New

PINNACLE

1st edition

Complete Arithmetic and Advance maths

# Maths Formula Book

- Class notes
- Concepts
- Formula
- Short tricks
- Smart methods
- solved examples
- Exam hall approach
- Calculation tricks

English Medium

Useful for SSC, Railway, Banking, Police, CET, and other exams

piracy check

each book has multipurpose unique ID

PINNACLE Publications

# INDEX

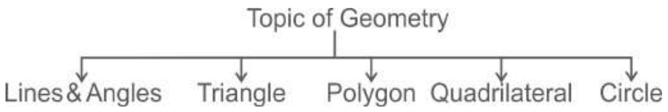
## Advance Math



## Arithmetic

## Calculation Math

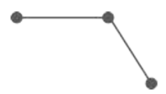
# Geometry




## Lines

**POINT :-** Point is a circle having zero radius. It is usually represented by a dot (.) .

### Type of Points

| Collinear Points | Non - Collinear Point |
|---|---|
| Points are said to be collinear when a set of three or more points exist on the same straight line. | The points do not exist on the same straight line. |
|  |  |

**Note :-** From a point, an infinite number of lines can be drawn.

**LINE :-** A line is a straight one - dimensional figure that does not have a thickness, and it extends endlessly in both directions.

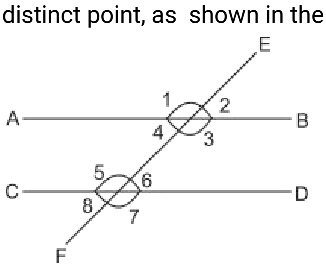
### Types of Line

**1. Parallel Line :-** A pair of two lines that are on the same plane and the distance between them is equal and remains constant.

**Note:**These lines do not intersect with each other.

**i.e.**

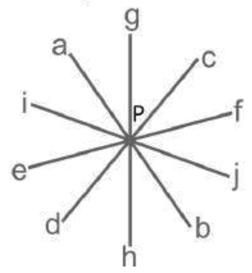
**2. Transversal Lines :-** A line which cuts parallel lines at a distinct point, as shown in the figure below.



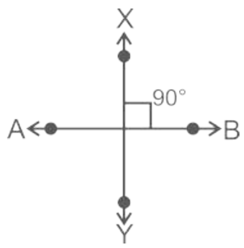

- EF is the transversal line. AB and CD are parallel lines. Symbol of parallel lines is '||'. To denote a line symbols like $\overline{AB}$ or simply AB are used.

**3. Intersecting Line :-** When two or more pairs of lines are on the same plane and intersect each other at one given point, they are known as intersecting lines.

P is the point of intersection



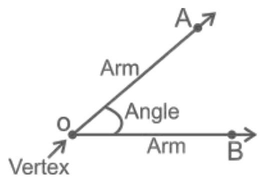

**4. Perpendicular Line :-** When two lines on the same plane intersect each other and form a 90° angle at the point of intersection, they are known to be perpendicular lines.



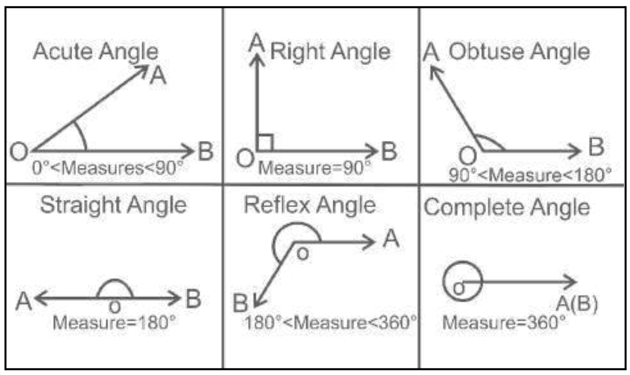

## Angles

The measure of the 'opening' between two lines/rays is called an 'angle'. It is represented by the symbol "∠".

**Parts of Angle :-**

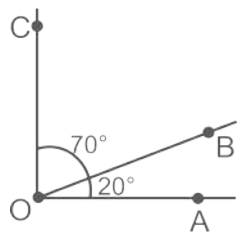



### Types of Angle

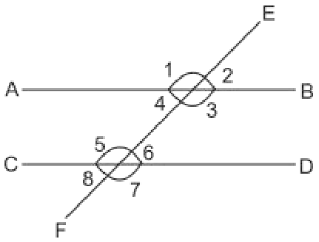



### Pair of Angles

**1) Complementary Angles :-** Sum of two angles is equal to 90°.

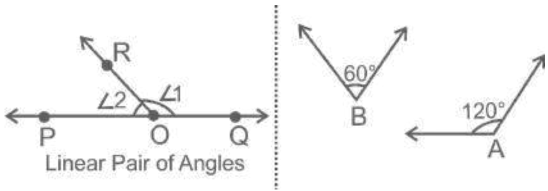



**2) Supplementary Angles :-** Sum of the two angles is equal to 180°. E.g. $\angle 1 + \angle 2 = 180°$, $\angle 4 + \angle 3 = 180°$ etc.

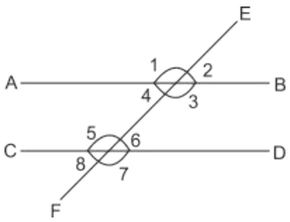



**3) Linear pair of Angles :-** A linear pair of angles is a pair of adjacent angles formed when two lines intersect each other at a point.

**Linear pair of angles V/S Supplementary Angles :-**




**When a transversal line intersects two parallel lines :-**




**1) Corresponding Angles :-**
$\angle 1 = \angle 5,\ \angle 2 = \angle 6, \angle 4 = \angle 8,\ \angle 3 = \angle 7$

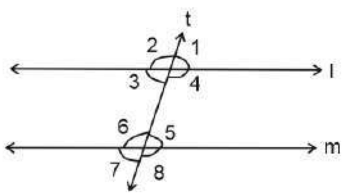
**2) Alternate Angles :-**
$(\angle 1 = \angle 7),\ (\angle 2 = \angle 8), (\angle 4 = \angle 6),\ (\angle 3 = \angle 5)$

**3) Vertically opposite angles :-**
$(\angle 1 = \angle 3), (\angle 2 = \angle 4), (\angle 6 = \angle 8), (\angle 5 = \angle 7)$

**4) Adjacent angles:-** $\angle 1 + \angle 2 = 180°$ in this case as these are linear pairs. It is not necessary that their sum should be $180°$.

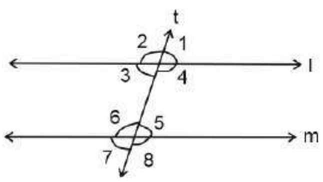
**5) Co-Interior angles :-** Sum of Interior angles on same side $= 180°$. E.g. $\angle 3 + \angle 6 = 180°$ , $\angle 4 + \angle 5 = 180°$. The angle made by bisectors of interior angle will be $90°$.

**Example :-** In the given figure, $l \parallel$ m and t is a transversal. If $\angle 1$ and $\angle 2$ are in the ratio 4 : 11 , the measures of the angles $\angle 7$ and $\angle 8$ , respectively, are :
RRC Group D 09/09/2022 (Morning)




(a) 110° and 70° (b) 87° and 93°
(c) 132° and 48° (d) 65° and 115°

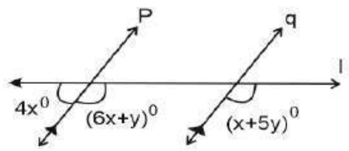
**Solution :-**




Ratio of $\angle 1$ and $\angle 2$ = 4 : 11

$4x + 11x = 180$
$\Rightarrow 15x = 180 \Rightarrow x = 12$
$\angle 1 = 48°$, and $\angle 2 = 132°$
$\angle 7 = \angle 3 = \angle 1 = 48°$
$\angle 8 = \angle 4 = \angle 2 = 132°$

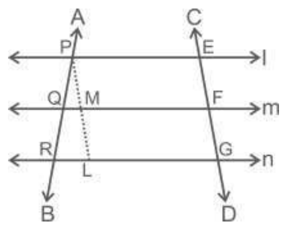
**Example :-** In the given figure, p || q and l is a transversal cutting p and q with angles as specified. The value of (5x - y) is :




RRC Group - D 06/09/2022 (Evening)
(a) 60 (b) 40 (c) 96 (d) 116

**Solution :-** When two lines are parallel then corresponding angles are equal
So, $6x + y = x + 5y$
$5x = 4y \Rightarrow x = \frac{4y}{5}$
On a straight line sum of angles $= 180°$
So, $4x + 6x + y = 180°$
$10x + y = 180°$ .....(1)
By putting value of $x$ in equation (1)
$8y + y = 180°$ so, $y = 20°$
And $x = \frac{4 \times 20°}{5} = 16°$
Then the value of $5x - y = 16° \times 5 - 20° = 60°$

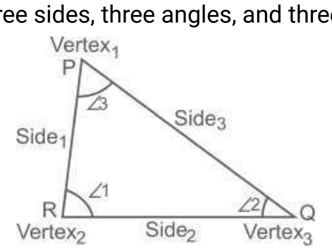
**If three or more parallel lines are intersected by two transversals -**



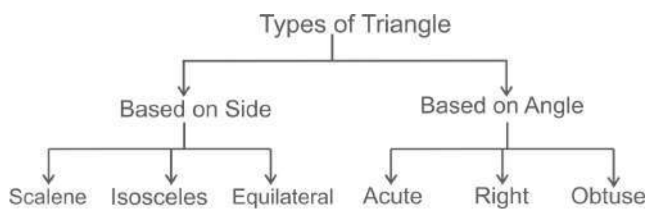

Results $\rightarrow \frac{PQ}{QR} = \frac{EF}{FG}$

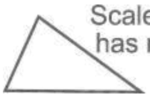
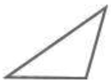
## Triangles

A triangle has three sides, three angles, and three vertices.




