n = 5, then

$2^5 - 5^2 = 5 + 2$

$7 = 7$

Thus option (a) and (b) are wrong and option (c) is correct.

CHAPTER 05

REMAINDER THEOREM

Ex:- What remainders can be possible when 25 is divided by 7

$+4 \nwarrow \frac{25}{7} \nearrow -3$

$7\,)\,25\,(\,3$
$\underline{21}$
$+4 \longrightarrow$ Actual Remainder

or

$7\,)\,25\,(\,4$
$\underline{-28}$
$-3 \longrightarrow$ Negative Remainder

Remainder is always positive but some times we use negative remainder for our convenience if 25 is divided by 7 then actual remainder will be + 4 but – 3 can be used for convenience for actual remainder multiple of 7, less than 25 is 21 hence actual remainder will be + 4 and for negative remainder we have to see the multiple of 7 greater than 25, which is 28 so – 3 will be the remainder

Ex:- What will be the remainder when 37 is divided by 9

$9\,)\,37\,(\,4$
$\underline{-36}$
$+1 \longrightarrow$ Actual Reminder

or

$9\,)\,37\,(\,5$
$\underline{-45}$
$-8 \longrightarrow$ Negative Remainder

When 37 is divided 9, then the multiple of 9 smaller than 37 is 36. Hence actual remainder will be +1 It we want a negative remainder we have to see the multiple of 9 greater than 37 which is 45, hence – 8 will be the negative Remainder.

$+1 \nwarrow \frac{37}{9} \nearrow -8$

Ex:- $+6 \nwarrow \frac{55}{7} \nearrow -1$

Ex:- $+5 \nwarrow \frac{167}{6} \nearrow -1$

* '0' is the smallest divisible number when 0 is divided by any number always remainder will be 0

Ex:- $\frac{0}{7}$ $\quad 7\,)\,0\,(\,0$
$\underline{-0}$
0

* when 0 is divided by 7, then 0^{th} multiple of 7 is $(7 \times 0 = 0)$ then 0 is subtracted from 0, we will get zero.

Ex:- When 45 is divided by 14 then

$14\,)\,45\,(\,3$
$\underline{-42}$
$+3 \longrightarrow$ Actual Remainder

or

$14\,)\,45\,(\,4$
$\underline{-56}$
$-11 \longrightarrow$ Negative Remainder

$+3 \nwarrow \frac{45}{14} \nearrow -11$

Ex:- $+1 \nwarrow \frac{73}{8} \nearrow -7$

Ex:- $+3 \nwarrow \frac{111}{4} \nearrow -1$

Ex:- $\frac{0}{100}$, Remainder = 0

$100\,)\,0\,(\,0$
$\underline{0}$
$0 \longrightarrow$ Remainder

Ex:- $+13 \nwarrow \frac{13}{15} \nearrow -2$

$15\,)\,13\,(\,0$
$\underline{0}$
13

or

$15\,)\,13\,(\,1$
$\underline{15}$
-2

* When 13 is divided by 15, then the multiple of 15 which is less than 13 is 0. which is 0^{th} multiple of 15. Hence actual remainder will be +13 and for the negative remainder we have to see the multiple of 15 which should be greater than 13, Now 15 is the multiple of 15 greater than 13, so remainder will be - 2

Ex:- $+2 \nwarrow \frac{2}{3} \nearrow -1$

$3\,)\,2\,(\,0$
$\underline{-0}$
$+2 \longrightarrow$ Actual Remainder

$3\,)\,2\,(\,1$
$\underline{-3}$
$-1 \longrightarrow$ Negative Remainder

Ex:-

$+4 \nwarrow \frac{4}{7} \nearrow -3$

$7\,)\,4\,(\,0$
$\underline{0}$
$+4$

$7\,)\,4\,(\,1$
$\underline{-7}$
-3

TYPE – 1

Ex.1 what will be the remainder when 23×34 is divided by 9

Sol. $\frac{23\times 34}{9}$

when 23 is divided by 9 the remainder is

+5   -4

$\frac{23}{9}$

When 34 is divided by 9, the remainder is

+7   -2

$\frac{34}{9}$

then $\frac{\overset{+5}{23} \times \overset{+7}{34}}{9}$ $=5\times7= 35$

The sign will be the same between remainders as in the process. For Ex (23 × 34). Here we see that the sign b/w 23 and 34 is (×), So, the sign b/w remainders will be (×). If the product of remainder is greater than divisor, we have to divide it again to get the remainder

In this process when 23 is divided by 9, remainder +5 has been used and when 34 is divided by 9 remainder +7 has been used we can see that the sign between the proces is (×), then the product of remainders is (5 × 7) = 35,

Which is greater than 9. Now again we have to divide 35 by 9 we will get + 8 or - 1 as remainder. If the remainder is negative (-1) it should be deducted from divisor, so we will get positive (+ve) remainder)

$\frac{\overset{+5}{23} \times \overset{+7}{34}}{9} = \frac{5\times 7}{9}$

+8   -1

$= \frac{35}{9} = \boxed{+8 \text{ (Remander)}}$

or

$\boxed{9 - 1 = 8}$

Alternate II

$\frac{\overset{-4}{23}\times \overset{-2}{34}}{9} = \frac{-4\times -2}{9} = 8$

+5   -4   +7   -2

$\frac{23}{9}$ $\frac{34}{9}$

Now this time we have used negative remainder. If 23 is divided by 9, the remainder will be -4 and if 34 is divided by 9, the remainder will be - 2 As there is (×) sign in the process, the product of the remainders is (+8) As the product is less than divisor so there is no need to divide it again.

Alternate III

$\frac{\overset{-4}{23}\times \overset{+7}{34}}{9}$

$= -4 \times 7 = -28$

After dividing 23 by 9 remainder - 4 has taken and after dividing 34 by 9, remainder (+7) has taken. Now the product of the remainders are (-28). We will neglect the (-ve) sign and again will get the remainder by dividing fist process. After that we will put (-ve) sign. It the remainder is negative, then we will get (+ve) remainder by adding divisor into it.

$\frac{\overset{-4}{23}\times \overset{+7}{34}}{9} = -28$

Negelecting (-ve) sign

$= \frac{\overset{+1}{28}}{9}$ Now dividing by general process

= +1 (Again putting (-ve) sign)

= – 1 = 9 – 1

(To get (+ve) remainder) = 8

Same Remainder in each process

Ex.2 What will be the remainder when 43 × 83 is divided by 21?

Sol.

$\frac{43\times 83}{21}$

+1   -20   +20   -1

$\frac{43}{21}$ $\frac{83}{21}$

Wheather remainder is + ve or negative smaller remainder should be used for the easier calculation. If 43 is divided by 21, the smaller remainder will be (+1) and If 83 is divided by 21 the smaller remainder will be – 1,

$\frac{\overset{+1}{43} \times \overset{-1}{83}}{21} = \frac{1\times -1}{21} = -1+21$

= 20

Ex.3 What will be the remainder when $\frac{121+93}{8}$

+1   -7   +5   -3

$\frac{121}{8}$ $\frac{93}{8}$

By using smaller remainders

$\frac{\overset{+1}{121} + \overset{-3}{93}}{8} = \frac{1-3}{8} = -2$

= 8 – 2 = 6

In this operation we have used (+ve) sign. So the same sign (+) will be used b/w the remainders. (1 –3) = (–2) the remainder is (-ve). So, to get actual remainder we have to add 8 hence actual remainder will be 6.

Ex.4 What will be the remainder when $\frac{130+147}{11}$

Sol. +9   -2   +4   -7

$\frac{130}{11}$ $\frac{147}{11}$

By using smaller remainder

$\frac{\overset{-2}{130} + \overset{+4}{147}}{11} = \frac{-2+4}{11} = 2$

So, remainder is 2

Ex.5 When 127 × 139 × 12653 × 79 × 18769 is divided by 5, the remainder will be.

$$\frac{\overset{+2}{12\underline{7}} \times \overset{-1}{13\underline{9}} \times \overset{-2}{1265\underline{3}} \times \overset{-1}{7\underline{9}} \times \overset{-1}{1876\underline{9}}}{5}$$

Divisiblity of 5 can be examined by dividing the last digit of the number

$$\Rightarrow \frac{2\times -1\times -2\times -1\times -1}{5} = \frac{4}{5} = 4$$

Hence remainder is 4

Ex.6 What will be the remainder when $127+139+12653+79+18769$ is divided by 5

Sol.

$$\frac{\overset{+2}{12\underline{7}} + \overset{-1}{13\underline{9}} + \overset{-2}{1265\underline{3}} + \overset{-1}{7\underline{9}} + \overset{-1}{1876\underline{9}}}{5}$$

$$\frac{2-1-2-1-1}{5} = \frac{-3}{5} = -3$$

$= 5 - 3 = 2$

Ex.7 What will be the remainder when 195×1958×1975×170 is divided by 19.

$$= \frac{\overset{+5}{195} \times \overset{+1}{1958} \times \overset{-1}{1975} \times \overset{-1}{170}}{19}$$

$$= \frac{5\times 1\times -1\times -1}{19} = 5$$

Ex.8 What will be the remainder when 1750 × 1748 × 1753 × 70 × 35 is divided by 17

Sol.

$$\frac{\overset{-1}{175\underline{0}} \times \overset{-3}{17\underline{4}8} \times \overset{+2}{17\underline{5}3} \times \overset{+2}{\underline{70}} \times \overset{+1}{\underline{35}}}{17}$$

$$\frac{-1\times -3\times 2\times 2\times 1}{17} = 12$$

Hence Remainder is 12

Ex.9 What will be the remainder when (1750 + 1748 + 1752 + 70 + 35) is divided by 17 ?

