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NDA / NA

National Defence Academy & Naval Academy Entrance Examination

Mathematics

Chapterwise Previous Years'

3000+ OBJECTIVE QUESTIONS

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National Defence Academy & Naval Academy Entrance Examination

ABOUT THE EXAMINATION

Education is the glorious route through which anyone can attain the goal of success. And if education has been acquired through a renowned institution, it leads to achieve glorious heights in career. National Defence Academy (NDA) is one such institution which propels the students in the arena of life and contributes to a very successful and fulfilled career. But to get enrolled in this institution means goal directed study for passing in a competitive examination which is conducted by Union Public Service Commission nationwide. For recruitment to Army, Navy and Air Force wings of Indian Army, there is prestigious National Defence Academy Entrance Examination. The examination is conducted twice a year, and the duration of training is three years.

Though the candidate may give his preference for a particular wing of the Armed Forces, the final selection depends upon his performance and place in the merit list.

EDUCATIONAL QUALIFICATIONS

(i) For Army wing of National Defence Academy, Class XII pass of the 10+2 pattern of school education or equivalent examination conducted by a State Education Board or a University.

(ii) For Air Force and Naval wings of National Defence Academy and for the 10+2 (Executive Branch) Course at the Naval Academy, Class XII pass of the 10+2 pattern of school education or equivalent with Physics and Mathematics conducted by a State Education Board or a University.

SYLLABUS

Algebra Concept of a set, operations on sets, Venn diagrams. De-Morgan laws. Cartesian product, relation, equivalence relation. Representation of real numbers on a line. Complex numbers – basic properties, modulus, argument, cube roots of unity. Binary system of numbers. Conversion of a number in decimal system to binary system and vice-versa. Arithmetic, Geometric and Harmonic progressions. Quadratic equations with real coefficients. Solution of linear inequations of two variables by graphs. Permutation and Combination. Binomial theorem and its application. Logarithms and their applications.

Matrices and Determinants Types of matrices, operations on matrices Determinant of a matrix, basic properties of determinant. Adjoint and inverse of a square matrix, Applications – Solution of a system of linear equations in two or three unknowns by Cramer's rule and by Matrix Method.

Trigonometry Angles and their measures in degrees and in radians. Trigonometrical ratios. Trigonometric identities Sum and difference formulae. Multiple and Sub-multiple angles. Inverse trigonometric functions. Applications – Height and distance, properties of triangles.

Analytical Geometry of Two and Three Dimensions Rectangular Cartesian Coordinate system. Distance formula. Equation of a line in various forms. Angle between two lines. Distance of a point from a line. Equation of a circle in standard and in general form. Standard forms of parabola, ellipse and hyperbola. Eccentricity and axis of a conic. Point in a three dimensional space, distance between two points. Direction Cosines and direction ratios. Equation of a plane and a line in various forms. Angle between two lines and angle between two planes. Equation of a sphere.

Differential Calculus Concept of a real valued function – domain, range and graph of a function. Composite functions, one to one, onto and inverse functions. Notion of limit, Standard limits – examples. Continuity of functions – examples, algebraic operations on continuous functions. Derivative of a function at a point, geometrical and physical interpretation of a derivative – applications. Derivatives of sum, product and quotient of functions, derivative of a function with respect of another function, derivative of a composite function. Second order derivatives. Increasing and decreasing functions. Application of derivatives in problems of maxima and minima.

Integral Calculus and Differential Equations Integration as inverse of differentiation, Integration by substitution and by parts, Standard integrals involving algebraic expressions, trigonometric, exponential and hyperbolic functions. Evaluation of definite integrals – determination of areas of plane regions bounded by curves – applications. Definition of order and degree of a differential equation, formation of a differential equation by examples. General and particular solution of a differential equation, solution of first order and first degree differential equations of various types – examples. Application in problems of growth and decay.

Vector Algebra Vectors in two and three dimensions, magnitude and direction of a vector. Unit and null vectors, addition of vectors, scalar multiplication of vector, scalar product or dot product of two-vectors. Vector product and cross product of two vectors. Applications-work done by a force and moment of a force and in geometrical problems.

Statistics and Probability Statistics: Classification of data, Frequency distribution, Cumulative frequency distribution – examples Graphical representation – Histogram, Pie Chart, Frequency Polygon – examples. Measures of Central tendency – Mean, Median and Mode. Variance and standard deviation – determination and comparison. Correlation and regression.

Probability Random experiment, outcomes and associated sample space, events, mutually exclusive and exhaustive events, impossible and certain events. Union and Intersection of events. Complementary, elementary and composite events. Definition of probability – classical and statistical – examples. Elementary theorems on probability – simple problems. Conditional probability, Bayes' theorem – simple problems. Random variable as function on a sample space. Binomial distribution, examples of random experiments giving rise to Binomial distribution.

CONTENTS

SOLVED PAPER
2021 (II & I)

NDA/NA

National Defence Academy/Naval Academy

SOLVED PAPER 2021 (II)

PAPER I : Mathematics

1. If $x^2 + x + 1 = 0$, then what is the value of $x^{199} + x^{200} + x^{201}$?

(a) -1 (b) 0
(c) 1 (d) 3

(*b*) Given that,

$$x^2 + x + 1 = 0 \quad \text{...(i)}$$

$$\therefore x^{199} + x^{200} + x^{201} = x^{199}(1 + x + x^2) = x^{199} \times 0 = 0$$

2. If x, y, z are in GP, then which of the following is/are correct?

1. $\ln(3x), \ln(3y), \ln(3z)$ are in AP.
2. $xyz + \ln(x), xyz + \ln(y), xyz + \ln(z)$ are in HP.

Select the correct answer using the code given below.

(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

(*a*) Given that x, y, z are in GP.

$$\Rightarrow \quad y^2 = xz \quad \text{...(i)}$$

(1) If $\log(3x), \log(3y), \log(3z)$ are in AP

Then, $2\log(3y) = \log(3x) + \log(3z)$

$$9y^2 = (9xz)$$
$$9y^2 = (9xz)$$
$$y^2 = xz$$

Hence, statement (1) is correct.