## Types of Triangles



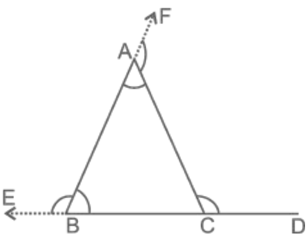
| By Side | By Angle |
|---|---|
|  Scalene Triangle has no equal sides |  Acute Triangle has three angles<90° |
|  Isoceles Triangle has two equal sides |  Right Triangle has one angle=90° |
|  Equilateral Triangle has three equal sides |  Obtuse Triangle has one angle>90° |

**Example:-** In a triangle ABC, the three angles are x, y and y + 10. Also, 2x - 4y = 20°. Which type of triangle is ABC?
SSC CGL 26/07/2023 (3rd shift)
(a) Equilateral (b) Obtuse (c) Acute (d) Right-angled
**Solution :-** According to question,
$x + y + (y + 10) = 180°$
$x + 2y = 170°$ .......(1)
$2x - 4y = 20° \Rightarrow x - 2y = 10°$ ........(2)
By solving both equations, we get
$x = \frac{170° + 10°}{2} = 90°$
So, triangle ABC is a right angled triangle.

**Special cases of a Right-angled triangle :-**

| Isosceles Right - angled triangle | Scalene Right - angled triangle |
|---|---|
| • 45° - 45° - 90° triangle | • 30° - 60° - 90 triangle |
| • The angles of this triangle are in the ratio – 1 : 1 : 2<br>• The sides of this triangle will be in the ratio – 1 : 1 : √2 respectively. | • The angles of this triangle are in the ratio – 1: 2: 3<br>• The sides opposite to these angles will be in the ratio – 1 : √3 : 2 respectively |

## Properties of Triangles




**Angle Sum Property :-** The sum of angles of a triangle is always 180°.
• ∠ABC + ∠BAC + ∠ACB = 180° (Angle sum property)

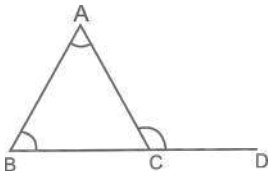
**Triangle inequality :-**
**(a)** The sum of the length of any two sides of a triangle is greater than the length of the third side.
• AB + BC > AC
• AB + AC > BC
• BC + AC > AB

**(b)** The difference between the lengths of any two sides of a triangle is always less than the length of the third side
• AB - BC < AC
• AB - AC < BC
• BC - AC < AB

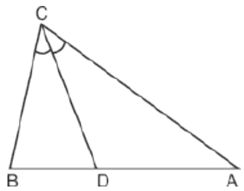
**(c)** Let ABC be a triangle with sides a, b, c.
If $c^2 = a^2 + b^2$, then the angle at C is a right angle.
If $c^2 < a^2 + b^2$, then the angle at C is acute angle.
If $c^2 > a^2 + b^2$, then the angle at C is obtuse angle.

**Angle Side Inequality:-**
**(a)** The side opposite to the largest angle of a triangle is the largest side.
**(b)** The side opposite to the smallest angle is the shortest side.

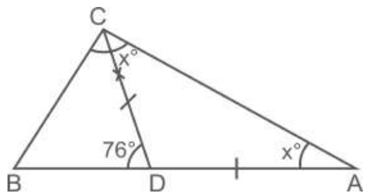
**Exterior Angle property:-** Any exterior angle of the triangle is equal to the sum of its interior opposite angles. This is called the exterior angle property of a triangle.
• ∠ACD = ∠ABC + ∠CAB (Exterior Angle Property)




**Example :-** In the given triangle, CD is the bisector of ∠BCA. CD = DA. If ∠BDC = 76°, what is the degree measure of ∠CBD ?

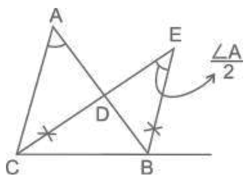



SSC CGL 01/12/2022 (4th Shift)
(a) 32° (b) 76° (c) 80° (d) 66°
**Solution :-**
Exterior angle of a triangle is equal to Sum of two opposite interior angles.

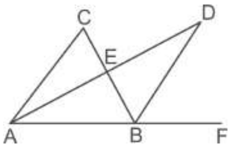



In $\Delta CDA$
$\angle DCA = \angle DAC$ ........( DC = DA )
∠DCA + ∠DAC = 76° ........(exterior angle )
x° + x° = 76° ⇒ x° = 38°
Now, $\angle DCA = \angle DCB = 38°$ .... ( CD is angle bisector of ∠C )
Now , In $\Delta CBD$
$\angle DCB + \angle CBD + \angle BDC = 180°$
$\Rightarrow 38° + \angle CBD + 76° = 180° \Rightarrow \angle CBD = 66°$

• In a triangle ABC, angle bisector of interior angle C and exterior angle B meet at E, then $\angle BEC = \frac{\angle A}{2}$



**Example :-** In the given figure, AD is bisector of angle ∠CAB and BD is bisector of angle ∠CBF. If the angle at C is 34°, the angle ∠ADB is:




SSC CHSL 13/10/2020 (Morning)

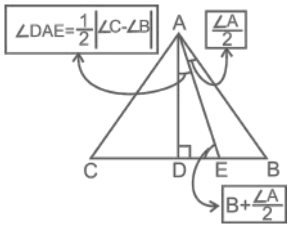
(a) 34° (b) 32° (c) 17° (d) 16°

**Solution :-** $\angle ADB = \frac{1}{2}\angle ACB$

$\angle ADB = \frac{1}{2} \times 34° = 17°$

- In Δ ABC, AE is angle Bisector of ∠A and AD ⊥ BC

then $\angle DAE = \frac{1}{2}|\angle C - \angle B|$

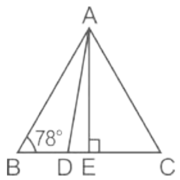



**Example :-** In ΔABC, ∠B = 78°, AD is a bisector of ∠A meeting BC at D, and AE⊥BC at E. If ∠DAE = 24°, then the measure of ∠ACB is:

SSC CGL Tier II (29/01/2022)

(a) 32° (b) 38° (c) 30° (d) 42°

**Solution :-**




As we know,

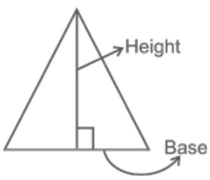
$\frac{1}{2}(\angle ABC - \angle ACB) = \angle DAE$

$\Rightarrow \frac{1}{2}(78° - \angle ACB) = 24°$

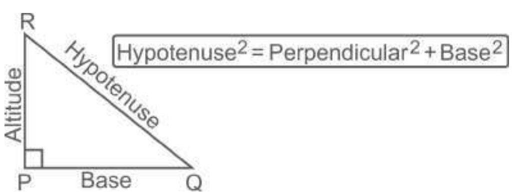
$78° - \angle ACB = 48°$

$\angle ACB = 78° - 48° = 30°$

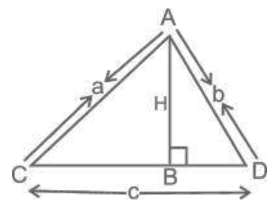
**Height and Base of Triangle :-** The height of a triangle is equal to the length of the perpendicular dropped from a vertex to its opposite side, and this side is considered the base.




**Pythagoras Theorem :-** In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.




**Area of Triangle :-** The area of a triangle is the region occupied by the triangle in 2d space.




Area = $\frac{1}{2}$ × Base × Height

In ΔABD,

$\sin D = \frac{H}{AD}$ or $AD = \frac{H}{\sin D}$

**General Formula:**

Area = $\frac{1}{2}$ × b × c × sinD

= $\frac{1}{2}$ × a × c × sinC

= $\frac{1}{2}$ × a × b × sinA

**Example :-** Find the area of a triangle whose length of two sides are 4 cm and 5 cm and the angle between them is 45°.

SSC CPO 04/10/2023 (2nd Shift)

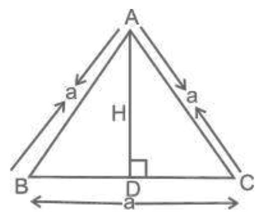
(a) $7\sqrt{2}\ cm^2$ (b) $4\sqrt{2}\ cm^2$ (c) $6\sqrt{2}\ cm^2$ (d) $5\sqrt{2}\ cm^2$

**Solution :-** According to the question,

Sine rule = $\frac{1}{2}$ × a × b × sinθ (By using the formula )

Area of triangle = $\frac{1}{2} \times 4 \times 5 \times \frac{1}{\sqrt{2}} = 5\sqrt{2}$

**Formulas of triangle :-**




**In case of equilateral triangle,**

- Area(A) = $\frac{\sqrt{3}}{4}(\text{side})^2$
- Inradius(r) = $\frac{side}{2\sqrt{3}}$
- circumradius(R) = $\frac{side}{\sqrt{3}}$
- height(H) = $\frac{\sqrt{3}}{2}$ × side

**Example:-** The perimeter of an equilateral triangle is 36 m and the length of its altitude is $6\sqrt{3}$ m. The area of triangle is:

RRB NTPC CBT - I 03/02/2021 (Morning)

(a) $36\sqrt{3}m^2$ (b) $18m^2$ (c) $24m^2$ (d) $12m^2$

**Solution :-**

Perimeter of equilateral triangle = 36 m

Each Side of triangle = $\frac{36}{3} = 12\ m,$

Area of triangle = $\frac{\sqrt{3}}{4}(\text{side})^2 = \frac{\sqrt{3}}{4} \times 12 \times 12 = 36\sqrt{3}m^2$

**Example :-** An equilateral triangle has sides of 18 cm each. The ratio of the inradius to circumradius of the triangle is:

SSC CHSL 14/08/2023 (4th Shift)

(a) 2 : 1 (b) 3 : 2 (c) 3 : 4 (d) 1 : 2

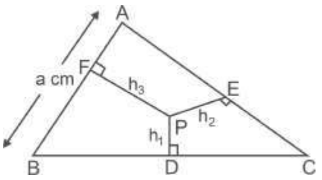
**Solution :-**

Inradius of equilateral Δ = $\frac{side}{2\sqrt{3}}$

circumradius of equilateral Δ = $\frac{side}{\sqrt{3}}$

Required ratio = 1 : 2

• If P is any point inside an equilateral triangle, then the sum of perpendiculars drawn from point P to sides AB, BC and AC is equal to the height of the triangle.