Sol.

$$\frac{\overset{-1}{175\underline{0}} + \overset{-3}{17\underline{4}8} + \overset{+1}{17\underline{5}2} + \overset{+2}{70} + \overset{+1}{35}}{17}$$

$$= \frac{-1-3+1+2+1}{17} = \frac{0}{17} = 0$$

Hence remainder is 0

Ex.10 When $\lfloor 1+\lfloor 2+\lfloor 3+\lfloor 4+\lfloor 5 \ldots\ldots + \lfloor 1000$ is divided by 10 the remainder will be

$\lfloor\ \rightarrow$ It is sign of Factorial.
$\lfloor 1 \rightarrow 1$
$\lfloor 2 \rightarrow 1 \times 2 = 2$
$\lfloor 3 \rightarrow 1 \times 2 \times 3 = 6$
$\lfloor 4 \rightarrow 1 \times 2 \times 3 \times 4 = 24$
$\lfloor 5 \rightarrow 1 \times 2 \times 3 \times 4 \times 5 = 120$
$\lfloor 0 \rightarrow$ value is 1

$\lfloor 1 = \frac{1}{10}$ (+1, −9)

$\lfloor 2 = 1\times 2 = \frac{2}{10}$ (+2, −8)

$\lfloor 3 = 1\times 2\times 3 = \frac{6}{10}$ (+6, −4)

$\lfloor 4 = 1\times 2\times 3\times 4 = \frac{24}{10}$ (+4, - 6)

$\lfloor 5 = 5\times 4\times 3\times 2\times 1 = \frac{120}{10}$

=remainder =0

Value of $\lfloor 5$ is 120 which is completly divisible by 10, Hence the reamainder (In the same way) will be 0.

In the same way

$\lfloor 6 = 6\times 5\times 4\times 3\times 2\times 1 = \frac{720}{10}$

= Reamainder = 0

* $\lfloor 7$, $\lfloor 8$.........$\lfloor 1000$ is divided by 10, 0 will be the remainder in each case. So by using smaller remainder

$$\frac{\overset{+1}{\lfloor 1} + \overset{+2}{\lfloor 2} + \overset{-4}{\lfloor 3} + \overset{+4}{\lfloor 4} + \overset{0}{\lfloor 5} + \overset{0}{\lfloor 6} \ldots\ldots\ldots \overset{0}{\lfloor 1000}}{10}$$

$$= \frac{1+2-4+4}{10} = \frac{3}{10} = \boxed{3}$$

Ex.11 What will be the remainder when $\lfloor 1+\lfloor 2+\lfloor 3+\lfloor 4+ \ldots\ldots\ldots \lfloor 1000$ is divided by 12 ?

$\lfloor 1 = 1 = \frac{1}{12}$ (1, -11)

$\lfloor 2 = 1\times 2 = \frac{2}{12}$ (+2, -10)

$\lfloor 3 = 1\times 2\times 3 = \frac{6}{12}$ (+6, -6)

$\lfloor 4 = 1\times 2\times 3\times 4 = \frac{24}{12}$

= Remainder = 0

Hence all the factorial next to will be completly divisible by 12 So, '0' will be the remainder is each case

$$\Rightarrow \frac{1+2+6}{12} = \frac{9}{12} = 9$$

Remainder = 9

Ex.12 Which of the following will completly divide

$\lfloor 1+\lfloor 2+\lfloor 3+\lfloor 4+\lfloor 5+\lfloor 6 \ldots\ldots\ldots \lfloor 1000$

(a) 10 (b) 9

(c) 12 (d) 8

Sol. In such type of question you can take the help of Options to save your valuable time

Option 'b'

$$\frac{\overset{+1}{\lfloor 1} + \overset{+2}{\lfloor 2} + \overset{-3}{\lfloor 3} + \overset{-3}{\lfloor 4} + \overset{+3}{\lfloor 5} + \overset{0}{\lfloor 6} + \overset{0}{\lfloor 7} \ldots\ldots \overset{0}{\lfloor 1000}}{9}$$

$$= \frac{1+2-3-3+3}{9} = 0$$

Hence 0 is the remainder

Hence this number is divisible by 9

* The number is divided by 10 to get unit digit
* The number is divided by100 (10^2) to get last two digits
* The number is divided by[$(10)^3$]to get last three digit
* This process will continues as it is

Last Two Digit (अन्तिम 2 अंक) ⟶

Ex.13 Find the last two digit of the product

23 × 13999 × 497 × 73 × 96

Sol. This number should be divided by 100 to get last two digit.

$$\frac{123\times 13999\times 497\times 73\times \cancel{96}^{\,24}}{\cancel{100}_{\,25}}$$

In such type of process we simplify the operation firstly. The number by which we simplify the operation, the same number is multiplied in the last. In this case 96 and 100 are simplified. So, we multiply by 4 in the last

$$\frac{1\overset{-2}{\underline{23}}\times 13\overset{-1}{\underline{99}}\times 4\overset{-3}{\underline{97}}\times \overset{-2}{\underline{73}}\times \overset{-1}{\underline{24}}}{25}$$

$$=\frac{-12}{25}=-12$$

$\Rightarrow \quad 25-12=13$

To get last two digit we multiply it by 4.

13 × 4 = 52

So, the last two digit is 52 (5 and 2)

* divisibility of 25 → last 2 digits divisible by 25

Ex.14 39 × 55 × 57 × 24 × 13872 × 9871 Find the last two digits

Sol. $$=\frac{39\times \overset{11}{\cancel{55}}\times 57\times \overset{6}{\cancel{24}}\times 13872\times 9871}{\cancel{100}\ \cancel{20}\ 5}$$

Simplifying two times by 4 and 5. So, to get last two digit we have to multiply 20 (4 × 5)

$$\frac{\overset{-1}{39}\times \overset{+1}{11}\times \overset{+2}{57}\times \overset{+1}{6}\times 138\overset{+2}{72}\times 98\overset{+1}{71}}{5}$$

* divisibility of 5 → last 1 digit divisible by 5

$$=\frac{-1\times 1\times 2\times 1\times 2\times 1}{5}=\frac{-4}{5}=-4$$

$=5-4=1$

So, actual last two digits

1 × 20 = 20 (2 and 0)

Ex.15 173 × 192 × 99 × 96 find the last two digits

Sol. $$\frac{173\times 192\times 99\times \overset{24}{\cancel{96}}}{\cancel{100}\ 25}$$

Simplifying by 4

$$\frac{\overset{-2}{173}\times \overset{-8}{192}\times \overset{-1}{99}\times \overset{-1}{24}}{25}$$

$$=\frac{-2\times -8\times -1\times -1}{25}=\frac{16}{25}=16$$

So, Actual last two digit

= 16 × 4 = 64 (6 and 4)

Ex.16 87× 92 × 194 × 44 Find the last two digits ?

Sol. $$\frac{87\times \overset{23}{\cancel{92}}\times 194\times 44}{\cancel{100}\ 25}$$

Simplifying by 4

$$\frac{\overset{-13}{87}\times \overset{-2}{23}\times \overset{-6}{194}\times \overset{-6}{44}}{25}$$

$$\frac{\underline{-13\times -2}\times \underline{-6\times -6}}{25}$$

$$\frac{\overset{+1}{26}\times \overset{+11}{36}}{25}$$

$$\frac{1\times 11}{25}=11$$

So, Actual last two digits

$=11\times 4=\boxed{44}$

Ex.17 What will be the remainder when 25 is divided by 13

Sol. 12 ← $\frac{25}{13}$ → -1 = -1

= 13 - 1 = 12

$\boxed{\text{Remainder is always positive}}$

Ex.18 Find the remainder $\frac{(25)^{48}}{13}$?

$$\frac{\overset{-1}{(25)}^{48}}{13}=(-1)^{48}=1$$

In such type of operations we try to get the multiple of divisor near to the dividends actual number So, that the difference b/w then will be 1. In this case the multiple of 13 near to 25 is 26. and the difference b/w 25 and 26 is 1 and power of even So, the remainder will be (+ve)

Ex.19 $\frac{(36)^{13}}{7}$ Find the remainder?

Sol. $$\frac{\overset{+1}{(36)}^{13}}{7}=(+1)^{13}=1$$

In this operation the multiple of 7 near to 36 is 35 and the difference between 36 and 35 is 1

Ex.20 $\frac{2^{18}}{9}$ Find the remainder?

Sol. In such type of Operations, the power is simplified in such a way that the difference b/w divisor and the number made by breaking of power is minimum So, the number near to 9 should be 8 or 10

9 < 8, 10

$$\frac{2^{3\times 6}}{9}=\frac{(2^3)^6}{9}=\frac{\overset{-1}{(8)}^6}{9}$$

$$=\frac{(-1)^6}{9}=\frac{1}{9}=1$$

So Remainder = 1

Ex.21 What will be the remainder when 2^{21} is divided by 9

Sol. $$\frac{2^{21}}{9}=\frac{(2^3)^7}{9}=\frac{\overset{-1}{(8)}^7}{9}$$

$$=\frac{(-1)^7}{9}=-1=9-1=8$$

Ex.22 What will be the remainder when 2^{22} is divided by 9

Sol. $$=\frac{2^{22}}{9}=\frac{(2^3)^7\times 2}{9}=\frac{\overset{-1}{(8)}^7\times \overset{+2}{2}}{9}$$

$$=\frac{(-1)^7\times 2}{9}$$

$$\Rightarrow \quad \frac{-1\times 2}{9}=\frac{-2}{9}=-2$$

$\Rightarrow \quad 9-2=7$

Ex.23 What will be the remainder when $(35)^{37}$ is divided by 9 ?

Sol. $$\frac{(35)^{37}}{9}$$

The multiple of 9 near to 35 is 36

$$\Rightarrow \frac{\overset{-1}{(35)}^{37}}{9}=\frac{(1)^{37}}{9}=-1$$

Remainder = 9 - 1 = 8

Ex.24 What will be the remainder when 7^{40} is divided by 400

Sol.

7^1 = 7
7^2 = 49
7^3 = 343
7^4 = 2401

$\frac{7^{40}}{400}$ $\frac{\left(7^4\right)^{10}}{400}$

$\Rightarrow \frac{(2401)^{10}}{400} = \frac{(1)^{10}}{400} = 1$

(power has broken in such a way that $7^4 = 2401$, which is near to the 2400 a multiple of 400)

Ex.25 What will be the remainder when 2^{42} is divided by 33

Sol. $\frac{2^{42}}{33}$ $\quad 33 \begin{cases} 32 \\ 34 \end{cases}$

32 and 34 are near to the 33 the difference is 1. Hence Power is to be broken in such a way that we can a get 32 and 34

$\boxed{\begin{array}{l} 2^1 = 2 \\ 2^2 = 4 \\ 2^3 = 8 \\ 2^4 = 16 \\ 2^5 = 32 \end{array}}$

$= \frac{2^2 \times 2^{40}}{33} = \frac{4 \times \left(2^5\right)^8}{33}$

$\Rightarrow \frac{\overset{+4}{\overset{\uparrow}{4}} \times \overset{-1}{\overset{\uparrow}{(32)}}{}^8}{33}$

$= \frac{+4 \times (-1)^8}{33} = \frac{4 \times 1}{33} = 4$

remainder = 4

Ex.26 What will be the remainder when 3^{55} is divided by 82

Sol. $\boxed{\begin{array}{l} 3^1 = 3 \\ 3^2 = 9 \\ 3^3 = 27 \\ 3^4 = 81 \end{array}}$

$\frac{3^{55}}{82} \Rightarrow \frac{3^3 \times 3^{52}}{82}$ $\quad 82 \begin{cases} 81 \\ 83 \end{cases}$

$\Rightarrow \frac{3^3 \times \left(3^4\right)^{13}}{82}$

$= \frac{\overset{+27}{\overset{\uparrow}{27}} \times \overset{-1}{\overset{\uparrow}{(81)}}}{82} = \frac{27 \times -1}{82} = -27$

remainder = 82 - 27 = 55

Ex.27 What will be the remainder when 2^{68} is divided by 65 ?