Hence, we can say if x, y, z are in GP.

$\therefore \log x, \log y, \log z$ are in AP.

$\Rightarrow xyz + \log x, xyz + \log y, xyz + \log z$ are in AP.

Hence, Statement (2) is wrong.

$\therefore$ Option (a) is correct.

3. If $\log_{10} 2, \log_{10}(2^x - 1), \log_{10}(2^x + 3)$ are in AP, then what is x equal to?

(a) 0 (b) 1
(c) $\log_2 5$ (d) $\log_5 2$

(*c*) Given that, $\log_{10} 2, \log_{10}(2^x - 1)$, $\log_{10}(2^x + 3)$ are in AP.

$$\therefore 2\log_{10}(2^x - 1) = \log_{10} 2 + \log_{10}(2^x + 3)$$
$$\log_{10}(2^x - 1)^2 = \log_{10} 2(2^x + 3)$$
$$\Rightarrow \quad 2^{2x} + 1 - 2 \cdot 2^x = 2 \cdot 2^x + 6$$
$$\Rightarrow (2^x)^2 - 4(2^x) - 5 = 0$$

Let $2^x = y$

$$\Rightarrow \quad y^2 - 4y - 5 = 0$$
$$(y - 5)(y + 1) = 0$$
$$\Rightarrow \quad y = 5 \text{ or } y = -1$$

(Ignore because 2^x cannot be negative)

$$\Rightarrow \quad y = 5 \Rightarrow 2^x = 5$$
$$x = \log_2 5$$

Hence, option (c) is correct.

4. Let $S = \{2, 3, 4, 5, 6, 7, 9\}$. How many different 3-digit numbers (with all digits different) from S can be made which are less than 500?

(a) 30 (b) 49
(c) 90 (d) 147

(*c*) Let $S = \{2, 3, 4, 5, 6, 7, 9\}$

$$\Rightarrow \quad n(S) = 7$$

Three digit number less than

$$500 = \boxed{3}\,\boxed{6}\,\boxed{5}$$

$$= 3 \times 6 \times 5 = 90$$

$\therefore$ Option (c) is correct.

Note Hundreds digit can be filled with 3 choices that are 2, 3, 4.
Similarly, tens digit can be filled with 6 ways and unit digit can be filled with 5 ways.

5. If $p = (1111 \ldots$ up to n digits), then what is the value of $9p^2 + p$?

(a) $10^n p$ (b) $2p \cdot 10^n$
(c) $10^n p - 1$ (d) $10^n p + 1$

(*a*) Given that,

$$p = (1111\ldots \text{ upto } n \text{ digits})$$
$$= 1 + 10 + 10^2 + \ldots + 10^{n-1}$$
$$= \frac{1(10^n - 1)}{10 - 1}$$

$$\left[\because a + ar + ar^2 + \ldots + ar^{n-1} = \frac{a(r^n - 1)}{r - 1}\right]$$

$$\Rightarrow \quad p = \frac{10^n - 1}{9}$$
$$\Rightarrow \quad 9p = 10^n - 1$$
$$\Rightarrow \quad 9p + 1 = 10^n$$
$$\Rightarrow \quad 9p^2 + p = 10^n \cdot p$$

$\therefore$ Hence, option (a) is correct.

6. The quadratic equation $3x^2 - (k^2 + 5k)x + 3k^2 - 5k = 0$ has real roots of equal magnitude and opposite sign. Which one of the following is correct?

(a) $0 < k < \frac{5}{3}$

(b) $0 < k < \frac{3}{5}$ only

(c) $\frac{3}{5} < k < \frac{5}{3}$

(d) No such value of k exists.

(*d*) Since, we know that if a quadratic equation $ax^2 + bx + c = 0$ has real roots of equal magnitude and opposite sign.

Then, $b = 0$ ...(i)

and product of roots < 0 ...(ii)

In the given quadratic equation,

$3x^2 - (k^2 - 5k)x + 3k^2 - 5k = 0$

$a = 3, b = -(k^2 + 5k), c = 3k^2 - 5k$

By Eq. (i), $b = 0$

$\Rightarrow \quad -(k^2 + 5k) = 0$

$\Rightarrow \quad k(k+5) = 0$

$\therefore \quad k = 0, -5$

By Eq. (ii), Product of roots < 0

$$\frac{c}{a} < 0$$

$$\Rightarrow \quad \frac{3k^2 - 5k}{3} < 0$$

$\Rightarrow \quad k(3k - 5) < 0$

$$\therefore \quad 0 < k < \frac{5}{3}$$

From (i) and (ii) no such values of k exists.

Hence, option (d) is correct.

7. If $a_n = n(n!)$, then what is $a_1 + a_2 + a_3 + \ldots + a_{10}$ equal to?

(a) $10! - 1$ (b) $11! + 1$

(c) $10! + 1$ (d) $11! - 1$

(***d***) Given, $a_n = n(n!)$

$= (n + 1 - 1)(n!)$

$= (n+1)n! - n!$

$= (n+1)! - n!$

$\therefore \quad a_1 = 2! - 1!$

$a_2 = 3! - 2!$

... ...

$a_{10} = 11! - 10!$

$\therefore a_1 + a_2 + a_3 + \ldots + a_{10}$

$= 2! + 1! + 3! - 2! + 4! - 3! + \ldots + 11! - 10!$

$= 11! - 1!$

$= 11! - 1$

$\therefore$ Option (d) is correct.

8. If p and q are the non-zero roots of the equation $x^2 + px + q = 0$, then how many possible values can q have?

(a) Nil (b) One

(c) Two (d) Three

(***b***) Given quadratic equation

$$x^2 + px + q = 0$$

and roots are p and q (non zero)

$$\therefore \quad \text{Sum of roots} = \frac{-\text{coefficient of } x}{\text{coefficient of } x^2}$$

$p + q = -p$ ...(i)

$$\therefore \text{Product of roots} = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

$pq = q$ ...(ii)

$\Rightarrow \quad pq - q = 0$

$q(p-1) = 0$

$\because \quad q \neq 0 \Rightarrow p - 1 = 0$

$p = 1$

From Eq. (i)

$p + q = -p$

$q = -2p = -2(1)$

$q = -2$

$\therefore$ Option (b) is correct.