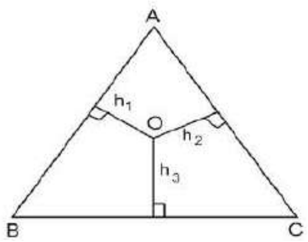



$h_1 + h_2 + h_3 = \frac{\sqrt{3}}{2}a$

Where a = side of equilateral triangle

**Example:-**'O' is a point in the interior of an equilateral triangle. The perpendicular distance from 'O' to the sides are √3 cm , 2√3cm, 5√3 cm. The perimeter of the triangle is:
SSC CGL 13/12/2022 (4th Shift)
(a) 48 cm (b) 32 cm (c) 24 cm (d) 64 cm

**Solution :-**

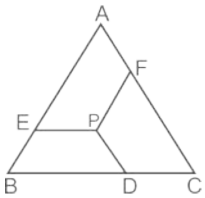



Side (a) = $\frac{2}{\sqrt{3}}(h_1 + h_2 + h_3)$

= $\frac{2}{\sqrt{3}}(\sqrt{3} + 2\sqrt{3} + 5\sqrt{3})$ = 16 cm

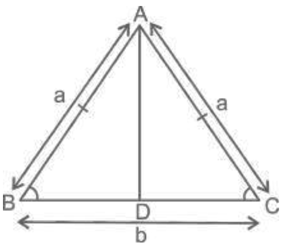
Perimeter of equilateral triangle = 3 × a = 48 cm

• If P is any point inside the equilateral triangle and PF II AB, PD II AC, PE II CB.




PD + PE + PF = AB = BC = CA.

**In case of isosceles triangle :-**




• Area(A) = $\frac{BC}{4}\sqrt{4AB^2 - BC^2}$

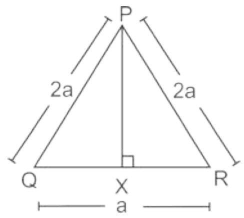
• Inradius(r) = $\frac{area\ of\ triangle}{Semi\ perimeter}$

• circumradius(R) = $\frac{abc}{4 \times Area\ of\ triangle}$

• height(h) = $\sqrt{a^2 - \frac{b^2}{4}}$

**Example :-** ΔPQR is an isosceles triangle and PQ = PR = 2a unit, QR = a unit. Draw PX ⊥QR, and find the length of PX.
SSC CGL 06/12/2022 (4th Shift)
(a) $\sqrt{5}$a (b) $\frac{\sqrt{5}a}{2}$ (c) $\frac{\sqrt{10}\,a}{2}$ (d) $\frac{\sqrt{15}\,a}{2}$

**Solution :-**




In an isosceles triangle , perpendicular drawn on the base from the opposite vertex bisect the base.

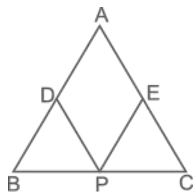
QX = XR = $\frac{a}{2}$

In right angle triangle PQX,

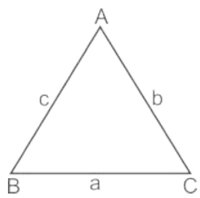
PX = $\sqrt{(2a)^2 - (\frac{a}{2})^2} = \sqrt{\frac{15a^2}{4}}$

$= \frac{\sqrt{15}\,a}{2}$

• If P is a point on the side BC and DP II AC & EP II AB then DP + EP = AB = AC



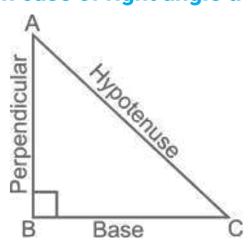

**In case of scalene triangle :-**




Semi-perimeter (s) = $\frac{a + b + c}{2}$

• Area(A) = $\sqrt{s(s - a)(s - b)(s - c)}$

**In case of right angle triangle :-**

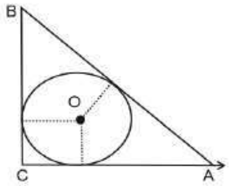



• Area(A) = $\frac{1}{2}$ × Base × height

• Inradius(r) = $\frac{Perpendicular + base - hypotenuse}{2}$

• circumradius(R) = $\frac{hypotenuse}{2}$

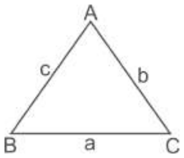
**Example:-** A circle of radius 4 cm is drawn inscribed in a right angle triangle ABC, right angled at C. If AC = 12 cm, then the value of CB is:




SSC CGL 25/07/2023 (4th shift)
(a) 8 cm (b) 12 cm (c) 20 cm (d) 16 cm

**Solution :-**
Pythagoras triplets :- (12, 16, 20)
Using hit and trial method ,
AC = 12 cm , BC = 16 cm and AB = 20 cm
Inradius = $\frac{16+12-20}{2}$ = 4 cm ....(satisfy)

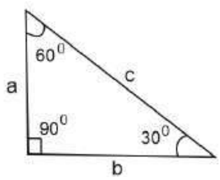
**Sine rule :-**




$\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC}$ = 2R
Where R = circumradius of triangle

**Example :-** In a triangle, if the angles are in the ratio 1 : 2 : 3, then the ratio of the corresponding sides is:
Selection Post 04/08/2022 (4th Shift)
(a) 1 : 1 : 2 (b) 1 : 2 : √3 (c) 1 : √3 : 2 (d) 1 : 2 : 3

**Solution :-** Let the angles of a triangle be x, 2x and 3x , ATQ, x + 2x + 3x = 180°
6x = 180°⇒ x = $\frac{180}{6}$ = 30°
Now,
2x = 2 × 30° = 60°⇒ 3x = 3 × 30° = 90°
So, we have following triangle;

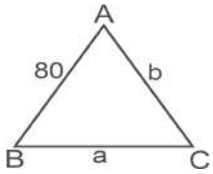



Using sine rule, we have;
$\frac{a}{sin\ 30°} = \frac{b}{sin\ 60°} = \frac{c}{sin\ 90°}$ = K
a : b : c = $\frac{1}{2}$K : $\frac{\sqrt{3}}{2}$K : K = 1 : $\sqrt{3}$ : 2
So, the required ratio = 1 : $\sqrt{3}$ : 2

**Cosine rule :-**
CosA = $\frac{c^2 + b^2 - a^2}{2bc}$
CosB = $\frac{c^2 + a^2 - b^2}{2ac}$
CosC = $\frac{a^2 + b^2 - c^2}{2ab}$

**Example:-** Side AB of a triangle ABC is 80 cm long, whose perimeter is 170 cm. If angle ABC = 60°, the shortest side of triangle ABC measures_____cm.
SSC CPO 16/03/2019 (Evening)
(a) 17 (b) 15 (c) 25 (d) 21

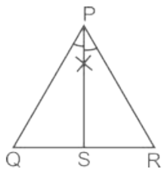
**Solution :-**
a + b + 80 = 170




a + b = 90 ⇒ a = 90 - b
From cosine rule
Cos60 ° = $\frac{a^2 + 80^2 - b^2}{2 \times 80 \times a}$
80(90 - b) = $(90 - b)^2 + 80^2 - b^2$
7200 - 80b = 8100 - 180b + 6400
100b = 7300 ⇒ b = 73
a = 90 - 73 = 17

## Some Important Result of Angle Bisector

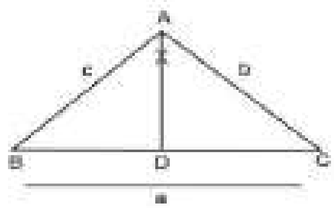
**When bisect internally :**




$\frac{PQ}{PR} = \frac{QS}{SR}$ or $\frac{PQ}{QS} = \frac{PR}{SR}$
Here PS is the internal angle bisector and a common side of the triangles $\Delta\ PQS\ and\ \Delta PSR$.

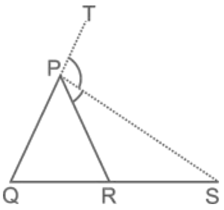
**Example:-** In △ABC, AD, the bisector of ∠A, meets BC at D. If BC = a, AC = b and AB = c, then BD - DC =
SSC CHSL 03/07/2019 (Afternoon)
(a) $\frac{ac}{b+c}$ (b) $\frac{a(c+b)}{c-b}$ (c) $\frac{a(c-b)}{c+b}$ (d) $\frac{ab}{b+c}$

**Solution:-** BC = a cm
Let BD = x, DC = a - x




We know that, if AD is bisector of ∠BAC then,
$\frac{AB}{AC} = \frac{BD}{DC}$ ⇒ $\frac{c}{b} = \frac{x}{a-x}$
⇒ ac - cx = bx ⇒ x = $\frac{ac}{b+c}$
⇒ DC = a - x ⇒ DC = a - $\frac{ac}{b+c} = \frac{ab}{b+c}$
BD - DC = $\frac{ac}{b+c} - \frac{ab}{b+c} = \frac{a(c-b)}{c+b}$

**When bisect externally :-**




$\frac{PQ}{PR} = \frac{QS}{RS}$

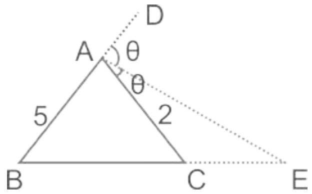
PS is the external angle bisector.