Sol. $\boxed{\begin{array}{c} 65 \begin{cases} 64 \\ 66 \end{cases} \\ \boxed{2^6 = 64} \end{array}}$

$\frac{2^{68}}{65}$

$= \frac{2^2 \times \left(2^6\right)^{11}}{65}$

$= \frac{\overset{+4}{\overset{\uparrow}{4}} \times \overset{-1}{\overset{\uparrow}{(64)}}{}^{11}}{65} \Rightarrow \frac{+4 \times (-1)^{11}}{65}$

$\Rightarrow \frac{4 \times -1}{65} = \frac{-4}{65}$

remainder = 65 - 4 = 61

Ex.28 What will be the remainder when 4^{19} is divided by 33

Sol. $\frac{4^{19}}{33}$ $\quad 33 \begin{cases} 32 \\ 34 \end{cases}$

$4^{19} = (2^2)^{19} = 2^{38}$

So $\frac{2^{38}}{33}$ $\because$ $\boxed{2^5 = 32}$

$\Rightarrow \frac{2^3 \times 2^{35}}{33} = \frac{8 \times \left(2^5\right)^7}{33}$

$\Rightarrow \frac{\overset{+8}{\overset{\uparrow}{8}} \times \overset{-1}{\overset{\uparrow}{(32)}}{}^7}{33} = \frac{+8 \times (-1)^7}{33}$

$= \frac{8 \times -1}{33} = -8$

remainder =33 - 8 = $\boxed{25}$

TYPE - 2

Ex.29 When 20 is divided by 8 the remainder will be

Sol. 8)20(2
-16
4 ⟶ Remainder

$= \frac{\cancel{20}^{\,5}}{\cancel{8}_{\,2}}$ $\quad = \frac{\overset{+1}{\overset{\uparrow}{5}}}{2} = 1$

When 20 is divided by 8 we get '4' remainder.

If $\frac{20}{8}$ is simplified by 4 we get $\frac{5}{2}$. Now 5 is divided by 2 we get remainder 1, In means that the divisor should be multiplied by remainder to get actual remainder

$= \frac{5}{2} = 1 \times \underline{4} = 4$

(Actual remainder)

Ex.30 What will be the remainder when 2^{35} is by 10 ?

Sol. $\frac{2^{35}}{10} = \frac{2^{35}}{2 \times 5} = \frac{\cancel{2} \times 2^{34}}{\cancel{2} \times 5}$

This Fraction is simplified by 2

$= \frac{2^{34}}{5} = \frac{\left(2^2\right)^{17}}{5}$

$= \frac{\overset{-1}{\overset{\uparrow}{(4)}}{}^{17}}{5}$

$= \frac{(-1)^{17}}{5} = -1 = 5 - 1 = 4$

Actual Remainder = 4 × $\underline{2}$ = 8

As this number was simplified by 2, So to get actual remainder we have to multiply it by 2

Ex.31 What will be the remainder when 5^{500} is divided by 500

Sol. $\frac{5^{500}}{500} = \frac{5^3 \times 5^{497}}{125 \times 4}$

$= \frac{\cancel{5^3} \times 5^{497}}{\cancel{5^3} \times 4}$

5^3 = simplifying by 125

$\Rightarrow \frac{\overset{+1}{\overset{\uparrow}{(5)}}{}^{497}}{4} = \frac{(+1)^{497}}{4} = 1$

Actual Remainder = 1 × 125 = 125

Ex.32 What will be the remainder when 37^{100} is divided by 7 ?

Sol. $\frac{\overset{+2}{\overset{\uparrow}{(37)}}{}^{100}}{7} = \frac{(+2)^{100}}{7} = \frac{2 \times \left(2^3\right)^{33}}{7}$

$\boxed{2^3 = 8}$

2^{100} remainder is far greater than 7, So, we have to divide remainder again.

$$\Rightarrow \frac{\overset{+2}{\uparrow}\!\!\!2\times(\overset{+1}{\uparrow}\!\!\!8)^{33}}{7}$$

$$= \frac{+2\times(+1)^{33}}{7}$$

$$= \frac{2\times +1}{7} = \frac{+2}{7}$$

So, Remainder = 2

<u>Cyclicity</u>:- Happening again and again In the same order or period

Ex.33 Find the remainder when 11^{77} is divided by 7

Sol. $\frac{11^{77}}{7}$

The Remainder when 11^1 is divided by 7 = $= \frac{11}{7} = 4$

$$11^2 = \frac{\overset{+4}{\overset{\uparrow}{11}}\times\overset{+4}{\overset{\uparrow}{11}}}{7} = \frac{4\times4}{7} = \frac{16}{7} = 2$$

$$11^3 = \frac{\overset{+4}{\overset{\uparrow}{11}}\times\overset{+4}{\overset{\uparrow}{11}}\times\overset{+4}{\overset{\uparrow}{11}}}{7} = \frac{4\times4\times4}{7} = \frac{64}{7} = 1$$

$$11^4 = \frac{\overset{+4}{\overset{\uparrow}{11}}\times\overset{+4}{\overset{\uparrow}{11}}\times\overset{+4}{\overset{\uparrow}{11}}\times\overset{+4}{\overset{\uparrow}{11}}}{7} = \frac{4\times4\times4\times4}{7}$$

$$\frac{\overset{2}{\overset{\uparrow}{16}}\times\overset{2}{\overset{\uparrow}{16}}}{7} = 4$$

$$11^5 = \frac{\overset{+4}{\overset{\uparrow}{11}}\times\overset{+4}{\overset{\uparrow}{11}}\times\overset{+4}{\overset{\uparrow}{11}}\times\overset{+4}{\overset{\uparrow}{11}}\times\overset{+4}{\overset{\uparrow}{11}}}{7} = \frac{4\times4\times4\times4\times4}{7}$$

$$= \frac{\overset{2}{\overset{\uparrow}{16}}\times\overset{2}{\overset{\uparrow}{16}}\times\overset{+4}{\overset{\uparrow}{4}}}{7} = \frac{4\times4}{7} = 2$$

$$11^6 = \frac{\overset{+4}{\overset{\uparrow}{11}}\times\overset{+4}{\overset{\uparrow}{11}}\times\overset{+4}{\overset{\uparrow}{11}}\times\overset{+4}{\overset{\uparrow}{11}}\times\overset{+4}{\overset{\uparrow}{11}}\times\overset{+4}{\overset{\uparrow}{11}}}{7}$$

$$= \frac{64\times64}{7} = \frac{1\times1}{7} = 1$$

you are seeing that after three steps the cycle of remainders is repeating, which is generaly know as 'Pattern method So break the power of multiple of 3

$$\frac{(11)^{77}}{7} = \frac{(11)^{75}\times(11)^2}{7}$$

$$= \frac{(\overset{+1}{\overset{\uparrow}{11^3}})^{25}\times\overset{+2}{\overset{\uparrow}{121}}}{7}$$

$$\frac{(+1)^{25}\times2}{7} = \frac{1\times2}{7} = 2$$

Ex.34 Find the remainder when 5^{135} is divided by 7.

Sol. The Remainder when 5^1 is divided by 7 = $\frac{5^1}{7}$ = R = **5**.

$$\frac{5^2}{7} = \frac{\overset{+4}{\overset{\uparrow}{25}}}{7} = R = \mathbf{4}$$

$$\frac{5^3}{7} = \frac{\overset{+6}{\overset{\uparrow}{125}}}{7} = R = \mathbf{6}$$

$$\frac{5^4}{7} = \frac{5\times5\times5\times5}{7}$$

$$= \frac{\overset{-3}{\overset{\uparrow}{25}}\times\overset{-3}{\overset{\uparrow}{25}}}{7} = \frac{9}{7} = R = \mathbf{2}$$

$$= \frac{5^5}{7} = \frac{5\times5\times5\times5\times5}{7}$$

$$= \frac{\overset{-3}{\overset{\uparrow}{25}}\times\overset{-3}{\overset{\uparrow}{25}}\times\overset{+5}{\overset{\uparrow}{5}}}{7} = \frac{9\times5}{7} = R = \mathbf{3}$$

$$= \frac{5^6}{7} = \frac{5\times5\times5\times5\times5\times5}{7}$$

$$= \frac{\overset{-3}{\overset{\uparrow}{25}}\times\overset{-3}{\overset{\uparrow}{25}}\times\overset{-3}{\overset{\uparrow}{25}}}{7} = \frac{-27}{7} = -6$$

$\therefore$ R = –6 + 7 = 1

$$\frac{5^7}{7} = \frac{5\times5\times5\times5\times5\times5\times5}{7}$$

$$\frac{\overset{-3}{\overset{\uparrow}{25}}\times\overset{-3}{\overset{\uparrow}{25}}\times\overset{-3}{\overset{\uparrow}{25}}\times\overset{-2}{\overset{\uparrow}{5}}}{7} = \frac{9\times6}{7} = \mathbf{5}$$

$$\frac{5^8}{7} = \frac{5\times5\times5\times5\times5\times5\times5\times5}{7}$$

$$= \frac{\overset{-3}{\overset{\uparrow}{25}}\times\overset{-3}{\overset{\uparrow}{25}}\times\overset{-3}{\overset{\uparrow}{25}}\times\overset{-3}{\overset{\uparrow}{25}}}{7} = \frac{9\times9}{7} = \mathbf{4}$$