9. If $\Delta = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$

then what is $\begin{vmatrix} 3d + 5g & 4a + 7g & 6g \\ 3e + 5h & 4b + 7h & 6h \\ 3f + 5i & 4c + 7i & 6i \end{vmatrix}$ equal to?

(a) Δ (b) 7Δ

(c) 72Δ (d) -72Δ

(***d***) Given, $\Delta = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$

$$= \begin{vmatrix} 3d + 5g & 4a + 7g & 6g \\ 3e + 5h & 4b + 7h & 6h \\ 3f + 5i & 4c + 7i & 6i \end{vmatrix}$$

$$= 6\begin{vmatrix} 3d + 5g & 4a + 7g & g \\ 3e + 5h & 4b + 7h & h \\ 3f + 5i & 4c + 7i & i \end{vmatrix}$$

By $C_1 \to C_1 - 5C_3, C_2 \to C_2 - 7C_3$

$$= 6\begin{vmatrix} 3d & 4a & g \\ 3e & 4b & h \\ 3f & 4c & i \end{vmatrix}$$

$$= 6 \times 3 \times 4 \begin{vmatrix} d & a & g \\ e & b & h \\ f & c & i \end{vmatrix} \text{ By } C_2 \leftrightarrow C_1$$

$$= -72 \begin{vmatrix} a & d & g \\ b & e & h \\ c & f & i \end{vmatrix} \quad \text{By } R \leftrightarrow C$$

$$= -72 \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = -72\Delta$$

Hence, option (d) is correct.

10. If $\dfrac{1}{b+c}, \dfrac{1}{c+a}, \dfrac{1}{a+b}$ are in HP, then which of the following is/are correct?

1. a, b, c are in AP
2. $(b+c)^2, (c+a)^2, (a+b)^2$ are in GP.

Select the correct answer using the code given below.

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

(***a***) Given that,

$$\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b} \text{ are in HP.}$$

$\Rightarrow b + c, c + a, a + b$ are in AP.

$\Rightarrow (a + b + c) - (b + c), (a + b + c) - (c + a), (a + b + c) - (a + b)$ are in AP.

$\Rightarrow a, b, c$ are in AP.

2. From 1; a, b, c are in AP.

$\therefore \quad b = a + d, c = a + 2d$

where, d is common difference.

$\therefore (b + c)^2 = (a + d + a + 2d)^2$

$= (2a + 3d)^2$

$(c + a)^4 = (a + 2d + a)^4 = (2a + 2d)^4$

$(a + b)^2 = (a + a + d)^2 = (2a + d)^2$

Here, $(c + a)^4 = (b + c)^2 . (a + b)^2$

So, $(b+c)^2, (c+a)^2, (a+b)^2$ are not in G.P.

Hence, option (a) is correct.

11. If $A = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$,

where $a \in \mathbb{N}$, then what is $A^{100} - A^{50} - 2A^{25}$ equal to?

(a) $-2I$ (b) $-I$

(c) $2I$ (d) I

where I is the identity matrix.

(***a***) $A = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} a \in \mathbb{N}$

The sequence for given matrix A is

$$A^n = \begin{bmatrix} 1 & na \\ 0 & 1 \end{bmatrix}$$

$$\therefore A^{100} - A^{50} - 2A^{25} = \begin{bmatrix} 1 & 100a \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 50a \\ 0 & 1 \end{bmatrix} - 2\begin{bmatrix} 1 & 25a \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 - 1 - 2 & 100a - 50a - 50a \\ 0 - 0 - 0 & 1 - 1 - 2 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} = -2I$$

Hence, option (a) is correct.

12. If $\begin{vmatrix} a & -b & a - b - c \\ -a & b & -a + b - c \\ -a & -b & -a - b + c \end{vmatrix} - kabc = 0$

$(a \neq 0, b \neq 0, c \neq 0)$

then what is the value of k?

(a) -4 (b) -2

(c) 2 (d) 4

(***a***) Given that,

$$\begin{vmatrix} a & -b & a - b - c \\ -a & b & -a + b - c \\ -a & -b & -a - b + c \end{vmatrix} - kabc = 0$$

$(a \neq 0, b \neq 0, c \neq 0)$

$R_1 \to R_1 + R_2$

$$\begin{vmatrix} 0 & 0 & -2c \\ -a & b & -a + b - c \\ -a & -b & -a - b + c \end{vmatrix} - kabc = 0$$

$-2c[(-a)(-b) - (-a)b] - kabc = 0$

$-2c(2ab) - kabc = 0$

$-kabc = 4abc$

$\Rightarrow \quad k = -4$

Hence, option (a) is correct.

13. What is $\sum_{n=1}^{8n+7} i^n$ equal to, where $i = \sqrt{-1}$?

(a) -1 (b) 1
(c) i (d) $-i$

(a) Let $S = \sum_{n=1}^{8n+7} i^n$

$$S = i + i^2 + i^3 + \dots + i^{8n+7}$$
$$= i\left[\frac{(i)^{8n+7} - 1}{i - 1}\right] = i\left[\frac{i^{4(2n+1)+3} - 1}{i - 1}\right]$$
$$= i\left(\frac{i^3 - 1}{i - 1}\right) \quad [\because i^{4n+r} = i^r]$$
$$= i\left[\frac{-i - 1}{i - 1}\right] = \frac{-i^2 - i}{i - 1}$$
$$= \frac{1 - i}{i - 1} = -1$$

14. If $z = x + iy$, where $i = \sqrt{-1}$, then what does the equation $z\bar{z} + |z|^2 + 4(z + \bar{z}) - 48 = 0$ represent?