**Example :-** In a triangle ABC AB : AC = 5 : 2. BC = 9 cm. BA is produced to D, and the bisector of the Angle CAD meets BC produced at E. What is the length (in cm) of CE?

SSC CGL 13/08/2021 (Afternoon)

(a) 9 (b) 10 (c) 6 (d) 3

**Solution :-**



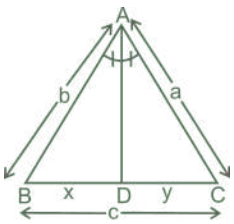

As AE is the external angle bisector so it divide the opposite sides in the ratio of the corresponding sides

Therefore,

$\frac{AB}{AC} = \frac{BE}{CE} \Rightarrow \frac{5}{2} = \frac{CE+9}{CE}$ ⇒ CE = 6 cm

**How to find length of angle bisector :-**

**(i)**




AD = angle bisector of ∠A

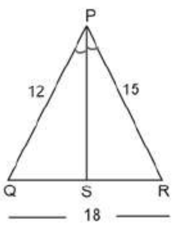
$AD^2$ = AB × AC - BD × DC

**Example :-** The bisector of ∠QPR of △PQR meets the side QR at S. If PQ = 12 cm, PR = 15 cm and QR = 18 cm, then the length of SR and PS is:-

NTPC CBT II (14/06/2022) 2nd Shift

(a) 10, 10 (b) 12, 5 (c) 8, 7 (d) 13, 15

**Solution :-**




By angle bisector theorem,

$\frac{PQ}{QS} = \frac{PR}{SR}$

$\frac{12}{x} = \frac{15}{18-x}$

216 - 12x = 15x

27x = 216 ⇒ x = 8

So, length of SR = 18 - 8 = 10 cm

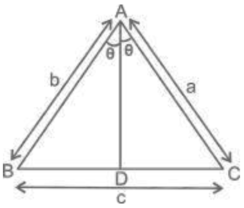
length of angle bisector →

$PS^2$ = PQ × PR - QS × SR

= 12 × 15 - 8 × 10 ⇒ $PS^2$ = 180 - 80

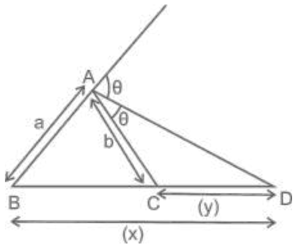
⇒ PS = $\sqrt{100}$ ⇒ PS = 10 cm

**(ii)**




Length of angle bisector(AD) = $\frac{2ab.\cos\theta}{a+b}$

**(iii)**



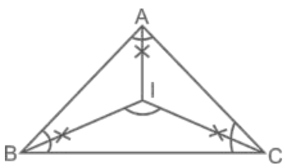

AD = angle bisector of ∠ A

$AD^2$ = BD × DC - AB × AC

## Centers of The Triangle

**Incentre(r):** The point of intersection of angle bisectors of a triangle.

**1.**



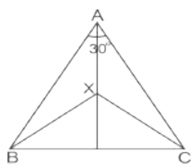

∠BIC = 90° + $\frac{\angle A}{2}$, ∠AIC = 90° + $\frac{\angle B}{2}$ ∠AIB = 90° + $\frac{\angle C}{2}$

**Example :-** In ΔABC, the internal bisectors of ∠ABC and ∠ACB meet at X and ∠BAC = 30°. The measure of ∠BXC is:

SSC CHSL 03/08/2023 (4th Shift)

(a) 120° (b) 115° (c) 105° (d) 150°

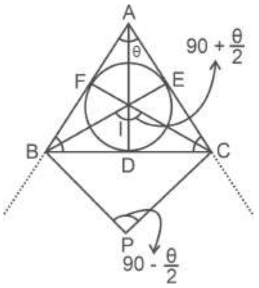
**Solution :-**



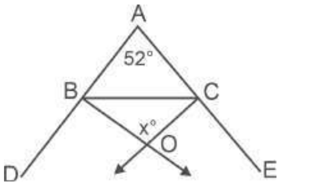

We know that :-

∠BXC = 90° + $\frac{\angle BAC}{2}$ = 90° + 15° = 105°

**2.**



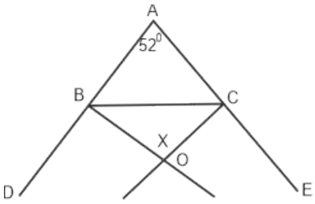

**Example:-** In the above figure, BO and CO are the angular bisectors of $\angle DBC$ and $\angle BCE$ respectively. The measure of x is _______.




RRB NTPC 01/04/2021 (Evening)

(a) 52° (b) 58° (c) 64° (d) 54°

**Solution :-**




Given , ∠BAC = 52°, BO and CO are the angle bisectors of $\angle DBC$ and $\angle BCE$

Now , $\angle X = (90° - \frac{52°}{2}) = (90° - 26°) = 64°$

**3. Inradius (r):**

- For any triangle :

$r = \frac{A}{S}$, (where A = area of triangle , S = semi perimeter of triangle)

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

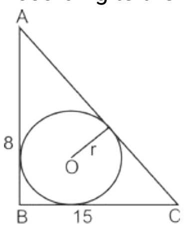
**Example :-** In a triangle ABC, right angled at B, BC=15cm, and AB=8 cm. A circle is inscribed in triangle ABC. Then the radius of the circle is:

RRB ALP 13/08/2018 (Evening)

(a) 2 cm (b) 4 cm (c) 3 cm (d) 1 cm

**Solution :-**

According to the question,




By using triplet (8, 15, 17) = AC = 17 cm

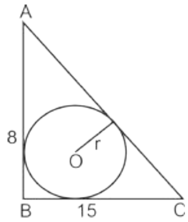
$\Rightarrow S = \frac{AB + BC + AC}{2} = \frac{8 + 15 + 17}{2} = \frac{40}{2} = 20$ cm

Now,

Inradius of circle (r) = $\frac{A}{S}$

$= \frac{\frac{1}{2} \times 8 \times 15}{20} = \frac{4 \times 15}{20} = 3$ cm

**Exam hall approach:-**



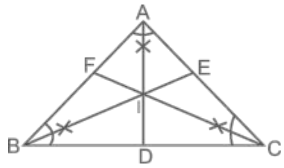

Using pythagoras theorem, we have :

AC = $\sqrt{15^2 + 8^2} = \sqrt{289} = 17$ cm

Inradius = $\frac{AB + BC - AC}{2} = \frac{15 + 8 - 17}{2}$

$= \frac{6}{2} = 3$ cm

**Concept :-** Incentre divides each angle bisector in the ratio of length of sum of two adjacent sides and opposite side.

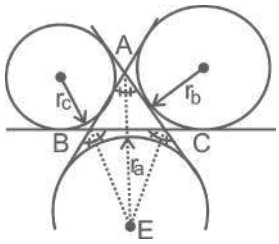



AI : ID = AB + AC : BC

BI : IE = AB + BC : AC

CI : IF = AC + BC : AB

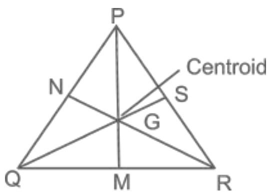
**Excentre :-** The intersection point of the internal angle bisector of one angle and bisectors of the other two opposite exterior angles.




(i) $\angle BEC = 90° - \frac{\angle A}{2}$

(ii) $r_a = \frac{\Delta}{s-a}$, $r_b = \frac{\Delta}{s-b}$, $r_c = \frac{\Delta}{s-c}$; where S is semiperimeter of triangle

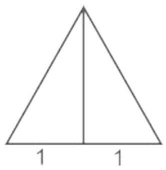
**Centroid (G) :-** It is the intersection point of all medians of a triangle. It is also called the center of mass.



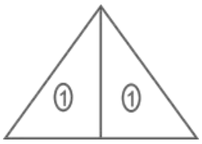

PM, QS, RN are median

**Properties: of medians :-**

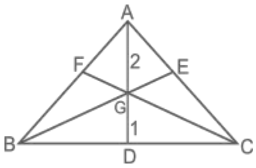
**a)** Median divides any side into two equal parts.




**b)** Median divides the triangle into two equal areas.

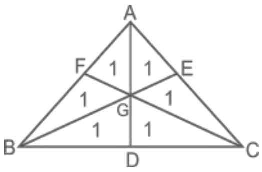



**c)** Centroid divides each median in the ratio of 2 : 1.

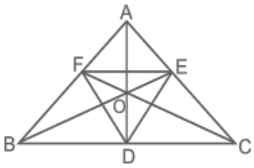



AG : GD = BG : GE = CG : FG = 2 : 1

**d)** Medians divide the triangle into 6 equal Parts.




**e).** area of triangle formed by joining midpoints of two sides and centroid is $\frac{1}{12}$ of area of triangle.

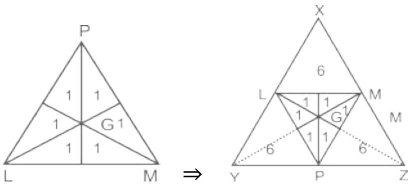



ar ΔOFE = arΔ OFD = arΔOED = $\frac{1}{12}$ area of ΔABC

**Example :-** XYZ is a triangle. If the medians ZL and YM intersect each other at G, then (Area of ΔGLM : Area of ΔXYZ)
SSC CHSL 12/10/2020 (Afternoon)
(a) 1 : 14 (b) 1 : 12 (c) 1 : 11 (d) 1 : 10

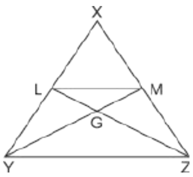
**Solution :-**

Median divides triangle in two equal parts



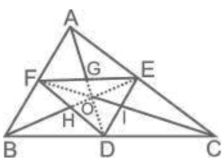

Area of △ GLM: Area of △ XYZ = 2 : 24 = 1 : 12

**Exam hall approach :-**




We know,
Area of ΔGLM : Area of ΔXYZ = 1 : 12

**f).** The line segment joining the midpoints of two sides divides the line joining of vertex in between line to the centroid in the ratio 3 : 1.