So we see that the cyclic period of remainder is 6, since after 6 steps the remainder start repeating Now we divide the power by 6

$$= \frac{5^{135}}{7} = \frac{(5^{6\times22})\times5^3}{7}$$

$$= \frac{(\overset{1}{\overset{\uparrow}{5^6}})^{22}\times125}{7}$$

$$= \frac{(1)^{22}\times6}{7} = \frac{6}{7}$$

Remainder = 6

Ex.35 Find the Remainder when 143^{321} is divided by 5

Sol. when 143 is divided by 5 we get remainder 3 thus 143^{321} is divided by 5 then remainder 3^{321}. this remainder is very large divisor so again divided

$$\frac{(143)^{321}}{5} = \frac{(3)^{321}}{5}$$

The remainder when 3^1 is divided by 5 $= \frac{3^1}{5} = R = 3$

$$= \frac{3^1}{5} = R = 3$$

$$\frac{3^2}{5} = \frac{9}{5} = R = 4$$

$$\frac{3^3}{5} = \frac{27}{5} = R = 2$$

$$\frac{3^4}{5} = \frac{81}{5} = R = 1$$

$$\frac{3^5}{5} = \frac{243}{5} = R = 3$$

$$\frac{3^6}{5} = \frac{729}{5} = R = 4$$

$$\frac{3^{321}}{5} = \frac{(3)^{4\times80}\times3}{5}$$

$$= \frac{(\overset{+1}{\overset{\uparrow}{3^4}})^{80}\times\overset{+3}{\overset{\uparrow}{3}}}{5}$$

$= \frac{(+1)^{80}\times 3}{5} = \frac{1\times 3}{5} = 3$

Remainder = 3

Ex.36 find the remainder when 3^{6773} divide by 80

Sol. $= \frac{3^{6773}}{80}$

we know that = $3^4 = 81$

$\frac{3^{6773}}{80} = \frac{3^{6772}\times 3^1}{80}$

$= \frac{(3^4)^{1693}\times 3^1}{80}$

$= \frac{(\overset{\overset{+1}{\uparrow}}{81})^{1693}\times 3^1}{80}$

$= \frac{(1)^{1693}\times 3}{80} = \frac{1\times 3}{80} = 3$

Hence Remainder = 3

Ex.37 Find the Remainder of $(32^{32})^{32}$ when divided by 7.

Sol. $\frac{(32^{32})^{32}}{7}$

$= \frac{(4^{32})^{32}}{7}$

$4 = 2^2$

$\Rightarrow \frac{(2^{2\times 32})^{32}}{7}$

$\Rightarrow \frac{(2^{64})^{32}}{7}$

$\because \quad 2^3 = 8$

$= \frac{(2^{63}\times 2^1)^{32}}{7}$

$= \frac{((2^3)^{21}\times 2^1)^{32}}{7} = \frac{((\overset{\overset{+1}{\uparrow}}{8})^{21}\times 2^1)^{32}}{7}$

$= \frac{(1\times 2)^{32}}{7} = \frac{2^{32}}{7}$

Again $\because \quad 2^3 = 8$

$\frac{2^{30}\times 2^2}{7} = \frac{(2^3)^{10}\times 4}{7}$

$= \frac{(\overset{\overset{+1}{\uparrow}}{8})^{10}\times 4}{7}$

$= \frac{(1)^{10}\times 4}{7} = \frac{1\times 4}{7}$

Remainder = 4

Ex.38 What will be the remainder when $[48 + (62)^{117}]$ is divided by 9 ?

Sol. $\frac{\overset{\overset{+3}{\uparrow}}{48}+(\overset{\overset{-1}{\uparrow}}{62})^{117}}{9}$

$= \frac{+3+(-1)^{117}}{9} = \frac{3-1}{9} = \frac{2}{9} = \boxed{2}$

Hence Remainder = 2

Ex.39 when $[51 + (67)^{99}]$ is divided by 68, find the remainder

Sol. $\frac{\overset{\overset{-17}{\uparrow}}{51}+(\overset{\overset{-1}{\uparrow}}{67})^{99}}{68}$

$\Rightarrow \frac{-17+(-1)^{99}}{68} = \frac{-17-1}{68} = \frac{-18}{68}$

$= -18$

Remainder = 68 - 18 = 50

Remainder of Algebraic Function

When F (x) is divided by $(x-a)$ the remainder is F (a)

$\because$ $(x - a)$ is a factor of F (x) then f (a) = 0

Ex.40 Is $(x - 2)$ a factor of f (x) $x^2 + x - 5$?

Sol. $(x - 2) = 0$

$\boxed{x = 2}$

x value f (x)

F (2) = $(2)^2 + (2) - 5$

= 4 + 2 - 5

= 6 - 5 = 1 $\neq$ 0

$(x - 2)$ is not a factor of $x^2 - x + 5$

If F (2) = 0, we can say $(x - 2)$, it is a factor of f (x)

Ex.41 $x^{29} - x^{26} - x^{23} + 1$

(a) $(x - 1)$ but not $(x + 1)$

(b) $(x + 1)$ but not $(x - 1)$

(c) both $(x + 1)$ & $(x - 1)$

(d) Neither $(x + 1)$ not $(x - 1)$

Sol. (c) If $(x - 1)$, is a factor then, $f(x) = 0$,

and $x - 1 = 0$

$x = 1$

f (1) = 0

f $(x) = x^{29} - x^{26} - x^{23} + 1$

$f(1) = 1 - 1 - 1 + 1 = 0$

$f(1) = 0$,

we can say $(x - 1)$ is a factor of f (x)

$x + 1 = 0$

$x = -1$

$x^{29} - x^{26} - x^{23} + 1$

$-1 - 1 + 1 + 1 = 0$

$(x + 1)$ is a factor of f (x)

Both $(x + 1)$ & $(x - 1)$ is a factor of $x^{29} - x^{26} - x^{23} + 1$

Ex.42 If $(x - 2)$ is a factor of Polynomial $x^2 + kx + 4$. Find the value of k.

Sol. $(x - 2)$ is a factor of $x^2 + kx + 4$ when $(x - 2) = 0$

$x = 2$

f (2) = $(2)^2 + 2k + 4 = 0$

$2k = -8$

$k = -4$

Ex.43 If $(x + 1)$ & $(x - 1)$ are the Factor of the Polynomial $ax^3 + bx^2 + 3x + 5$. find the value of a and b

Sol. If $(x - 1)$ is factor of $f(x)$ then,

$x - 1 = 0$

$x = 1$

$f(x) = ax^3 + bx^2 + 3x + 5$

$f(1) = a(1)^3 + b(1)^2 + 3(1) + 5 = 0$

$a + b = -8$ ..... (i)

If $(x + 1)$, is a factor of f (x)

Then,

$(x + 1) = 0$

$x = -1$

$f(-1) = a(-1)^3 + b(-1)^2 + 3(-1) + 5 = 0$

$-a + b - 3 + 5 = 0$

$-a + b = -2$

$a - b = 2$ ..... (ii)

from (i) & (ii)

$\boxed{a = -3}$, $\boxed{b = -5}$

Ex.44 Find the remainder when $x^3 + 5x^2 + 7$ is divided by $(x - 2)$

Sol. $x - 2 = 0$

$x = 2$

f $(x) = x^3 + 5x^2 + 7$

f (2) = $(2)^3 + 5(2)^2 + 7$

= 8 + 20 + 7 = 35

Remainder = 35

Ex.45 Find the remainder when $x^2 - 7x + 15$ is divided by $x - 3$

Sol. $x - 3 = 0$

$x = 3$

Put the value of $x = 3$

F $(x) = x^2 - 7x + 15$

F (3) = $(3)^2 - 7(3) + 15$

= 9 − 21 + 15 = 3

Remainder 3

Ex.46 $x^{51} + 16$ when divided by $x + 1$ find the Remainder.

Sol. $(x + 1) = 0$

$x = -1$

$f(x) = x^{51} + 16$

$f(-1) = (-1)^{51}+16=-1 + 16 = 15$

Remainder = 15

Ex.47 If $x^2 + 4x + k$ when divided by $x - 2$ leave remainder $2x$. find the value of k.

Sol. $x^2 + 4x + k$

$x - 2 = 0$

$x = 2$

$f(x) = 2x$

$f(2) = 2 + 2 = 4$

$f(2) = (2)^2 + 4 \times 2 + k = 4$

$4 + 8 + k = 4$

$\boxed{K = -8}$

TYPE – 3

Ex.48 777777........... 129 Times is divided by 37 the remainder will be ?

Sol. If any number is made by repeating a digit 6 times the number will be divisible by 7, 11, 13 and 37.

So, 777777 .... 126 times is divisible by 37 because 126 is the multiple of 6. So, the remaining three digits will be divided by 37 to get the remainder

$$\Rightarrow \frac{77777777......126\text{ Times},777}{37}$$

$$37)\,777\,(21 \quad \frac{777}{37} = 0$$

74

37

37

×

Remainder = 0

Hence, the number is divisible by 37.

Ex.49 When 444444444 is divided by 13 the remainder will be ?

Sol. 4 is repeating 9 times in this number As we know that any number repeating 6 times is divisible by 13. So the remaining three digit will be divided by 13 to get the will be divided by 13 to get the remainder

$$\frac{444444,444}{13} = \frac{444}{13}$$

$$=13)\,444\,(34$$

39

54

52

2 ⟶ Remainder

Ex.50 What will be the remainder when 123456789 is divided by 8 ?

Sol. (Divisibility Rule)

$2^1 = 2 \longrightarrow$ Last digit divisibile by 2

$2^2 = 4 \longrightarrow$ Last two digits divisibile by 4

$2^3 = 8 \longrightarrow$ Last three digits divisibile by 8

$2^4 = 16 \longrightarrow$ Last four digits divisibile by 16

$2^5 = 32 \longrightarrow$ Last five digits divisibile by 32

So, for the divisibility of 8 the last three digit of the number should be divisible by 8. In this way we get 5 as the remainder

$$\frac{123456\underline{789}}{8}$$

$$8)\,789\,(98$$

72

69

64

5 ⟶ Remainder

So, the remainder is 5.

Ex.51 What will be remainder when 12345678910111213141516 divided by 16.