(a) Straight line
(b) Parabola
(c) Circle
(d) Pair of straight lines

(c) Given, $z = x + iy$

$\therefore$ $\bar{z} = x - iy$

$\therefore$ $z + \bar{z} = 2x$

and $|z|^2 = x^2 + y^2$

$\therefore$ $z\bar{z} + |z|^2 + 4(z + \bar{z}) - 48 = 0$

$$(x + iy)(x - iy) + x^2 + y^2 + 4(2x) - 48 = 0$$
$$x^2 + y^2 + x^2 + y^2 + 8x - 48 = 0$$
$$2x^2 + 2y^2 + 8x - 48 = 0$$
$$x^2 + y^2 + 4x - 24 = 0$$

which represents circle.

Hence, option (c) is correct.

15. Which one of the following is a square root of $2a + 2\sqrt{a^2 + b^2}$, where $a, b \in \mathbb{R}$?

(a) $\sqrt{a + ib} + \sqrt{a - ib}$
(b) $\sqrt{a + ib} - \sqrt{a - ib}$
(c) $2a + ib$
(d) $2a - ib$, where $i = \sqrt{-1}$

(a) $2a + 2\sqrt{a^2 + b^2}$

$$= 2a + ib - ib + 2\sqrt{a^2 - i^2b^2}$$
$$= (a + ib) + (a - ib) + 2\sqrt{(a + ib)(a - ib)}$$
$$= (\sqrt{a + ib} + \sqrt{a - ib})^2$$

Hence, square root of

$$2a + 2\sqrt{a^2 + b^2} = \sqrt{a + ib} + \sqrt{a - ib}$$

16. If $\sin\theta$ and $\cos\theta$ are the roots of the equation $ax^2 + bx + c = 0$, then which one of the following is correct?

(a) $a^2 + b^2 - 2ac = 0$
(b) $-a^2 + b^2 + 2ac = 0$
(c) $a^2 - b^2 + 2ac = 0$
(d) $a^2 + b^2 + 2ac = 0$

(c) Given, equation $ax^2 + bx + c = 0$ ...(i)

$\because$ Roots are $\sin\theta$ and $\cos\theta$

$\therefore$ $\sin\theta + \cos\theta = -\frac{b}{a}$

and $\sin\theta \cdot \cos\theta = \frac{c}{d}$

On squaring both sides, we get

$$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = \frac{b^2}{a^2}$$
$$1 + 2\frac{c}{a} = \frac{b^2}{a^2}$$
$$a(a + 2c) = b^2$$
$$\Rightarrow \quad a^2 - b^2 + 2ac = 0$$

$\therefore$ Option (c) is correct.

17. If $C(n, 4)$, $C(n, 5)$ and $C(n, 6)$ are in AP, then what is the value of n?

(a) 7 (b) 8 (c) 9 (d) 10

(a) ${}^nC_4 \cdot {}^nC_5$ and ${}^nC_6$ are in AP.

$$\Rightarrow \quad 2 \cdot {}^nC_5 = {}^nC_4 + {}^nC_6$$
$$\frac{2n!}{5!(n-5)!} = \frac{n!}{4!(n-4)!} + \frac{n!}{6!(n-6)!}$$
$$\frac{2(n!)}{5(4!)(n-5)(n-6)!}$$
$$= \frac{n!}{4!(n-6)!}\left[\frac{1}{(n-4)(n-5)} + \frac{1}{6 \times 5}\right]$$
$$\frac{2}{5(n-5)} = \frac{1}{(n-4)(n-5)} + \frac{1}{30}$$
$$\frac{2n - 8 - 5}{5(n^2 - 9n + 20)} = \frac{1}{30}$$
$$30(2n - 13) = 5n^2 - 45n + 100$$
$$5n^2 - 105n + 490 = 0$$
$$n^2 - 21n + 98 = 0$$
$$(n - 14)(n - 7) = 0$$
$$n = 14 \text{ or } n = 7$$

$\therefore$ Option (a) is correct.

18. How many 4-letter words (with or without meaning) containing two vowels can be constructed using only the letters (without repetition) of the word 'LUCKNOW'?

(a) 240 (b) 200
(c) 150 (d) 120

(a) In LUCKNOW, there are 2 vowels and 5 consonants.

$\therefore$ 4 letter words $= {}^5C_2 \cdot {}^2C_2 \cdot 4!$

$$= 10 \times 1 \times 24 = 240$$

$\therefore$ Option (a) is correct.

19. Suppose 20 distinct points are placed randomly on a circle. Which of the following statements is/are correct?

1. The number of straight lines that can be drawn by joining any two of these points is 380.
2. The number of triangles that can be drawn by joining any three of these points is 1140.

Select the correct answer using the code given below.

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

(b) Given, that there are 20 distinct points on a circle and we have to draw a straight line by joining any two of these points.

Hence, number of straight lines

$$= {}^{20}C_2 = \frac{20 \times 19}{2} = 190$$

$\therefore$ Statement (1) is wrong.

and Number of triangle

$$= {}^{20}C_3 = \frac{20 \times 19 \times 18}{1 \times 2 \times 3} = 1140$$

$\therefore$ Statement (2) is correct.

Hence, option (b) is correct.

20. How many terms are there in the expansion of $\left(\frac{a^2}{b^2} + \frac{b^2}{a^2} + 2\right)^{21}$ where $a \neq 0, b \neq 0$?

(a) 21 (b) 22
(c) 42 (d) 43

(d) $\left(\frac{a^2}{b^2} + \frac{b^2}{a^2} + 2\right)^{21}$

$$\Rightarrow \quad \left[\left(\frac{a}{b} + \frac{b}{a}\right)^2\right]^{21} = \left(\frac{a}{b} + \frac{b}{a}\right)^{42}$$

Since, we know that number of terms in the expansion of $(a + b)^n = n + 1$

Hence, total number of terms

$$= 42 + 1 = 43$$

$\therefore$ Option (d) is correct.

21. For what values of k is the system of equations $2k^2x + 3y - 1 = 0$, $7x - 2y + 3 = 0$, $6kx + y + 1 = 0$ consistent?