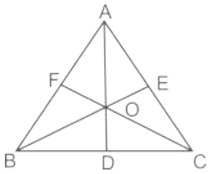

AG : GO = BH : HO = CI : IO = 3 : 1

**g)** Area of triangles formed by medians

$= \frac{4}{3}\sqrt{s(s-a)(s-b)(s-c)}$

S = $\frac{\text{sum of the medians}}{2}$

**h)** Sum of any two sides of a triangle is greater than twice the median with respect to the third side.




AB + AC > 2AD
BC + AC > 2CF
AB + BC > 2BE

So we can say that Sum of the perimeter of a triangle is greater than the sum of the medians.

AB + BC + AC > AD + BE + CF

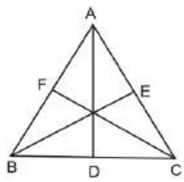
also

$3(AB^2 + BC^2 + CA^2) = 4(AD^2 + BE^2 + CF^2)$

**Example:-** If ΔABC is an equilateral triangle in which D, E and F are the points on sides BC, AC, and AB respectively such that AD⊥BC, BE⊥AC and CF⊥AB , which of the following is true ?
SSC CPO 11/12/2019 (Morning)

(a) $7AB^2 = 9AD^2$ (b) $2AB^2 = 3AD^2$
(c) $4AC^2 = 5BE^2$ (d) $3AC^2 = 4BE^2$

**Solution :-**




Since, ABC is an equilateral triangle
Altitude = Angle bisector = Median
We know that the following relation satisfies in case of an equilateral triangle.

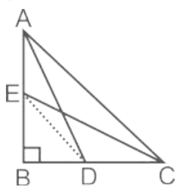
$3[AB^2 + BC^2 + AC^2] =$
$4[AD^2 + BE^2 + CF^2]$ .....eq .(1)

⇒ AB = BC = AC( ΔABC is an equilateral triangle)

Putting the values in eq .(1)

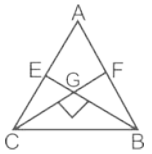
$3[3AC^2] = 4[3BE^2] \Rightarrow 3AC^2 = 4BE^2$

**i)**



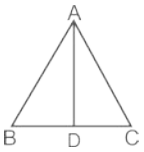

$4(AD^2 + CE^2) = 5(AC^2)$
$(AD^2 + CE^2) = (AC^2 + ED^2)$
$(AD^2 + CE^2) = 5(ED^2)$

**j)** If two medians bisect at 90°




$(AC^2 + AB^2) = 5(BC^2)$

**k) Apollonius theorem (used to evaluate length of median)**

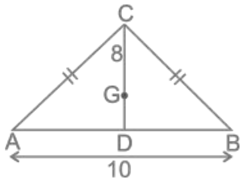



- $(AB^2 + AC^2) = 2(AD^2 + DC^2)$
- $(AB^2 + AC^2) = 2(AD^2 + \frac{BC^2}{4})$

**Example :-** In $\Delta ABC$, AC = BC, and the length of the base AB is 10 cm. If CG = 8 cm, where G is the centroid, then what is the length of AC (in cm) ?
SSC CHSL 12/08/2021 (Evening)

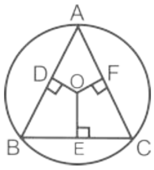
(a) 13 (b) 15 (c) $\sqrt{91}$ (d) 12

**Solution :-**



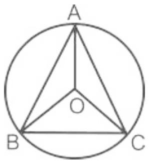

AC = BC
G is the centroid so GD = 4cm
Therefore CD = 8 + 4 = 12cm
By apollonius theorem,
$(AC^2 + BC^2) = 2(CD^2 + AD^2)$
$2AC^2 = 2(5^2 + 12^2) \Rightarrow AC^2 = 13^2 \Rightarrow AC = 13$

**Circumcentre (O) :** point of intersection of perpendicular bisector of sides.



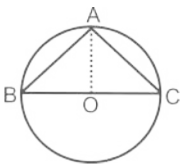

OD ⊥ AB, OF ⊥ AC, OE ⊥ BC
AD = BD, AF = CF, BE = EC

ii)



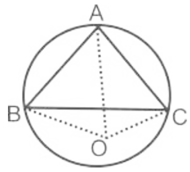

∠BOC = 2∠A, ∠AOB = 2∠C, ∠AOC = 2 ∠B

iii)




∠BOC = 2∠A, ∠AOB = 2∠C, ∠AOC = 2 ∠B

iv) In an obtuse angled triangle,




∠BOC = 2(180 -∠A) , ∠COA = 2∠B and ∠BOA = 2∠C

**Position of circumcentre in different triangles :-**

(i) In an acute angled triangle it lies inside the triangle.

(ii) In a right angled triangle it is the midpoint of the hypotenuse.

(iii) In an obtuse triangle, it lies outside and in front of the obtuse angle of that triangle .

**Circumradius (R) :-**

- For any triangle
  R = $\frac{abc}{4 \times area\ of\ triangle}$ where a,b,c is sides of the triangle
- For right angled triangle
  R = $\frac{H}{2}$ where H is hypotenuse
- For Equilateral triangle
  R = $\frac{a}{\sqrt{3}}$ where a = side
- Distance between circumcentre and incentre of any triangle
  $= \sqrt{R^2 - 2Rr}$

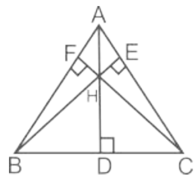
**Example :-** The circumradius of a triangle is 9 cm while the inradius of it is 4 cm. What is the distance between the circumcentre and the incentre of the triangle?

RRB NTPC 27/03/2021 (Evening)

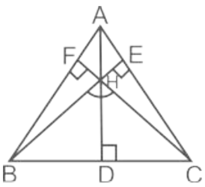
(a) 4 cm (b) 2 cm (c) 3 cm (d) 5 cm

**Solution :-** Given, Radius of triangle (R) = 9 cm
And the radius of the triangle (r) = 4 cm
Distance between circumcentre and center of triangle = $\sqrt{R^2 - 2Rr}$
= $\sqrt{9^2 - 2 \times 9 \times 4}$ = 3cm

**Orthocentre (H):-** point of intersection of altitudes of a triangle.




**Useful results :-**




(i) ∠BHC = 180° - ∠A , ∠BHA = 180° - ∠C , ∠CHA = 180° - ∠B
(ii) BH × EH = FH × HC ,
BF × EH = FH × CE

BF × HC = HB × CE (by similarity of ΔBFH & Δ CEH )

(iii) BD × DC = AD × AH
BE × EH = AE × CE
CF × FH = AF × BF

(iv) The perimeter of any triangle is greater than the sum of altitudes of the triangle.
AB + BC + AC > AD + BE + CF

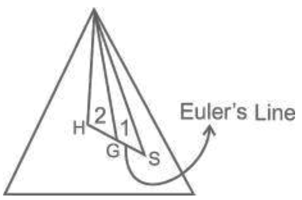
**Position of Orthocentre in different triangles :-**
(i) In an acute angled triangle it lies inside the triangle.

(ii) In a right angled triangle it is at the vertex of the right angled triangle.

(iii) In an obtuse angle triangle, it lies outside and in the back side of the obtuse angle of that triangle .

**Euler's Line :-** According to the Euler's theory, in a triangle, there exists a straight line called the Euler's line, which passes through the orthocenter, the circumcentre, and the centroid of the triangle. Hence, these given points of concurrencies of the triangle are the collinear points in a triangle.

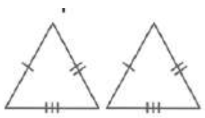
## Relation between orthocentre(s), centroid (G) and circumcentre(H)



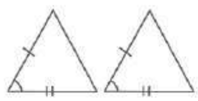

## Congruency of Triangles

Two triangles will be congruent if:

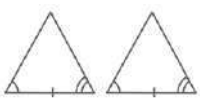
**1. SSS: (Side – Side – Side rule) :-** When all three corresponding sides are equal.


SSS Congruency

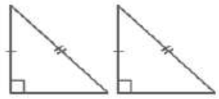
**2. SAS: (Side – Angle – Side rule) :-** When two corresponding sides and one corresponding angle are equal.


SAS Congruency

**3. ASA: (Angle – Side – Angle rule):** When two corresponding angles and one corresponding side are equal.

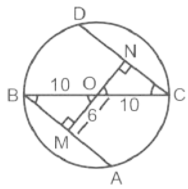

ASA Congruency

**4. RHS: (Right angle – Hypotenuse – Side rule) :-** When one corresponding side and corresponding hypotenuse of the right angled triangle are equal.


RHS Congruency

**5. AAS: ( angle – angle – Side rule) :-** When any two pairs of corresponding angles and corresponding sides are equal.