Sol. For the divisibility of 16, the last four digits of the number should be divisible by 16. In this way we get 12 as remainder

$$\frac{123456789101112131\underline{41516}}{16}$$

$$16)\,1516\,(94$$

144

76

64

12 ⟶ Remainder

Hence the remainder is 12.

Ex.52 $10^1 + 10^2 + 10^3 + ... 10^{99} + 10^{100}$ when divided by 6, the remainder will be?

Sol. $$\frac{10^1 + 10^2 + 10^3 + ...10^{99} + 10^{100}}{6}$$

$$\longrightarrow \frac{\overset{+4}{10}}{6} = \text{Remainder} = 4$$

$$\longrightarrow \frac{\overset{+4}{10}+\overset{+4}{10^2}}{6} = \frac{4+4}{6} = \frac{8}{6}$$

Remainder = 8 - 6 = 2

$$\longrightarrow \frac{\overset{+4}{10^1}+\overset{+4}{10^2}+\overset{+4}{10^3}}{6} = \frac{4+4+4}{6} = \frac{12}{6}$$

Remainder = 0

The remainder will be zero (0) after each three number So the remainder is 0 upto the 99th term. So the remaining 10^{100} term will be divided by 6 to get the remainder

$$\frac{\overset{+4}{10^1}+\overset{+4}{10^2}+\overset{+4}{10^3} + \overset{+4}{10^1}+\overset{+4}{10^2}+\overset{+4}{10^3} \ldots \; ..... \overset{+4}{10^{99}} + \overset{+4}{10^{100}}}{6}$$

$$\frac{\overset{+4}{10^{100}}}{6} = 4$$

Hence, the remainder is 4.

Ex.53 What will be the remainder when $10^1 + 10^2 + 10^3 + ... 10^{32}$ is divided by 6 ?

Sol. $$\frac{\overset{+4}{10^1}+\overset{+4}{10^2}+\overset{+4}{10^3} + ... \overset{+4}{10^{30}}+\overset{+4}{10^{31}}+\overset{+4}{10^{32}}}{6}$$

0 will be the remainder of ter each three term. So, o will be the remainder up to 30th term

$$\Rightarrow \frac{\overset{+4}{10^{31}}+\overset{+4}{10^{32}}}{6}$$

$$= \frac{4+4}{6} = \frac{8}{6} = 2$$

Hence remainder is 2

EXERCISE

1. Find the Remainder when 77 × 85 × 73 is divided by 9
(a) 1 (b) 2 (c) 4 (d) 7
2. Find the Remainder when 273 + 375 + 478 + 657 + 597 is divided by 25
(a) 5 (b) 10 (c) 9 (d) 8
3. Find the Remainder when 1330 × 1356 × 1363 × 1368 × 1397 is divided by 13
(a) 7 (b) 9 (c) 11 (d) 8
4. Find the Remainder when 2327 + 2372 + 2394 + 4624 + 4650 is divided by 23
(a) 12 (b) 14 (c) 13 (d) 10
5. Find the Remainder when 67^{32} is divided by 68
(a) 67 (b) 66 (c) 1 (d) 0
6. Find the Remainder when 99^{99} is divided by 100
(a) 99 (b) 98 (c) 1 (d) 3
7. Find the Remainder 197^{130} is divided by 196
(a) 1 (b) 195 (c) 7 (d) 5
8. Find the Remainder 6^{36} is divided by 215
(a) 214 (b) 6 (c) 5 (d) 1
9. Find the Remainder 75^{7575} is divided by 37
(a) 1 (b) 36 (c) 3 (d) 7
10. Find the Remainder 43^{197} is divided by 7
(a) 42 (b) 41 (c) 1 (d) 6
11. Find the Remainder when 17^{200} is divided by 18
(a) 17 (b) 16 (c) 1 (d) 4
12. Find the Remainder when $(12^{13} + 23^{13})$ is divided by 11
(a) 2 (b) 1 (c) 0 (d) 3
13. Find the remainder when $(7^{19} + 2)$ is divided by 6
(a) 3 (b) 1 (c) 5 (d) 2
14. Find the Remainder when 3^{21} is divided by 5 is
(a) 3 (b) 2 (c) 1 (d) 4
15. Find the Remainder when 2^{31} is divided by 5
(a) 1 (b) 2 (c) 3 (d) 4
16. Find the Remainder when 2^{591} is divided by 255
(a) 225 (b) 128 (c) 127 (d) 64
17. Find the Remainder when 51^{203} is divided by 7
(a) 4 (b) 2 (c) 1 (d) 6
18. The Remainder when $(2)^{243}$ is divided by 3^2 is
(a) 8 (b) 4 (c) 10
(d) None of these
19. Find the Remainder when $(59)^{28}$ is divided by 7
(a) 2 (b) 4 (c) 6 (d) 1
20. Find the Remainder when 41^{77} is divided by 17
(a) 2 (b) 1 (c) 6 (d) 4
21. Find the Remainder when 2^{49} is divided by 7
(a) 1 (b) 2 (c) 3 (d) 4
22. Find the Remainder when $(51^{203} + 2^{49})$ is divided by 17
(a) 4 (b) 5 (c) 6
(d) None of these
23. Find the Remainder when 12345678910111213 14is divided by 8
(a) 4 (b) 2 (c) 6 (d) 3
24. Find the Remainder when 41424344454647484950 is divided by 16
(a) 2 (b) 12 (c) 6 (d) 8
25. Find the Remainder when 21222324252627282930 is divided by 8
(a) 5 (b) 2 (c) 3 (d) 4
26. Find the Remainder when 919293949596979899 is divided by 16
(a) 3 (b) 13 (c) 11 (d) 8
27. Find the Remainder when 313233343536373839 is divided by 4
(a) 1 (b) 2 (c) 3 (d) N.O.T.
28. Find the Remainder when 1234.... 41digits is divided by 8
(a) 1 (b) 2 (c) 3 (d) 4
29. Find the Remainder when 1234....... 81digits is divided by 16
(a) 13 (b) 8 (c) 1 (d) 7
30. Find the Remainder when 8^{77} is divided by 17
(a) 8 (b) 9 (c) 13 (d) 7
31. Find the Remainder when $\lfloor 1+\lfloor 2+\lfloor 3+\lfloor 4 ....\lfloor 100$ is divided by 5 is
(a) 0 (b) 1 (c) 2 (d) 3
32. Find the Remainder when $\lfloor 1+\lfloor 2+\lfloor 3+\lfloor 4 ....\lfloor 100$ is divided by 6 is
(a) 3 (b) 4 (c) 2 (d) 1
33. Find the Remainder when $\lfloor 1+\lfloor 2+\lfloor 3+\lfloor 4 ....\lfloor 50$ is divided by 12 is
(a) 2 (b) 8 (c) 7 (d) 9
34. Find the Remainder when 9^{111} is divided by 11
(a) 2 (b) 9 (c) 7 (d) 6
35. Find the Remainder when 5^{2450} is divided by 126
(a) 5 (b) 25 (c) 125 (d) 1
36. Find the Remainder when 40^{1012} is divided by 7
(a) 5 (b) 4 (c) 3 (d) 2
37. Find the Remainder when $10^1 + 10^2 + 10^3 + ... 10^{100}$ is divided by 6
(a) 4 (b) 6 (c) 2 (d) 3
38. Find the Remainder when $10^1 + 10^2 + 10^3 + ... 10^{1000} + 10^{1001}$ is divided by 6
(a) 4 (b) 6 (c) 2 (d) 3
39. Find the Remainder when 666666 .... 134times is divided by 13
(a) 1 (b) 3 (c) 11 (d) 9
40. Find the Remainder when 555555 .... 244times is divided by 37
(a) 18 (b) 5 (c) 36 (d) 0
41. Find the Remainder when 777777 .... 363 times is divided by 11
(a) 0 (b) 7 (c) 1 (d) 3
42. Find the Remainder when 888888 .... 184 times is divided by 37
(a) 1 (b) 8 (c) 36 (d) 7
43. Find the Remainder when 999999999 is divided by 13
(a) 8 (b) 11 (c) 5 (d) 12
44. Find the Remainder when 7^{99} is divided by 2400
(a) 1 (b) 49 (c) 343 (d) 7
45. Find the Remainder when 3^{1989} is divided by 7
(a) 2 (b) 6 (c) 4 (d) 5
46. Find the Remainder when 54^{124} is divided by 17
(a) 4 (b) 5 (c) 3 (d) 15
47. Find the Remainder when 21^{875} is divided by 17
(a) 8 (b) 13 (c) 16 (d) 9
48. Find the Remainder when 83^{261} is divided by 17
(a) 13 (b) 9 (c) 8 (d) 2

49. Find the Remainder when $(32^{32})^{32}$ is divided by 9
(a) 4 (b) 7 (c) 1 (d) 2

50. Find the Remainder when $(32^{32})^{32}$ is divided by 7
(a) 4 (b) 7 (c) 2 (d) 1

51. Find the Remainder when $(33^{34})^{35}$ is divided by 7
(a) 5 (b) 4 (c) 6 (d) 2

52. Find the Remainder when $888^{222} + 222^{888}$ is divided by 5
(a) 0 (b) 1 (c) 3 (d) 4

53. Find the Remainder when $2222^{5555} + 5555^{2222}$ is divided by 7
(a) 0 (b) 2 (c) 4 (d) 5

54. Find the Remainder when $50^{51^{52}}$ is divided by 11
(a) 6 (b) 4 (c) 7 (d) 3

55. The Remaidner when $(20)^{23}$ is divided by 17 is
(a) 11 (b) 3 (c) 6
(d) Can't determind

56. If $(x - 2)$ is a factor of $(x^2 + 3qx - 2q)$, then the value of q is :
(a) 2 (b) -2 (c) -1 (d) 1

57. If $x^3 + 6x^2 + 4x + k$ is exactly divisible by $(x + 2)$ then value of k is : –
(a) -6 (b) -7 (c) -8 (d) -10

58. Value of k for with $(x - 1)$ is a factor of $(x^3 - k)$ is :
(a) - 1 (b) 1 (c) 8 (d) - 8

59. If $x^{100} + 2x^{99} + k$ is divisible by $(x + 1)$, then the value of k is:
(a) 1 (b) -3 (c) 2 (d) -2

60. If $(x^3 - 5x^2 + 4p)$ is divisible by $(x + 2)$, then the value of p is
(a) 7 (b) -2 (c) 3 (d) -7

61. If (x - a) is a factor of $(x^3 - 3x^2 a + 2a^2x + b)$, then the value of b is:
(a) 0 (b) 2 (c) 1 (d) 3

62. If $x^3 + 3x^2 + 4x + k$ contains $(x + 6)$ as a factor, the value of k is:
(a) 66 (b) 33 (c) 132 (d) 36

63. If $(x + 2)$ and $(x - 1)$ are the factors of $(x^3 + 10x^2 + mx + n)$, the values of m and n are :
(a) m = 5, n = -3
(b) m = 17, n = -8
(c) m = 7, n = -18
(d) m = 23, n = -19