(a) $\frac{3 \pm \sqrt{11}}{10}$ (b) $\frac{21 \pm \sqrt{161}}{10}$
(c) $\frac{3 \pm \sqrt{7}}{10}$ (d) $\frac{4 \pm \sqrt{11}}{10}$

(b) Given equations,

$$2k^2x + 3y - 1 = 0$$
$$7x - 2y + 3 = 0$$
$$6kx + y + 1 = 0$$

For consistency, determinant formed by the equations

$$\begin{vmatrix} 2k^2 & 3 & -1 \\ 7 & -2 & 3 \\ 6k & 1 & 1 \end{vmatrix} = 0$$

$$2k^2(-2-3) - 3(7-18k) - 1(7+12k) = 0$$

$$-10k^2 - 21 + 54k - 7 - 12k = 0$$

$$-10k^2 - 42k - 28 = 0$$

$$5k^2 - 21k + 14 = 0$$

$$k = \frac{21 \pm \sqrt{441-280}}{10}$$

$$k = \frac{21 \pm \sqrt{161}}{10}$$

Hence, option (b) is correct.

22. The inverse of a matrix A is given by $\begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$

What is A equal to?

(a) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -2 \\ -3 & 4 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}$

(***a***) $A = \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$

$\therefore \quad |A| = (-2)\left(-\frac{1}{2}\right) - \frac{3}{2} = -\frac{1}{2} \neq 0$

$A_{11} = -\frac{1}{2}, A_{12} = -\frac{3}{2}$

$A_{21} = -1, A_{22} = -2$

$\therefore \text{adj } A = \begin{bmatrix} -\frac{1}{2} & -1 \\ -\frac{3}{2} & -2 \end{bmatrix}$

$\therefore A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{-\frac{1}{2}} \begin{bmatrix} -\frac{1}{2} & -1 \\ -\frac{3}{2} & -2 \end{bmatrix}$

$= 2\begin{bmatrix} \frac{1}{2} & 1 \\ \frac{3}{2} & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Hence, option (a) is correct.

23. What is the period of the function $f(x) = \ln(2 + \sin^2 x)$?

(a) $\frac{\pi}{2}$ (b) π (c) 2π (d) 3π

(***b***) $f(x) = \ln(2 + \sin^2 x)$

$\because$ Period of $\sin^2 x$ is π.

and $f(\pi + x) = \log\{2 + \sin^2(\pi + x)\}$

$= \log\{2 + \sin^2 x\}$

$= f(x)$

Hence, period of $\ln(2 + \sin^2 x) = \pi$

$\therefore$ Option (b) is correct.

24. If $\sin(A + B) = 1$ and $2\sin(A - B) = 1$, where $0 < A, B < \frac{\pi}{2}$, then what is $\tan A : \tan B$ equal to?

(a) 1 : 2 (b) 2 : 1

(c) 1 : 3 (d) 3 : 1

(***d***) Given, $\sin(A + B) = 1$

and $2\sin(A - B) = 1$

$0 < A, B < \frac{\pi}{2}$

$\because \quad \sin(A + B) = 1 = \sin\frac{\pi}{2}$

$\Rightarrow \quad A + B = \frac{\pi}{2}$ ...(i)

$2\sin(A - B) = 1$

$\Rightarrow \quad \sin(A - B) = \frac{1}{2} = \sin\frac{\pi}{6}$

$\Rightarrow \quad A - B = \frac{\pi}{6}$ ...(ii)

Now, adding Eq. (i) and Eq. (ii), we get

$$2A = \frac{\pi}{2} + \frac{\pi}{6} = \frac{4\pi}{6}$$

$$A = \frac{\pi}{3}, B = \frac{\pi}{6}$$

$\therefore \quad \tan A : \tan B = \tan\frac{\pi}{3} : \tan\frac{\pi}{6}$

$= \sqrt{3} : \frac{1}{\sqrt{3}} = 3 : 1$

$\therefore$ Option (d) is correct.

25. Consider a regular polygon with 10 sides. What is the number of triangles that can be formed by joining the vertices which have no common side with any of the sides of the polygon?

(a) 25 (b) 50

(c) 75 (d) 100

(***b***) Given number of sides $(n) = 10$

Number of triangles which have no common side with any of the sides of the polygon $= \frac{n(n-4)(n-5)}{3!}$

$\therefore$ Number of triangles

$= \frac{10(10-4)(10-5)}{6}$

$= \frac{10 \times 6 \times 5}{6}$

$= 50$

Hence, option (b) is correct.

26. Consider all the real roots of the equation $x^4 - 10x^2 + 9 = 0$.

What is the sum of the absolute values of the roots?

(a) 4 (b) 6

(c) 8 (d) 10

(***c***) Given equation,

$$x^4 - 10x^2 + 9 = 0$$

Let $y = x^2$

$\therefore \quad y^2 - 10y + 9 = 0$

$(y - 9)(y - 1) = 0$

$\Rightarrow \quad y = 9$ or $y = 1$

$x^2 = 9$ or $y = 1$

$x^2 = 9$ or $x^2 = 1$

$x = \pm 3, x = \pm 1$

$\therefore \quad \text{Sum} = |3| + |-3| + |1| + |-1|$

$= 8$

Hence, option (c) is correct.

27. Consider the expansion of $(1 + x)^n$. Let p, q, r and s be the coefficients of first, second, nth and $(n+1)$th terms respectively. What is $(ps + qr)$ equal to?

(a) $1 + 2n$ (b) $1 + 2n^2$

(c) $1 + n^2$ (d) $1 + 4n$

(***c***) Given, $(1 + x)^n$

In the above expansion, $(r + 1)$th term

$T_{r+1} = {}^nC_r x^r$

$\therefore \quad T_1 = {}^nC_0 x^\circ, T_2 = {}^nC_1 x'$

$p = 1$ ...(i)

$\therefore \quad {}^nC_1 = q$

$n = q$ ...(ii)

$T_n = {}^nC_{n-1} x^{n-1}$,

$T_{n+1} = {}^nC_n x^n$

$\therefore \quad r = n$ ...(iii)

$\therefore \quad s = 1$ ...(iv)

$\therefore \quad (ps + qr) = 1 \cdot 1 + n \cdot n = 1 + n^2$

$= (1 + n^2)$

Hence, option (c) is correct.