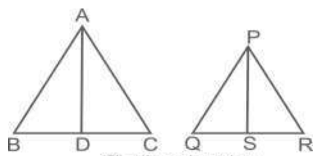
**Example :-** In a circle of diameter 20 cm, chords AB and CD are parallel to each other. BC is diameter. If AB is 6 cm from the centre of the circle, what is the length ( in cm) of the chord CD?
SSC CGL 13/04/2022 (Morning)
(a) 8 (b) 12 (c) 20 (d) 16
**Solution :-**




radius = $\frac{20}{2}$ = 10 cm.
As AB∥CD, ∠MBO = ∠NCO and
∠BOM = ∠CON (vertically opposite angles)
BO = CO = 10 (radius)
So, by ASA congruency ΔBOM ≅ ΔCON
Now, by c.p.c.t., ON = OM = 6 cm
And ∠OMB = ∠ONC = 90°
NC = $\sqrt{10^2 - 6^2}$ = 8 cm
CD = 2 × NC = 16 cm

## Similarity of Triangles

**1).** Two triangles are similar if they have the same shape but vary in size.
**2).** In congruency the triangles are mirror images of each other. So, We can say that all congruent triangles are similar but the vice versa is not true.



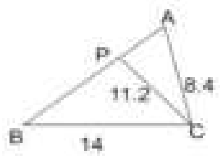
Similar triangles

**3).** Important properties of similar triangles :

$$\frac{Area\ of\ \Delta ABC}{Area\ of\ \Delta PQR} = \frac{AB^2}{PQ^2} = \frac{AC^2}{PR^2} = \frac{BC^2}{QR^2} = \frac{AD^2}{PS^2}$$

$$\frac{Perimeter\ of\ \Delta ABC}{Perimeter\ of\ \Delta PQR} = \frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR} = \frac{AD}{PS}$$

**Example :-** In $\Delta ABC$, AC = 8.4 cm, BC= 14 cm. P is a point on AB such that CP = 11.2 cm and ∠ACP = ∠B. What is the length (in cm) of BP ?
SSC CGL 04/03/2020 (Evening)
(a) 4.12 (b) 2.8 (c) 3.78 (d) 3.6
**Solution :-**
In $\Delta ABC$, ∠ACP = ∠B



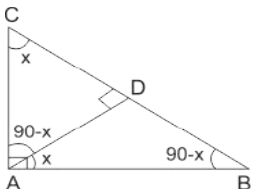

In $\Delta ABC$ and $\Delta ACP$:
$\angle ACP = \angle B$,
$\angle A$ and side AC is common

Hence, $\frac{AB}{AC} = \frac{BC}{PC} = \frac{AC}{AP}$

$\Rightarrow \frac{BC}{PC} = \frac{AC}{AP} \Rightarrow \frac{14}{11.2} = \frac{8.4}{AP} \Rightarrow$ AP = 6.72

Similarly, in $\Delta ABC$ and $\Delta CBP$:
$\Rightarrow$ AB × PC = AC × BC
$\Rightarrow$ AB × 11.2 = 8.4 × 14
$\Rightarrow$ AB = 10.5, BP = 10.5 - 6.72 = 3.78 cm

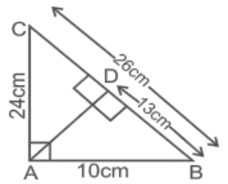
**4).** When a perpendicular is drawn in right angled triangle




- $AD^2 = BD \times DC$
- $AB^2 = BD \times BC$
- $AC^2 = CD \times BC$
- $AD = \frac{AC \times AB}{BC}$

**Example :-** In $\Delta CAB$, ∠CAB = 90° and AD ⊥ BC. If AC = 24 cm, AB = 10 cm, then find the value of AD (in cm).
SSC CHSL 17/08/2023 (1st Shift)
(a) 9.23 (b) 8.23 (c) 7.14 (d) 10.23

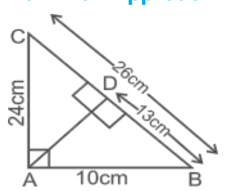
**Solution :-** According to the question,




By using triplet (10, 24 , 26) = CB = 26 cm

So, AD = $\frac{AC \times AB}{BC} = \frac{24 \times 10}{26}$ = 9.23 cm

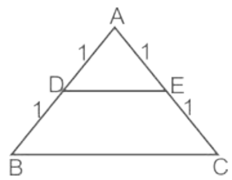
**Exam Hall Approach :-**




$\Delta CAB$ and ΔADB is similar

$\frac{26}{10} = \frac{24}{AD} \Rightarrow$ AD = $\frac{240}{26}$ = 9.23

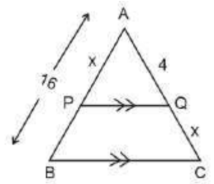
**[1.] Basic proportionality theorem :-** A line drawn parallel to one side divides the other two sides in the same ratio.




$\frac{AD}{DB} = \frac{AE}{EC}$ and $\frac{AD}{AB} = \frac{DE}{BC}$

**Example :-** In $\Delta ABC$, the straight line parallel to the side BC meets AB and AC at the points P and Q, respectively. If AP = QC, the length of AB is 16 cm and the length of AQ is 4 cm, then the length (in cm) CQ is:
SSC CHSL 02/08/2023 (2nd Shift)
(a) $(2\sqrt{21} + 2)$ (b) $(2\sqrt{18} - 2)$
(c) $(2\sqrt{17} - 2)$ (d) $(2\sqrt{19} + 2)$

**Solution :-**




By using Thales theorem

$\frac{AP}{AB} = \frac{AQ}{AC} \Rightarrow \frac{x}{16} = \frac{4}{x+4}$

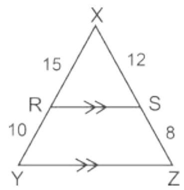
$= x^2 + 4x - 64 = 0$

$x = \frac{-4 \pm \sqrt{(4)^2 - 4 \times 1 \times (-64)}}{2 \times 1}$

$x = \frac{-4 \pm \sqrt{272}}{2 \times 1} = (2\sqrt{17} - 2)$

**Example :-** R and S are the points of the sides XY and XZ, respectively, of
ΔXYZ. Also, XR = 15 cm, XY = 25 cm, XS = 12 cm and XZ = 20 cm. RS is equal to :
Selection Post 27/06/2023 (4th Shift)
(a) $\frac{2}{5}$ YZ (b) $\frac{5}{3}$YZ (c) $\frac{3}{5}$YZ (d) $\frac{3}{4}$ YZ

**Solution :-**



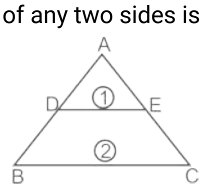

$\frac{XR}{XY} = \frac{XS}{XZ} \Rightarrow \frac{15}{25} = \frac{12}{20} \Rightarrow \frac{3}{5}$

So, RS ∥ YZ and $\Delta XRS \sim \Delta XYZ$

Using thales theorem, $\frac{XR}{XY} = \frac{RS}{YZ} = \frac{15}{25}$

$\frac{RS}{YZ} = \frac{3}{5} \Rightarrow RS = \frac{3}{5} \times YZ$

**[2.] Mid Point theorem:** The line segment joining the midpoints of any two sides is parallel to and half of the third side.

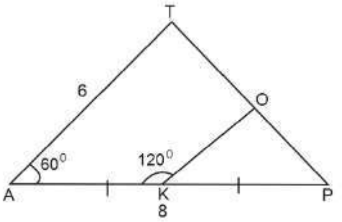


$DE = \frac{BC}{2}$

DE II BC.

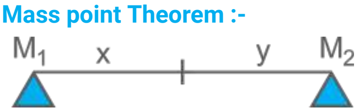
**Example :-** ΔTAP, ∠TAP = 60°, TA = 6cm, AP = 8 cm. K is the midpoint of AP. A line form k is produced to meet TP at O such that ∠AKO = 120°. Find the length of OK.
Graduate Level 01/08/2022 (Shift - 4)
(a) 3 cm (b) 4 cm (c) 5 cm (d) 6 cm

**Solution :-**




Since,
∠TAP + ∠OKA = 60° + 120° = 180°
(pair of interior angles on the same side of transversal is supplementary)
So,
OK ll AT and also K is the midpoint of AP. ⇒ OK = $\frac{AT}{2}$ ....(Mid point theorem)
So, OK = $\frac{6}{2}$ = 3 cm

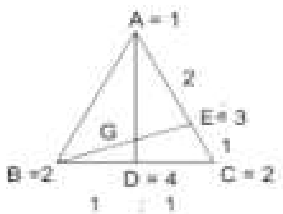
**Mass point Theorem :-**




$M_1X = M_2Y \Rightarrow \frac{M_1}{M_2} = \frac{Y}{X}$

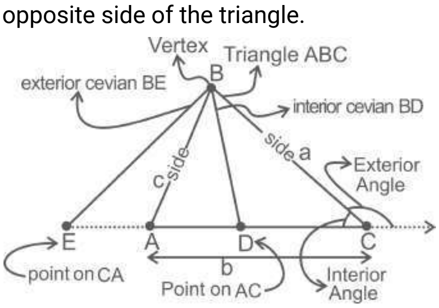
**Example :-**D is the midpoint of BC of $\Delta ABC$ . Point E lies on AC such that CE = $\frac{1}{3}$AC. BE and AD intersect at G. What is $\frac{AG}{GD}$?
SSC CGL 05/03/2020 (Morning)
(a) 8 : 3 (b) 4 : 1 (c) 5 : 2 (d) 3 : 1

**Solution :-** Apply mass point theorem in the following diagram:

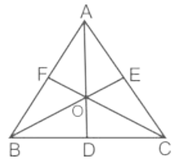



AG : GD = 4 : 1

**Cevians :** A line segment which joins a vertex of a triangle to a point on the opposite side of the triangle.



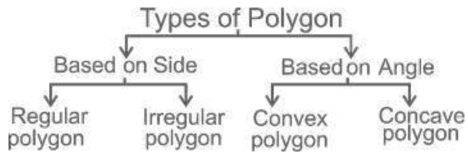

**Some important result :-**




$\frac{AF}{FB} \times \frac{BD}{DC} \times \frac{CE}{EA} = 1$

## Polygons

A two-dimensional closed shape bounded with minimum three sides. Triangle is the simplest form of polygon.