64. On dividing $(x^3 - 6x + 7)$ by $(x + 1)$, then remainder is :
(a) 2 (b) 12 (c) 0 (d) 7

65. If $(x^5 - 9x^2 + 12x - 14)$ is divided by $(x - 3)$, the remainder is :
(a) 184 (b) 56 (c) 2 (d) 1

66. When $(x^4 - 3x^3 + 2x^2 - 5x + 7)$ is divided by $(x-2)$, then remainder is :
(a) 3 (b) - 3 (c) 2 (d) 0

67. If $5x^3 + 5x^2 - 6x + 9$ is divided by $(x + 3)$, then remainder is :
(a) 135 (b) -135 (c) 63 (d) -63

68. If $(x^{11} + 1)$ is divided by $(x + 1)$, then remainder is :
(a) 2 (b) 0 (c) 11 (d) 12

69. If $2x^3 + 5x^2 - 4x - 6$ is divided by $2x + 1$, then remainder is :
(a) $-\frac{13}{3}$ (b) 3
(c) -3 (d) 6

70. If $x^3 + 5x^2 + 10k$ leaves remainder -2x when divided by $x^2 + 2$, then the value of k is:
(a) -2 (b) -1 (c) 1 (d) 2

ANSWER KEY

1. (b)	8. (d)	15. (c)	22. (d)	29. (a)	36. (d)	43. (b)	50. (a)	57. (c)	64. (b)
2. (a)	9. (a)	16. (b)	23. (b)	30. (b)	37. (a)	44. (c)	51. (b)	58. (b)	65. (a)
3. (b)	10. (c)	17. (a)	24. (c)	31. (d)	38. (c)	45. (b)	52. (a)	59. (a)	66. (b)
4. (b)	11. (c)	18. (a)	25. (b)	32. (a)	39. (a)	46. (a)	53. (a)	60. (a)	67. (d)
5. (c)	12. (a)	19. (b)	26. (c)	33. (d)	40. (b)	47. (b)	54. (d)	61. (a)	68. (b)
6. (a)	13. (a)	20. (c)	27. (c)	34. (b)	41. (b)	48. (d)	55. (a)	62. (c)	69. (c)
7. (a)	14. (a)	21. (b)	28. (a)	35. (b)	42. (b)	49. (a)	56. (c)	63. (c)	70. (c)

SOLUTION

1. (b)
$$\frac{\overset{+5}{77}\times\overset{+4}{85}\times\overset{+1}{73}}{9}$$
$$=\frac{5\times4\times1}{9}=\frac{20}{9}=\mathbf{2}$$

2. (a)
$$\frac{\overset{-2}{2\underline{73}}+\overset{0}{3\underline{75}}+\overset{+3}{4\underline{78}}+\overset{+7}{6\underline{57}}+\overset{-3}{5\underline{97}}}{25}$$
$$\frac{-2+0+3+7-3}{25}=\frac{5}{25}=\mathbf{5}$$

3. (b)
$$\frac{\overset{+4}{13\underline{30}}\times\overset{+4}{13\underline{56}}\times\overset{-2}{13\underline{63}}\times\overset{+3}{13\underline{68}}\times\overset{+6}{13\underline{97}}}{13}$$
$$\frac{4\times4\times-2\times3\times6}{13}=\frac{\overset{+3}{16}\times-36}{13}$$

* Avoid '–' (Negative) sign. Normally divided 36 by 13 remainder = -3. Now use '-' (Negative) sign

R = (-3) = 3
$$\frac{\overset{+3}{16}\times\overset{+3}{-36}}{13}=\frac{9}{13}=R=\mathbf{9}$$

4. (b)
$$\frac{\overset{+4}{23\underline{27}}+\overset{+3}{23\underline{72}}+\overset{+2}{23\underline{94}}+\overset{+1}{46\underline{24}}+\overset{+4}{46\underline{50}}}{23}$$
$$\frac{4+3+2+1+4}{23}=\frac{14}{23}$$
R = **14**

5. (c) -1 ↑

$$\frac{67^{32}}{68} = \frac{(-1)^{32}}{68} = \frac{1}{68}$$

R = **1**

6. (a) -1 ↑

$$\frac{99^{99}}{100} = \frac{(-1)^{99}}{100} = \frac{-1}{100}$$

R = 100 - 1 = **99**

7. (a) +1 ↑

$$\frac{197^{130}}{196} = \frac{(+1)^{130}}{196} = \frac{1}{196}$$

R = **1**

8. (d) $\frac{6^{36}}{215}$

∵ We know that $6^3 = 216$

∵ So break the power multiple 3

+1 ↑

$$= \frac{(6^3)^{12}}{215} = \frac{(216)^{12}}{215}$$

$$= \frac{(+1)^{12}}{215} = \frac{1}{215}$$

R = **1**

9. (a) +1 ↑

$$\frac{75^{7575}}{37} = \frac{(+1)^{7575}}{37} = \frac{1}{37}$$

R = **1**

10. (c)

+1 ↑

$$\frac{43^{197}}{7} = \frac{1^{197}}{7} = \frac{1}{7}$$

R = 1

11. (c) +1 ↑

$$\frac{17^{200}}{18} = \frac{(-1)^{200}}{18} = \frac{1}{18}$$

R = **1**

12. (a) +1 ↑ +1 ↑

$$\frac{12^{13} + 23^{13}}{11}$$

$$= \frac{(+1)^{13} + (+1)^{13}}{11} = \frac{1+1}{11} = \frac{2}{11}$$

R = **2**

13. (a) +1 ↑ +2 ↑

$$\frac{7^{19} + 2}{6} = \frac{(+1)^{19} + 2}{6}$$

$$= \frac{1+2}{6} = \frac{3}{6}$$

R = **3**

14. (a) $\frac{3^{21}}{5}$

∵ $3^2 = 9$

Break The power multiple of 2 form

-1 ↑ +3 ↑

$$= \frac{(3^2)^{10} \times 3^1}{5} = \frac{(9)^{10} \times 3}{5}$$

$$= \frac{(-1)^{10} \times 3}{5}$$

$$= \frac{1 \times 3}{5} = \frac{3}{5}$$

R = **3**

15. (c) $\frac{2^{31}}{5}$

∵ $2^2 = 4$

-1 ↑

$$\frac{(2^2)^{15} \times 2^1}{5} = \frac{(4)^{15} \times 2^1}{5}$$

$$= \frac{(-1)^{15} \times 2}{5} = \frac{-1 \times 2}{5} = \frac{-2}{5}$$

R = 5 - 2 = **3**

16. (b) $\frac{2^{591}}{255}$

∵ $2^8 = 256$

Now $\frac{(2^8)^{73} \times 2^7}{255}$

+1 ↑

$$= \frac{(256)^{73} \times 128}{255}$$

$$= \frac{(1)^{73} \times 128}{255} = \frac{128}{255}$$

Remainder = **128**

17. (a) +2 ↑

$$\frac{51^{203}}{7} = \frac{(2)^{203}}{7}$$

∵ $2^3 = 8$

+1 ↑ +4 ↑

$$= \frac{(2^3)^{67} \times 2^2}{7} = \frac{(8)^{67} \times 4}{7}$$

$$\Rightarrow \frac{(+1)^{67} \times 4}{7}$$

$$\Rightarrow \frac{1 \times 4}{7} = \frac{4}{7}$$

R = 4

18. (a) $\frac{2^{243}}{3^2} = \frac{2^{243}}{9}$

∵ $2^3 = 8$

-1 ↑

$$= \frac{(2^3)^{81}}{9} = \frac{(8)^{81}}{9} = \frac{(-1)^{81}}{9} = \frac{-1}{9}$$

Remainder = 9 – 1 = **8**

19. (b) +3 ↑

$$\frac{59^{28}}{7} = \frac{3^{28}}{7} = \frac{(3^3)^9 \times 3}{7}$$

∵ $3^3 = 27$

- 1 ↑ +3 ↑

$$\Rightarrow \frac{(27)^9 \times 3}{7}$$

$$\Rightarrow \frac{(-1)^9 \times 3}{7} = \frac{-1 \times 3}{7} = \frac{-3}{7}$$

R = 7 - 3 = 4

20. (c) +7 ↑

$$\frac{41^{77}}{17} = \frac{7^{77}}{17} = \frac{(7^2)^{38} \times 7^1}{17}$$

- 2 ↑

$$\Rightarrow \frac{(49)^{38} \times 7}{17} = \frac{(-2)^{38} \times 7}{17}$$

∵ There will be no effect of -ve sign because the power is even

$$\rightarrow \frac{2^{38} \times 7}{17} = \frac{(2^4)^9 \times 2^2 \times 7}{17}$$

∵ $2^4 = 16$

-1 ↑

$$\rightarrow \frac{(16)^9 \times 4 \times 7}{17} = \frac{(-1)^9 \times 28}{17} = \frac{-1 \times 28}{17}$$

$= \frac{-28}{17} = \frac{\overset{-6}{\overset{\uparrow}{(28)}}}{17}$ (Avoid -ve sign)

Now use -ve sign
R = - (-6) = **6**

21. (b) $\frac{2^{49}}{7}$ $\because 2^3 = 8$

$\Rightarrow \frac{(2^3)^{16} \times 2^1}{7} = \frac{\overset{+1}{\overset{\uparrow}{(8)}}{}^{16} \times 2}{7}$

$= \frac{(+1)^{16} \times 2}{7} = \frac{1 \times 2}{7} = \frac{2}{7}$

R = 2

22. (d) $\frac{\overset{0}{\overset{\uparrow}{51^{203}}} + 2^{49}}{17}$

51 is divisible by 17 So $(51)^{203}$ is divisible by 17 then remainder '0', Now only divide 2^{49}