28. Let $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$ for $0 \le x, y, z \le 1$. What is the value of $x^{1000} + y^{1001} + z^{1002}$?

(a) 0 (b) 1

(c) 3 (d) 6

(***c***) Let $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$

Which is only possible when,

$\sin^{-1} x = \frac{\pi}{2}, \sin^{-1} y = \frac{\pi}{2}$,

$\sin^{-1} z = \frac{\pi}{2}$

$\Rightarrow \quad x = 1, y = 1, z = 1$

$\therefore x^{1000} + y^{1001} + z^{1002}$

$= 1 + 1 + 1 = 3$

Hence, option (c) is correct.

29. Let $\sin x + \sin y = \cos x + \cos y$ for all $x, y \in \mathbb{R}$. What is $\tan\left(\frac{x}{2}+\frac{y}{2}\right)$ equal to?

(a) 1 (b) 2
(c) $\sqrt{2}$ (d) $2\sqrt{2}$

(***a***) Given that,

$\sin x + \sin y = \cos x + \cos y \ \forall\ x \in \mathbb{R}$

$$2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) = 2\cos\left(\frac{x+y}{2}\right)\cdot\cos\left(\frac{x-y}{2}\right)$$

$$\Rightarrow \frac{2\sin\left(\frac{x+y}{2}\right)\cdot\cos\left(\frac{x-y}{2}\right)}{2\cos\left(\frac{x+y}{2}\right)\cdot\cos\left(\frac{x-y}{2}\right)} = 1$$

$$\tan\left(\frac{x+y}{2}\right) = 1 \text{ or } \tan\left(\frac{x}{2}+\frac{y}{2}\right) = 1$$

Hence, option (a) is correct.

30. Let $A = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$ and $(mI + nA)^2 = A$, where m, n are positive real numbers and I is the identity matrix. What is $(m+n)$ equal to?

(a) 0 (b) $\frac{1}{2}$
(c) 1 (d) $\frac{3}{2}$

(***d***) Let $A = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$ and $(mI + nA)^2 = A$

where, I is Identify matrix

$$\because mI + nA = m\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + n\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$
$$= \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} + \begin{bmatrix} 0 & 2n \\ -2n & 0 \end{bmatrix}$$
$$= \begin{bmatrix} m & 2n \\ -2n & m \end{bmatrix}$$

$\therefore (mI + nA)^2 = A$

$$\begin{bmatrix} m & 2n \\ -2n & m \end{bmatrix}\begin{bmatrix} m & 2n \\ -2n & m \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} m^2 - 4n^2 & 4mn \\ -4mn & m^2 - 4n^2 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

$\Rightarrow \quad 4mn = 2$ and $m^2 - 4n^2 = 0$

$mn = \frac{1}{2}$ and $m = \pm 2n$

When, $m = 2n$

$(2n)(n) = \frac{1}{2}$

$n = \pm\frac{1}{2} \Rightarrow m = \pm 1$

$\therefore \quad m + n = 1 + \frac{1}{2} = \frac{3}{2}$

Hence, option (d) is correct.

31. What is the value of the following?

$$\cot\left[\sin^{-1}\left(\frac{3}{5}\right) + \cot^{-1}\left(\frac{3}{2}\right)\right]$$

(a) $\frac{6}{17}$ (b) $\frac{7}{16}$
(c) $\frac{16}{7}$ (d) $\frac{17}{6}$

(***a***) $\cot\left[\sin^{-1}\frac{3}{5} + \cot^{-1}\frac{3}{2}\right]$

$$\Rightarrow \cot\left[\cot^{-1}\left(\frac{\sqrt{1-\left(\frac{3}{5}\right)^2}}{\frac{3}{5}}\right) + \cot^{-1}\frac{3}{2}\right]$$

$$\left[\because \sin^{-1}x = \cot^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)\right.$$

$$\left.\text{and } \cot x + \cot^{-1}y = \cot^{-1}\left(\frac{xy-1}{x+y}\right)\right]$$

$$\Rightarrow \cot\left[\cot^{-1}\left(\frac{4}{3}\right) + \cot^{-1}\left(\frac{3}{2}\right)\right]$$

$$\Rightarrow \cot\left[\cot^{-1}\left(\frac{\frac{4}{3}\times\frac{3}{2}-1}{\frac{4}{3}+\frac{3}{2}}\right)\right]$$

$$\Rightarrow \frac{1}{\frac{17}{6}} = \frac{6}{17}$$

Hence, option (a) is correct.

32. Let $4\sin^2 x = 3$, where $0 \le x \le \pi$. What is $\tan 3x$ equal to?

(a) -2 (b) -1
(c) 0 (d) 1

(***c***) Given that $4\sin^2 x = 3, 0 \le x \le \pi$

$\therefore \quad \sin^2 x = \frac{3}{4}$

$\Rightarrow \quad \sin x = \frac{\sqrt{3}}{2} = \sin\frac{\pi}{3}$ or $\sin\frac{2\pi}{3}$

$\therefore \quad x = \frac{\pi}{3}, \frac{2\pi}{3}$

$\therefore \quad \tan 3x = \tan\frac{3\pi}{3}$

$= \tan\pi = 0$

Also $\quad \tan 3x = \tan 3\left(\frac{2\pi}{3}\right)$

$= \tan 2\pi = 0$

$\therefore$ Option (c) is correct.

33. Let p, q and 3 be respectively the first, third and fifth terms of an AP. Let d be the common difference. If the product (pq) is minimum, then what is the value of d?

(a) 1 (b) $\frac{3}{8}$
(c) $\frac{9}{8}$ (d) $\frac{9}{4}$

(***c***) Given that first term of AP = p

$\Rightarrow \quad a = p \quad$ ...(i)

Where, a denotes first term.

and $\quad a_3 = q, a_5 = 3$

$\Rightarrow \quad a + 2d = q \quad$ ...(ii)

$a + 4d = 3 \quad$ ...(iii)

$\therefore \quad pq = a(a + 2d)$

$= (3 - 4d)(3 - 4d + 2d)$

$= (3 - 4d)(3 - 2d)$

$= 9 - 18d + 8d^2$

Let $\quad f = 9 - 18d + 8d^2$

$f' = 0 - 18 + 16d$

$= -18 + 16d$

For maxima and minima

$f' = 0$

$\Rightarrow \quad -18 + 16d = 0$

$\Rightarrow \quad d = \frac{18}{16} = \frac{9}{8}.$

Now, $\quad f'' = 16$ (Positive)

So, f will be maximum at $d = \frac{9}{8}$.