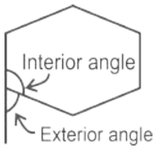



| By side | By Angle |
|---|---|
| Regular polygon - all the sides and interior angles of the polygon are equal. | Concave angle - one interior angle must be greater than 180° |
| Irregular Polygon - polygons that do not have equal sides and not equal angles. | Convex angle - whose all interior angles are less than 180°. |

## Types of Polygon

| | |
|---|---|
| **Triangle**<br>• Has 3 sides and 3 vertices<br>• Has no diagonals<br>• Sum of the interior angles is 180° | **Quadrilateral**<br>• Has 4 sides and 4 vertices<br>• Has two diagonals<br>• Sum of the interior angles is 360° |
| **Pentagon**<br>• Has 5 sides and 5 vertices<br>• Has 5 diagonals<br>• Sum of the interior angles is 540° | **Hexagon**<br>• Has 6 sides and 6 vertices<br>• Has 9 diagonals<br>• Sum of the interior angles is 720° |
| **Heptagon**<br>• Has 7 sides and 7 vertices<br>• Has 14 diagonals<br>• Sum of the interior angles is 900° | **Octagon**<br>• Has 8 sides and 8 vertices<br>• Has 20 diagonals<br>• Sum of the interior angles is 1080°<br>Area = $2a^2(1+\sqrt{2})$ |
| **Nonagon**<br>• Has 9 sides and 9 vertices<br>• Has 27 diagonals<br>• Sum of the interior angles is 1260° | **Decagon**<br>• Has 10 sides and 10 vertices<br>• Has 35 diagonals .<br>• Sum of the interior angles is1440° |

## Some Important Results of Polygon

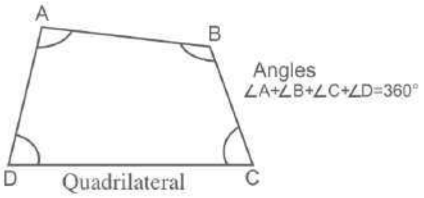



n = number of sides.

* Sum of all exterior angles = $360°$.
* Each exterior angle = $\frac{360°}{n}$.
* Exterior angle + Interior angle = 180°
* In case of convex polygon, sum of all interior angles = $(2n - 4) \times 90°$
* Number of diagonals = $\frac{n(n-3)}{2}$
* area of regular polygon = $\frac{na^2}{4}$ cot$\frac{\pi}{n}$
* inradius = $\frac{a}{2}$ cot$\frac{\pi}{n}$
* circumradius = $\frac{a}{2}$ cosec$\frac{\pi}{n}$

## Quadrilateral

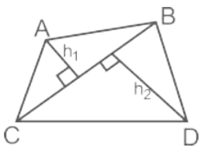
A two-dimensional shape with four sides, four vertices, and four angles.



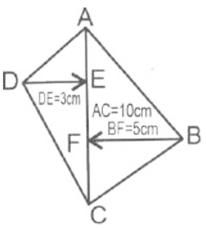

**Example :-** If the angles of a quadrilateral are in the ratio of 4 : 9 : 11 : 12, then the largest of these angles is:
RRB NTPC CBT - I 29/01/2021 (Evening)
(a) 168° (b) 72° (c) 166° (d) 120°
**Solution :-** Sum of angles of quadrilateral = 360°
$4x + 9x + 11x + 12x = 360°$
$36x = 360° \Rightarrow x = 10°$
Largest Angle(12x) = 120°

## Area of quadrilateral

(i) Area of quadrilateral = $\frac{1}{2}$ × diagonal × ($h_1 + h_2$)

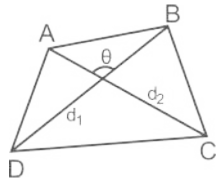



**Example :-** Find the area (in cm²) of the given quadrilateral ABCD.

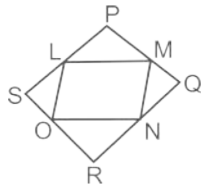



SSC CHSL 31/05/2022 (Afternoon)
(a) 50 (b) 35 (c) 45 (d) 40
**Solution :-** Area of quadrilateral ABCD =
$\Rightarrow \frac{1}{2} \times AC\,(DE + BF)$
$= \frac{1}{2} \times 10 \times (3 + 5) = 5 \times 8 = 40\ cm^2$

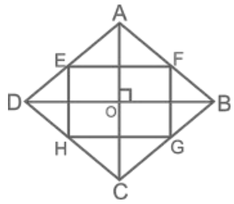
(ii) Area of quadrilateral = $\frac{1}{2} \times d_1 \times d_2 \times \sin(\theta)$



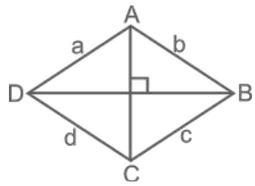

(iii) Line joining the midpoints of the adjacent sides of the quadrilateral formed a parallelogram.
Where L, M, N, O are the mid points of PS, PQ, QR and RS .



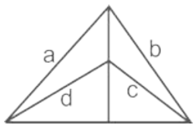

(iv) when the midpoints of adjacent sides of quadrilateral is joined and its diagonal intersects at $90°$ then a rectangle is formed.




**Special results of quadrilateral :-**

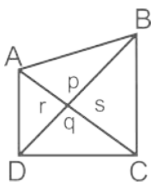



$a^2 + c^2 = b^2 + d^2$

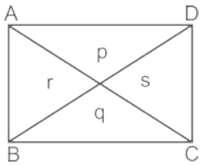



$a^2 + c^2 = b^2 + d^2$

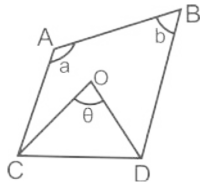
- If p, q ,r and s be the area of respective triangles then P × q = r × s



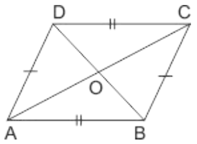

- If ABCD is a rectangle/square, then
- ar.(P) + ar.(q) = ar.(r) + ar.(s)



- If DO and CO are angle bisectors of ∠ADC and ∠BCD then,
- $\angle DOC = \frac{a+b}{2}$

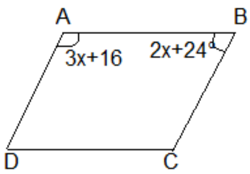


**1). Parallelogram :** Opposite sides and angles of a parallelogram are equal.



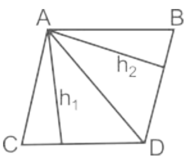
- AB = DC and AD = BC
- $\angle A + \angle B = \angle B + \angle C = \angle C + \angle D = \angle D + \angle A = 180°$

**Example :-** In a parallelogram ABCD, if m∠A =(3x + 16)° and m∠B = (2x + 24)°, then find m∠C.
RRC Group D 30/09/2022 (Evening)
(a) 56° (b) 28° (c) 100° (d) 110°
**Solution :-**

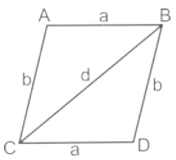



We know,
In parallelogram, sum of two adjacent angles = 180°
3x + 16° + 2x + 24° = 180°
5x + 40° = 180°
5x = 140° ⇒ x = 28°
∠A = 3x + 16° = 3 × 28 + 16 = 100°
∠A = ∠C = 100° (in parallelogram , opposite angles are equal)
∠C = 100°

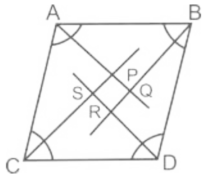
- $\angle A = \angle C$ and $\angle B = \angle D$
- The diagonals bisect each other, i.e. AO = OC and OB = OD
- Area of parallelogram = Base × Altitude



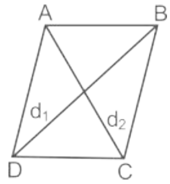
- Area of parallelogram = $2\sqrt{s(s-a)(s-b)(s-c)}$
- $S = \frac{a+b+c}{2}$



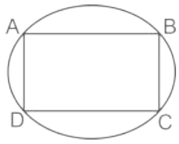
- Bisectors of a parallelogram(ABCD) form a rectangle(PQRS) .



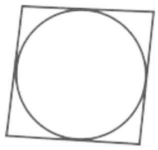
- Sum of squares of all four sides = Sum of squares of diagonals.
- $AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + AD^2$



- Parallelogram inscribed in a circle is a rectangle.

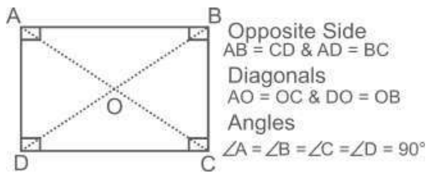


- Parallelogram that circumscribes a circle is a rhombus.



- Square, Rectangle and Rhombus are parallelogram.
- All rectangles are parallelogram but all parallelogram are not rectangles.