$= \frac{2^{49}}{17} = \frac{(2^4)^{12} \times 2^1}{17} = \frac{\overset{-1}{\overset{\uparrow}{(16)}}{}^{12} \times 2}{17}$

$= \frac{(-1)^{12} \times 2}{17} = \frac{1 \times 2}{17} = \frac{2}{17}$

R = **2**

23. (b) $\frac{1234567891011121314}{8}$

$\because$ divisibility by 8 $\rightarrow$ The Last Three digits are divisible by 8 So Now last 3 digits 314 divide by 8 we get remainder

$= \frac{314}{8}$, R = **2**

24. (c) $\frac{41424344.........4950}{16}$

divisibility by 16 $\rightarrow$ The last Four digits are divisible by 16
No : Last '4' digits $\underline{4}\ \underline{9}\ \underline{5}\ \underline{0}$

$\frac{4950}{16}$ = R = **6**

25. (b) $\frac{21222324252627282930}{8}$

Last '3' digits 930

Remainder $= \frac{930}{8}$

R = **2**

26. (c) $\frac{919293949596979899}{16}$

Last '4' digits $\underline{9}\ \underline{8}\ \underline{9}\ \underline{9}$

Remainder = 16) 9899 (
96
29
16
139
128
11

Remainder = **11**

27. (c) $\frac{313233........3839}{4}$

divisibility by 4 $\rightarrow$ The last '2' digits divisible by 4
Last '2' digits 39

R = $\frac{39}{4}$

R = **3**

28. (a) $\frac{12345..........41 \text{ digits}}{8}$

From 1 to 9 = 9 digits
Remainder = 41 - 9 = 32 digits

Number $= \frac{32}{2} = 16$

1, 2, 3, 4, ...9/ $\underline{10}$ $\underline{11}$ .... 41 digits
Total Number = 9 + 16 = 25
1 2 3 4 .... 23 $\underline{24}$ $\underline{25}$
Last '3' digits = 425

Remainder $= \frac{425}{8}$

R = **1**

29. (a) $\frac{1234.......81 \text{digits}}{16}$

from 1 to 9 = 9 digits
Remainder digits = 81 - 9 = 72 digits

Number $= \frac{72}{2} = 36$

1, 2, 3, ...9/ $\underline{10}$ $\underline{11}$ .. 81 digits
Total Number = 9 + 36 + 45
1, 2, 3, ... 43 44 45
Last '4' digits 4445 divide by 16 we get remainder

R = $\frac{4445}{16}$ = **13**

16) 4445 (
32
124
112
125
112
13 $\rightarrow$ Remainder

30. (b) $\frac{8^{77}}{17}$

$\because 8^3 = 512$

$\Rightarrow \frac{(8^3)^{25} \times 8^2}{17} = \frac{\overset{+2}{\overset{\uparrow}{(512)}}{}^{25} \times 64}{17}$

$\Rightarrow \frac{(2)^{25} \times 64}{17}$

$= \frac{(2^4)^6 \times 2^1 \times 64}{17}$

$= \frac{\overset{1}{\overset{\uparrow}{(16)}}{}^6 \times 2 \times \overset{-4}{\overset{\uparrow}{64}}}{17} = \frac{(-1)^6 \times 2 \times -4}{17}$

$= \frac{1 \times 2 \times -4}{17} = \frac{-8}{17}$

R = 17 - 8 = 9

31. (d) $\frac{\lfloor 1 + \lfloor 2 + \lfloor 3 + ...\lfloor 100}{5}$

$\lfloor 1 = 1 = \frac{\overset{+1}{\overset{\uparrow}{1}}}{5}$, R = 1

$\lfloor 2 = 1 \times 2 = \frac{\overset{+2}{\overset{\uparrow}{2}}}{5}$, R = 2

$\lfloor 3 = 1 \times 2 \times 3 = \frac{\overset{+1}{\overset{\uparrow}{6}}}{5}$, R = 1

$\lfloor 4 = 1 \times 2 \times 3 \times 4 = \frac{\overset{-1}{\overset{\uparrow}{24}}}{5}$, R = -1

$\lfloor 5 = 5 \times 4 \times 3 \times 2 \times 1 = \frac{120}{5}$, R = 0

$\lfloor 5$, $\lfloor 6$, $\lfloor 7$ .... $\lfloor 100$ is all perfect divisible by 5.So remainder '0'

$= \frac{\overset{+1}{\overset{\uparrow}{\lfloor 1}} + \overset{+2}{\overset{\uparrow}{\lfloor 2}} + \overset{+1}{\overset{\uparrow}{\lfloor 3}} + \overset{-1}{\overset{\uparrow}{\lfloor 4}} + \overset{0}{\overset{\uparrow}{\lfloor 5}} + \overset{0}{\overset{\uparrow}{\lfloor 6}} + ...\overset{0}{\overset{\uparrow}{\lfloor 99}} + \overset{0}{\overset{\uparrow}{\lfloor 100}}}{5}$

$\Rightarrow \frac{1 + 2 + 1 - 1}{5} = \frac{3}{5}$

R = **3**

32. (a) $\dfrac{\lfloor\underline{1}+\lfloor\underline{2}+\lfloor\underline{3}+\ldots\ldots\lfloor\underline{100}}{6}$

$\lfloor\underline{1} = 1 = \dfrac{\overset{+1}{1}}{6} = R = 1$

$\lfloor\underline{2} = 1\times2 = \dfrac{\overset{+2}{2}}{6} = R = 2$

$\lfloor\underline{3} = 1\times2\times3 = \dfrac{\overset{0}{6}}{6} = R = 0$

$\lfloor\underline{4} = 4\times3\times2\times1 = \dfrac{24}{6} = R = 0$

$= \dfrac{\overset{+1}{\lfloor\underline{1}}+\overset{+2}{\lfloor\underline{2}}+\overset{0}{\lfloor\underline{3}}+\overset{0}{\lfloor\underline{4}}+\overset{0}{\lfloor\underline{5}}+\ldots\ldots\ldots\overset{0}{\lfloor\underline{100}}}{6}$

$\dfrac{1+2}{6} = \dfrac{3}{6}$

R = **3**

33. (d) $\dfrac{\lfloor\underline{1}+\lfloor\underline{2}+\lfloor\underline{3}+\ldots\ldots\lfloor\underline{50}}{12}$

$\lfloor\underline{1} = 1 = \dfrac{\overset{+1}{1}}{12} = R = 1$

$\lfloor\underline{2} = 1\times2 = \dfrac{\overset{+2}{2}}{12} = R = 2$

$\lfloor\underline{3} = \dfrac{1\times2\times3}{12} = \dfrac{\overset{+6}{6}}{12} = R = 6$

$\lfloor\underline{4} = \dfrac{1\times2\times3\times4}{12} = \dfrac{\overset{0}{24}}{12} = R = 0$

$\lfloor\underline{5} = \dfrac{1\times2\times3\times4\times5}{12} = \dfrac{\overset{0}{120}}{12} = R = 0$

$= \dfrac{\overset{+1}{\lfloor\underline{1}}+\overset{+2}{\lfloor\underline{2}}+\overset{+6}{\lfloor\underline{3}}+\overset{0}{\lfloor\underline{4}}+\overset{0}{\lfloor\underline{5}}+\ldots\ldots\ldots\overset{0}{\lfloor\underline{50}}}{6}$

$\dfrac{1+2+6}{12} = \dfrac{9}{12} = 9$

R = 9

34.(b)

$\dfrac{9^{111}}{11} = \dfrac{(\overset{-2}{9})^{111}}{11}$

$= \dfrac{(-2)^{111}}{11} = -\dfrac{(2)^{111}}{11}$

Avoid -ve sign

$\dfrac{2^{111}}{11} = \dfrac{(2^5)^{22}\times2^1}{11}$

$\because\ 2^5 = 32$

$= \dfrac{(\overset{-1}{32})^{22}\times2}{11}$

$= \dfrac{(-1)^{22}\times2}{11} = \dfrac{1\times2}{11} = \dfrac{2}{11}$

Now use -ve sign

R = – 2

Actual Remainder = 11 – 2 = **9**

35. (b) $\dfrac{5^{2450}}{126}$

$\because\ 5^3 = 125$

$\dfrac{(5^3)^{816}\times5^2}{126} = \dfrac{(\overset{-1}{125})^{816}\times25}{126}$

$= \dfrac{(-1)^8\times25}{126} = \dfrac{1\times25}{126} = \dfrac{25}{126}$

R = 25

36. (d) $\dfrac{(\overset{-2}{40})^{1012}}{7} = \dfrac{(-2)^{1012}}{7}$

- ve sign will be no effect because power is even

$= \dfrac{(2)^{1012}}{7}$

$\because\ 2^3 = 8$

$= \dfrac{(2^3)^{337}\times2^1}{7}$

$= \dfrac{(\overset{+1}{8})^{337}\times2}{7} = \dfrac{(+1)^{337}\times2}{7}$

$= \dfrac{1\times2}{7} = \dfrac{2}{7}$

R = **2**

37. (a) $\dfrac{10^1+10^2+10^3+10^4\ldots\ldots.10^{100}}{6}$

$10^1 = \dfrac{10}{6} = R = 4$

$10^1+10^2 = \dfrac{\overset{4}{10}+\overset{4}{100}}{6} = \dfrac{\overset{+2}{8}}{6}$

R = 2

$10^1+10^2+10^3 = \dfrac{10+100+1000}{6}$

$= \dfrac{4+4+4}{6} = \dfrac{12}{6}$

Remainder = 0

'0' will be the remainder of each three terms

So '0' will be the remainder of 99th term.

$\Rightarrow \dfrac{\overset{+4}{10^{100}}}{6}$ R = **4**

* **10^n is divided by 6. We always get remainder 4, where n = natural number**

38. (c) $\dfrac{10^1+10^2+10^3\ldots\ldots\ldots.10^{1000}+10^{1001}}{6}$

$10^1 = \dfrac{\overset{+4}{10}}{6}$ R = 4

$10^1+10^2 = \dfrac{\overset{+4}{10}+\overset{+4}{100}}{6} = \dfrac{8}{6}$

R = 2

$10^1+10^2+10^3 = \dfrac{\overset{+4}{10}+\overset{+4}{100}+\overset{+4}{1000}}{6}$

$= \dfrac{4+4+4}{6} = \dfrac{12}{6}$

R = 0

'0' will be remainder of each three terms so '0' will be the remainder of 999th.