Hence, option (c) is correct.

34. Consider the following statements in respect of the roots of the equation $x^3 - 8 = 0$

1. The roots are non-collinear.
2. The roots lie on a circle of unit radius.

Which of the above statements is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

(***a***) $\quad x^3 - 8 = 0$

$\Rightarrow (x - 2)(x^2 + 2x + 4) = 0$

$x = 2, 2\omega, 2\omega^2$

Where, $\omega = \frac{-1 + \sqrt{3}i}{2}$

Hence, roots are non-collinear and will lie on a circle of 2 unit radius.

Hence, option (a) is correct.

35. Let the equation $\sec x \cdot \operatorname{cosec} x = p$ have a solution, where p is a positive real number. What should be the smallest value of p?

(a) $\frac{1}{2}$ (b) 1
(c) 2
(d) Minimum does not exist

(***c***) $\quad \sec x \cdot \operatorname{cosec} x = p$

$\Rightarrow \quad \frac{1}{\sin x \cdot \cos x} = p$

$\Rightarrow \quad \frac{2}{2\sin x\cos x} = p$

$\frac{2}{\sin 2x} = p$

Where, $\sin 2x \in [-1, 1]$

If $\sin 2x = 1$

Then $p = 2$ will be the smallest value.

Hence, option (c) is correct.

36. For what value of θ, where $0 < \theta < \frac{\pi}{2}$, does $\sin\theta + \sin\theta\cos\theta$ attain maximum value?

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

(**b**) Let $P = \sin\theta + \sin\theta \cdot \cos\theta$

$$\therefore \quad \frac{dP}{d\theta} = \cos\theta + \cos^2\theta - \sin^2\theta$$

For maxima-minima $= \frac{dP}{d\theta} = 0$

$$\cos\theta + \cos^2\theta - \sin^2\theta = 0$$

$$\cos + \cos^2\theta - 1 + \cos^2\theta = 0$$

$$2\cos^2\theta + \cos\theta - 1 = 0$$

$$(\cos\theta + 1)(2\cos\theta - 1) = 0$$

$$\Rightarrow \cos\theta = -1 \quad \text{or} \quad \cos\theta = \frac{1}{2}$$

$$\Rightarrow \quad \theta = \pi \quad \text{or} \quad \theta = \frac{\pi}{3}$$

$\theta = \pi$ can be neglected as $\theta \in \left(0, \frac{\pi}{2}\right)$.

$$\therefore \quad \theta = \frac{\pi}{3}$$

Hence, option (b) is correct.

37. Consider the following statements in respect of sets.

1. The union over intersection of sets is distributive.
2. The complement of union of two sets is equal to intersection of their complements.
3. If the difference of two sets is equal to empty set, then the two sets must be equal.

Which of the above statements are correct ?

(a) 1 and 2 (b) 2 and 3

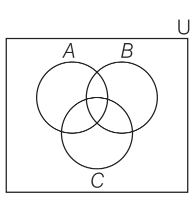
(c) 1 and 3 (d) 1, 2 and 3

(**a**) Since, we know that distributive property for sets A, B and C.

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

and $(A \cup B)' = A' \cap B'$

(By De Morgan's Law)



Also, if $A - B = \phi$

$\Rightarrow$ We cannot say $A = B$

e.g., if $A = \phi$ and $B = \{1, 2\}$

$\Rightarrow A - B = \phi$ and $A \neq B$

$\therefore$ Option (a) is correct.

38. Consider three sets X, Y and Z having 6, 5 and 4 elements respectively. All these 15 elements are distinct. Let $S = (X - Y) \cup Z$. How many proper subsets does S have?

(a) 255 (b) 256

(c) 1023 (d) 1024

(**c**) Given, $n(X) = 6, n(Y) = 5, n(Z) = 4$

$$S = (X - Y) \cup Z$$

Since, all 15 elements are different.

Hence, $n(X - Y) = 6$

and $n(S) = 6 + 4 = 10$

$\Rightarrow$ Number of proper subsets of S

$$= 2^{10} - 1$$
$$= 1024 - 1$$
$$= 1023$$

$\therefore$ Option (c) is correct.

39. Consider the following statements in respect of relations and functions.

1. All relations are functions but all functions are not relations.
2. A relation from A to B is a subset of Cartesian product $A \times B$.
3. A relation in A is a subset of Cartesian product $A \times A$.

Which of the above statements are correct?

(a) 1 and 2 (b) 2 and 3

(c) 1 and 3 (d) 1, 2 and 3

(**b**) Since, we know that relations can be function iff every element has unique image.

Hence, first statement is wrong.

If $R : A \to A$ then $R \subseteq A \times A$

and if $R : A \to B$ then $R \subseteq A \times B$

Hence, 2nd and 3rd statements are correct.

$\therefore$ Option (b) is correct.

40. If $\log_{10} 2 \log_2 10 + \log_{10}(10^x) = 2$, then what is the value of x?

(a) 0 (b) 1

(c) $\log_2 10$ (d) $\log_5 2$

(**b**) Given that,

$$\log_{10}2 \cdot \log_2 10 + \log_{10}(10)^x = 2$$

$$\log_{10}2 \times \frac{1}{\log_{10} 2} + x \log_{10} 10 = 2$$

$$1 + x = 2$$

$$\Rightarrow \quad x = 1$$

$\therefore$ Option (b) is correct.