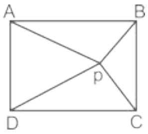
## Rectangle




- $\angle A = \angle B = \angle C = \angle D = 90°$
- Diagonals bisect each other i.e, OA = OC , OB = OD
- Opposite sides are equal i.e, AB = DC , AD = BC.
- Area = length × breadth
- Perimeter = 2 (length + breadth)

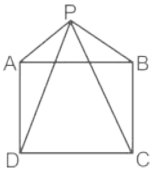
**Example :-** The two unequal sides of a rectangle are in the ratio 3:4. If the perimeter is 42 cm, then the length of the diagonal will be:
RRB NTPC CBT - I 02/02/2021 (Morning)
(a) 30 cm (b) 25 cm (c) 15 cm (d) 35 cm
**Solution :-**
Perimeter of rectangle
= 2 ( 3 + 4) = 14 unit
14 unit = 42 cm ⇒ 1 unit = 3 cm
Sides are 9 cm and 12 cm
Diagonal of rectangle
$= \sqrt{9^2 + 12^2} = \sqrt{81 + 144}$
$= \sqrt{225} = 15\ cm$

- If P be any point inside the rectangle.
$PA^2 + PC^2 = PB^2 + PD^2$

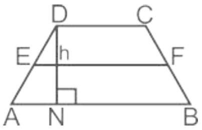



- If P be any point outside the rectangle.

$PA^2 + PC^2 = PB^2 + PD^2$




**2). Trapezium:** one pair of opposite sides are parallel.




Area of trapezium ABCD

$= \frac{1}{2} \times Sum\ of\ parallel\ side \times height$

$= \frac{1}{2} \times (AB + DC) \times h$

**Example :-** The lengths of a pair of parallel sides of a trapezium are 20 cm and 25 cm, respectively, and the perpendicular distance between these two sides is 14 cm. What is the area (in $cm^2$) of the trapezium?
SSC CHSL 02/06/2022 (Afternoon)
(a) 512 (b) 250 (c) 300 (d) 315

**Solution :-** Area of trapezium =

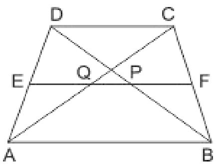
$\frac{1}{2}$ × (sum of parallel side) × perpendicular distance between them

$\Rightarrow \frac{1}{2} \times (20 + 25) \times 14 = \frac{1}{2} \times 45 \times 14$

$= 315\ cm^2$

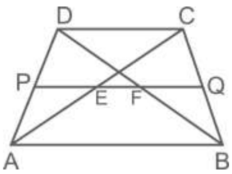
- If E is the midpoint of $\overline{AD}$ and F is the midpoint of $\overline{BC}$, then EF = $\frac{AB + DC}{2}$.
- If P is the midpoint of diagonal $\overline{BD}$ and Q is the midpoint of diagonal $\overline{AC}$, then

PQ = $\frac{AB - DC}{2}$



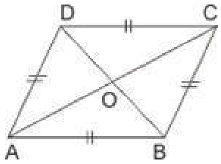

**Example:-** ABCD is a quadrilateral in which AB ∥ DC. E and F are the midpoints of the diagonals AC and BD, respectively. If AB = 18 cm and CD = 6 cm, then EF = ?
SSC CGL TIER 2 (29 / 01/22)
(a) 8 (b) 9 (c) 6 (d) 12

**Solution :-**




EF = $\frac{AB - DC}{2} = \frac{18 - 6}{2} = \frac{12}{2}$ = 6cm

**3). Rhombus:** All sides are equal and opposite sides are parallel to each other.



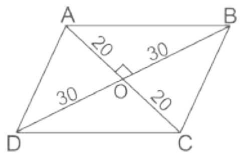

$\angle A + \angle B = \angle B + \angle C = \angle C + \angle D = \angle D + \angle A = 180°$

- $\angle A = \angle C$ and $\angle B = \angle D$
- $4a^2 = d_1^2 + d_2^2$

i.e. Sum of squares of sides = Sum of squares of diagonals.

**Example :-** The length of the diagonals of a rhombus is 40 cm and 60 cm. What is the length of the side of the rhombus ?
SSC CHSL 13/03/2023 (4th Shift)
(a) $50\sqrt{3}$ cm (b) $20\sqrt{3}$ cm (c) $10\sqrt{13}$ cm (d) $40\sqrt{13}$ cm

**Solution :-**




Length of the diagonal is 40cm and 60cm
Then, in ΔAOB

AB = $\sqrt{(20)^2 + (30)^2} = \sqrt{1300} = 10\sqrt{13}$ cm

So, the side of rhombus = $10\sqrt{13}$ cm

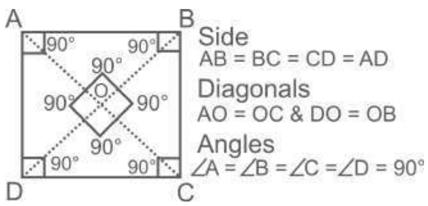
- Diagonals bisect each other at right angles and form four right angled triangles with equal areas.

$\Rightarrow Area\ of\ \Delta AOB = Area\ of\ \Delta BOC = Area\ of\ \Delta COD$

$= Area\ of\ \Delta DOA = \frac{1}{4} \times area\ of\ rhombus\ ABCD$

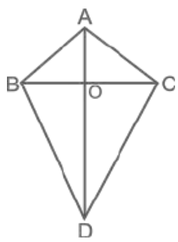
- The diagonals of Rhombus are not of equal length.

**4). Square:** All sides are equal in length and adjacent sides are perpendicular to each other.




- AB = BC = CD = AD and $\angle A = \angle B = \angle C = \angle D = 90°$
- Diagonals bisect each other at right angles and form four right angled isosceles triangles.
- Diagonals are of equal length **i.e.** AC = BD.

**5) KITE :**

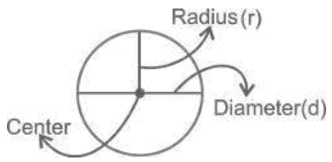



- adjacent sides are equal in length. i.e, AB = AC , BD = DC

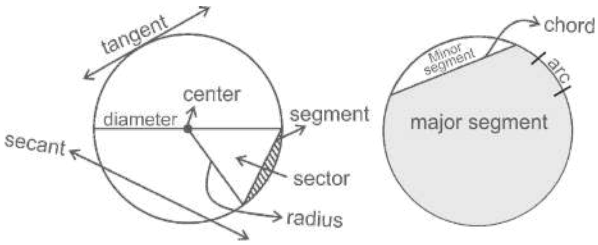
- Diagonal AD bisect BC but diagonal BC does not bisect diagonal AD **i.e,** BO = 0C
- It has one pair of opposite angles (which are obtuse) that are equal. Here, ∠B = ∠C
- Diagonals cuts at $90^\circ$
- Perimeter = 2 (a + b)
- Area = $\frac{1}{2} \times d_1 \times d_2$

## Circle

A **circle** is a two-dimensional figure formed by a set of points that are at a fixed distance (radius) from a fixed point (center) on the plane.




### Parts of Circle:




### Important Formulae:

**1)** Circumference of circle = 2πr,

**2)** Area of circle = $\pi r^2$

**3)** Length of Arc ($l$) = 2πr × $\frac{\theta}{360}$......(θ is in degree)

Or Length of Arc ($l$) = r × θ .....(θ is in radian)

Change radian to degree = θ in radian × $\frac{180^\circ}{\pi}$

**Example :-** In a circle of radius 14 cm, an arc subtends an angle of 90° at the center. The length of arc (in cm) is equal to:

Take ($\pi = \frac{22}{7}$)

SSC CHSL 02/08/2023 (3rd Shift)

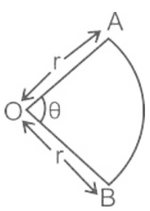
(a) 22 (b) 18 (c) 20 (d) 24

**Solution :-**Length of the arc = $\frac{\theta}{360^\circ} \times 2\pi r$

= $\frac{90^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 14$

= $\frac{1}{4} \times 2 \times \frac{22}{7} \times 14$ = 22 cm

### Sector :-




**1)** Perimeter of sector = [2r + $\frac{\theta}{360} \times (2\pi r)$]

**2)** Area of Sector = $\pi r^2 \times \frac{\theta}{360}$ ......(θ is in degree)

Or Area of Sector = $\frac{1}{2} \times l \times$ r ....($l$ is length of arc and r is radius)

**Example :-**The area of a sector of a circle with central angle 60° is A. The circumference of the circle is C. Then A is equal to :

SSC CHSL 10/07/2019 (Evening)

(a) $\frac{c^2}{6\pi}$ (b) $\frac{c^2}{18\pi}$ (c) $\frac{c^2}{24\pi}$ (d) $\frac{c^2}{4\pi}$

**Solution :-** We know that

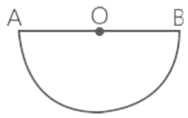
Circumference of a circle (C) = $2\pi r$

⇒ r = $\frac{c}{2\pi}$

Area of the sector (A) = $\frac{\theta}{360} \times \pi r^2$

⇒ A = $\frac{60}{360} \times \pi \times \frac{c}{2\pi} \times \frac{c}{2\pi} = \frac{c^2}{24\pi}$

### Semi - Circle :-

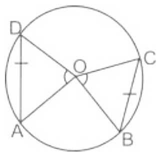



Diameter = 2 × radius, Perimeter = (π + 2)r, Area = $\frac{\pi r^2}{2}$

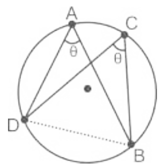
## Properties of circles

**1).** Equal chords of a circle subtend equal angle at the centre.

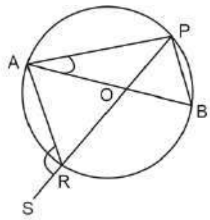
If AD = BC then, ∠ AOD = ∠COB




**2).** Angles in the same segment of a circle are equal.




**Example:-** In the given figure O is the centre of the circle. If $\angle PAB = 35°$, then find ∠ARS.




RRB NTPC 11/01/2021 (Evening)

(a) 125° (b) 55° (c) 65° (d) 115°

**Solution :-**