$\dfrac{\overset{+4}{10^{1000}}+\overset{4}{10^{1001}}}{6} = \dfrac{4+4}{6} = \dfrac{8}{6}$

R = 2

39.(a) $\dfrac{666666\ldots\ldots.134\text{ times}}{13}$

* If any digit is made by repeating a 6 times. The Number will be divisible by 3, 7, 11, 13, 37, 39. So, 666666 ……… 132 times is divisible by 13. because 132 is the multiple of 6. So, The Remaining 2 digits will be. divideded by 13 to get the Remainder

$$=\frac{666666........132\text{times}\underline{\underline{66}}}{13}$$

$$=\frac{66}{13}=R=1$$

40. (b) $\frac{555555......244\text{ times}}{37}$

555555……240 times is divisible by 37 becaused any digit is made by repeating 6 times. The number will be divisible by 37.

$$\frac{555555........240\text{times},5555}{37}$$

Remainder $\frac{5555}{37}$

$$37\overline{)5555}(15$$
$$\underline{37}$$
$$185$$
$$\underline{185}$$
$$5 \longrightarrow \text{Remainder}$$

R = 5

41. (b) $\frac{777777.............363\text{Times}}{11}$

$$=\frac{777777.............360\text{ Times},777}{11}$$

$$=\frac{777}{11}$$

Remainder = 7

42. (b) $\frac{888888.........184\text{ times}}{37}$

$$\Rightarrow \frac{888888.........180\text{ times},8888}{37}$$

$$\Rightarrow \frac{8888}{37}$$

$$37\overline{)8888}($$
$$\underline{74}$$
$$148$$
$$\underline{148}$$
$$8 \longrightarrow \text{Remainder}$$

$\because$ So Remainder = 8

43. (b) $\frac{999999\underline{\underline{999}}}{13}$

6 times '9' is divisible by 13, remainder will be 0, Remaining digits divide by 13 we get Remainder

$$\frac{999}{13}$$

R = **11**

44. (c) $\frac{7^{99}}{2400}$

$$\boxed{\begin{array}{l}7^1=7\\7^2=49\\7^3=343\\7^4=2401\end{array}}$$

Break the power multiple of '4' form

$$\Rightarrow \frac{(7^4)^{24}\times 7^3}{2400}$$

$$\Rightarrow \frac{(2401)^{24}\times 343}{2400}$$

$$\Rightarrow \frac{(\overset{+1}{\uparrow}{2401})^{24}\times 343}{2400}=\frac{(+1)^{24}\times 343}{2400}$$

$$=\frac{1\times 343}{2400}=\frac{343}{2400}$$

R = 343

45. (b) $\frac{3^{1989}}{7}$

$\because\ 3^3=27$

$$\frac{(3^3)^{663}}{7}=\frac{(\overset{-1}{\uparrow}{27})^{663}}{7}$$

$$=\frac{(-1)^{663}}{7}=-1$$

R = 7 – 1 = 6

46. (a)

$$\frac{54^{124}}{17}=\frac{(\overset{+3}{\uparrow}{54})^{124}}{17}$$

$$=\frac{(3)^{124}}{17}$$

$$\boxed{\begin{array}{l}3^1=3\\3^2=9\\3^3=27\\3^4=81\end{array}}$$

$\because\ 3^4=81$

$$=\frac{(3^4)^{31}}{17}=\frac{(\overset{-4}{\uparrow}{81})^{31}}{17}$$

$$=\frac{(-4)^{31}}{17}=-\frac{(4)^{31}}{17}$$

Avoid -ve sing. $=\frac{4^{31}}{17}$

$\because\ 4^2=16$

$$=\frac{(4^2)^{15}\times 4^1}{17}=\frac{(16)^{15}\times 4}{17}$$

$$=\frac{(-1)^{15}\times 4}{17}=\frac{-1\times 4}{17}=\frac{-4}{17}=-4$$

Now use -ve sign = – (–4)

Remainder = **4**

47. (b)

$$\frac{(21)^{875}}{17}=\frac{(\overset{+4}{\uparrow}{21})^{875}}{17}$$

$$=\frac{(4)^{875}}{17}\quad \because 4^2=16$$

$$=\frac{(4^2)^{437}\times 4^1}{17}$$

$$=\frac{(\overset{-1}{\uparrow}{16})^{437}\times 4^1}{17}=\frac{(-1)^{437}\times 4}{17}$$

$$=\frac{-1\times 4}{17}=\frac{-4}{17}$$

R = 17 – 4 = 13

48. (d)

$$\frac{83^{261}}{17}=\frac{(\overset{-2}{\uparrow}{83})^{261}}{17}$$

$$=\frac{(-2)^{261}}{17}=\frac{-(2)^{261}}{17}$$

Avoide -ve sign.

$$=\frac{2^{261}}{17}=\frac{(2^4)^{65}\times 2^1}{17}$$

$\because\ 2^4=16$

$$=\frac{(\overset{-1}{\uparrow}{16})^{65}\times 2}{17}=\frac{(-1)^{65}\times 2}{17}$$

$$=\frac{-1\times 2}{17}=-2$$

Now use sign -(-2) = 2

Remainder = 2

49. (a) $\frac{(32^{32})^{32}}{9}$

Cyclicity

$\frac{\overset{+5}{32^1}}{9} = R = 5$

$\frac{32^2}{9} = \frac{\overset{+5}{32}\times\overset{+5}{32}}{9} = \frac{5\times5}{9} = \frac{25}{9} = R = 7$

$\frac{32^3}{9} = \frac{\overset{+5}{32}\times\overset{+5}{32}\times\overset{+5}{32}}{9} = \frac{5\times5\times5}{9}$

$= \frac{125}{9} = R = 8$

$\frac{32^4}{9} = \frac{\overset{+5}{32}\times\overset{+5}{32}\times\overset{+5}{32}\times\overset{+5}{32}}{9} = \frac{5\times5\times5\times5}{9}$

$= \frac{\overset{-2}{25}\times\overset{-2}{25}}{9} = \frac{-2\times-2}{9} = \frac{4}{9}$, R= 4

$\frac{32^5}{9} = \frac{\overset{+5}{32}\times\overset{+5}{32}\times\overset{+5}{32}\times\overset{+5}{32}\times\overset{+5}{32}}{9} = \frac{\overset{-2}{25}\times\overset{-2}{25}\times\overset{5}{5}}{9}$

$= \frac{-2\times-2\times5}{9} = \frac{20}{9}$, R = 2

$\frac{32^6}{9} = \frac{\overset{+5}{32}\times\overset{+5}{32}\times\overset{+5}{32}\times\overset{+5}{32}\times\overset{+5}{32}\times\overset{+5}{32}}{9} = \frac{\overset{-2}{25}\times\overset{-2}{25}\times\overset{-2}{25}}{9}$

$= \frac{-2\times-2\times-2}{9} = \frac{-8}{9}$, R = 9 − 8 = 1

After this It repeated so,
Cyclicity = 6

So, $\frac{(32)^6}{9}$ R = 1

Now $(32^{32})^{32} = \left[(32^6)^5\times32^2\right]^{32}$

$= \frac{((1)^5\times32^2)^{32}}{9} = \frac{(32^2)^{32}}{9}$

∵ $\frac{32^2}{9} = R = 7$

(above explain In Solution)

$= \frac{(7)^{32}}{9} = \frac{(7^2)^{16}}{9} = \frac{(\overset{+4}{49})^{16}}{9}$

$= \frac{4^{16}}{9} = \frac{(4^3)^5\times4^1}{9} = \frac{(\overset{+1}{64})^5\times4}{9}$

$\frac{(+1)^5\times4}{9} = \frac{1\times4}{9} = \frac{4}{9}$

R = 4

50. (a) $\frac{(32^{32})^{32}}{7}$

When 32 is divided by 7 then Remainder 4
So, 32^{32} is divided by 7 remainder = 4^{32}

$= \frac{(4^{32})^{32}}{7}$

$4 = 2^2$

$\Rightarrow \frac{(2^{2\times32})^{32}}{7}$

$\Rightarrow \frac{(2^{64})^{32}}{7}$

∵ $2^3 = 8$

$= \frac{(2^{63}\times2^1)^{32}}{7}$

$= \frac{((2^3)^{21}\times2^1)^{32}}{7} = \frac{((\overset{+1}{8})^{21}\times2^1)^{32}}{7}$

$= \frac{(1\times2)^{32}}{7} = \frac{2^{32}}{7}$

Again ∵ $2^3 = 8$

$\frac{2^{30}\times2^2}{7} = \frac{(2^3)^{10}\times4}{7}$

$= \frac{(\overset{+1}{8})^{10}\times4}{7} = \frac{(1)^{10}\times4}{7} = \frac{1\times4}{7}$

Remainder = 4

51. (b) $\frac{(33^{34})^{35}}{7}$

we solve $\frac{33^{34}}{7} = \frac{(\overset{-2}{33})^{34}}{7}$

$= \frac{(-2)^{34}}{7}$

No effect of -ve sign. Because power is even.

$= \frac{2^{34}}{7} = \frac{(2^3)^{11}\times2^1}{7}$

$= \frac{(\overset{+1}{8})^{11}\times2}{7} = \frac{(+1)^{11}\times2}{7} = \frac{1\times2}{7}$

$= \frac{2}{7}$

Now :- $\frac{(33^{34})^{35}}{7} = \frac{(2)^{35}}{7} = \frac{(2^3)^{11}\times2^2}{7}$

$= \frac{(\overset{+1}{8})^{11}\times4}{7} = \frac{1\times4}{7} = \frac{4}{7}$

R = 4

52. (a) $\frac{888^{222}+222^{888}}{5}$

$= \frac{888^{222}}{5} + \frac{222^{888}}{5}$

$= \frac{3^{222}}{5} + \frac{2^{888}}{5}$

$= \frac{(3^4)^{55}\times3^2}{5} + \frac{(2^4)^{222}}{5}$

$= \frac{1\times9}{5} + \frac{1}{5}$

$= \frac{4}{5} + \frac{1}{5} = \frac{4+1}{5} \Rightarrow \frac{5}{5}$

Thus the remainder is zero.

Alternatively:

[To check the divisibility by 5 just see the sum of the unit digits which is 10 (=4+6)

∵ $8^{222} \to 4$(units digit)

and $2^{888} \to 6$(units digit)

Hence it is divisible. So there is no remainder]