41. Let ABC be a triangle. If $\cos 2A + \cos 2B + \cos 2C = -1$, then which one of the following is correct?

(a) $\sin A \sin B \sin C = 0$

(b) $\sin A \sin B \cos C = 0$

(c) $\cos A \sin B \sin C = 0$

(d) $\cos A \cos B \cos C = 0$

(**d**) Given that, ABC is a triangle and

$$\cos 2A + \cos 2B + \cos 2C = -1$$

$$\Rightarrow 1 + \cos 2A + \cos 2B + \cos 2C = 0$$

$$\Rightarrow 2\cos^2 A + 2\cos\left(\frac{2B + 2C}{2}\right) \cdot \cos\left(\frac{2B - 2C}{2}\right) = 0$$

$$\Rightarrow 2\cos^2 A + 2\cos(B + C) \cdot \cos(B - C) = 0$$

$$\{\because A + B + C = 180\}$$

$$\Rightarrow 2\cos^2 A + 2\cos(180° - A) \cdot \cos(B - C) = 0$$

$$\Rightarrow 2\cos^2 A - 2\cos A \cdot \cos(B - C) = 0$$

$$\Rightarrow 2\cos A[\cos A - \cos(B - C)] = 0$$

$$\Rightarrow 2\cos A[\cos(180° - (B + C)) - \cos(B - C)] = 0$$

$$\Rightarrow -2\cos A[\cos(B + C) + \cos(B - C)] = 0$$

$$\Rightarrow -2\cos A\left(2\cos\frac{B + C + B - C}{2} \cdot \cos\frac{B + C - B + C}{2}\right) = 0$$

$$-4\cos A \cdot \cos B \cdot \cos C = 0$$

$$\Rightarrow \quad \cos A \cdot \cos B \cdot \cos C = 0$$

$\therefore$ Option (d) is correct.

42. What is the value of the following determinant?

$$\begin{vmatrix} \cos C & \tan A & 0 \\ \sin B & 0 & -\tan A \\ 0 & \sin B & \cos C \end{vmatrix}$$

(a) -1

(b) 0

(c) $2\tan A \sin B \sin C$

(d) $-2\tan A \sin B \sin C$

(**b**) Let $\Delta = \begin{vmatrix} \cos C & \tan A & 0 \\ \sin B & 0 & -\tan A \\ 0 & \sin B & \cos C \end{vmatrix}$

$$\Delta = \cos C\,[0 + \sin B \tan A] - \tan A\,[\sin B \cos C - 0]$$

$$= \tan A \sin B \cos C - \tan A \sin B \cos C$$

$$\therefore \Delta = 0$$

Hence, option (b) is correct.

43. Suppose set A consists of first 250 natural numbers that are multiples of 3 and set B consists of first 200 even natural numbers. How many elements does $A \cup B$ have?

(a) 324 (b) 364
(c) 384 (d) 400

(**c**) Given that, A consists of first 250 natural numbers that are multiple of 3.

$\therefore \quad A = \{3, 6, 9, 12, ..., 750\}$,

$n(A) = 250$

Set B consists of first 200 even natural numbers.

$\therefore \quad B = \{2, 4, 6, 8, ..., 400\}$

$\therefore \quad A \cap B = \{6, 12, ..., 750\}$

$\because \quad n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 250 + 200 - 66$

$n(A \cup B) = 384$

Hence, option (c) is correct.

44. Let S_k denote the sum of first k terms of an AP. What is $\frac{S_{30}}{S_{20} - S_{10}}$ equal to?

(a) 1 (b) 2
(c) 3 (d) 4

(**c**) Let's take first K terms are first K natural numbers.

$$\therefore \quad S_K = \frac{K(K+1)}{2}$$

$$\text{Consider } \frac{S_{30}}{S_{20} - S_{10}} = \frac{\frac{30(31)}{2}}{\frac{20(21)}{2} - \frac{10(11)}{2}}$$

$$= \frac{930}{310} = 3$$

$\therefore$ Option (c) is correct.

45. If the roots of the equation $4x^2 - (5k+1)x + 5k = 0$ differ by unity, then which one of the following is a possible value of k?

(a) -3 (b) -1
(c) $-\frac{1}{5}$ (d) $-\frac{3}{5}$

(**c**) Given equation,

$4x^2 - (5K + 1)x + 5K = 0 \quad$ ...(i)

Let the roots are α and β.

$$\alpha + \beta = \frac{-(-(5K+1))}{4} = \frac{5K+1}{4}$$

$$\alpha \cdot \beta = \frac{5K}{4}$$

Given that, $\alpha - \beta = 1$

$$\sqrt{(\alpha+\beta)^2 - 4\alpha\beta} = 1$$

$$\left(\frac{5K+1}{4}\right)^2 - 4\left(\frac{5K}{4}\right) = 1$$

$$\frac{25K^2 + 1 + 10K}{16} = 1 + 5K$$

$$25K^2 + 10K + 1 = 80K + 16$$

$$\Rightarrow \quad 25K^2 - 70K - 15 = 0$$

$$5K^2 - 14K - 3 = 0$$

$$5K^2 - 15K + K - 3 = 0$$

$$5K(K-3) + 1(K-3) = 0$$

$$(K-3)(5K+1) = 0$$

$$\Rightarrow \quad K = 3 \text{ or } -\frac{1}{5}$$

Hence, option (c) is correct.

46. Consider the digits 3, 5, 7, 9. What is the number of 5-digit numbers formed by these digits in which each of these four digits appears?

(a) 240 (b) 180
(c) 120 (d) 60

(**a**) Given digits are 3, 5, 7, 9.

Since, the number of ways to find 5-digit numbers = 5!

but using 3, 5, 7, 9 every time one-digit will be repeated.

Hence number of 5-digit numbers with digit 3 repeated = $\frac{5!}{2!}$

Number of 5-digit numbers with digit 5 repeated = $\frac{5!}{2!}$

Number of 5-digit numbers with digit 7 repeated = $\frac{5!}{2!}$

Number of 5-digit numbers with digit 9 repeated = $\frac{5!}{2!}$

$\therefore$ Total 5-digit numbers

$$= \frac{5!}{2!} + \frac{5!}{2!} + \frac{5!}{2!} + \frac{5!}{2!}$$

$$= 4 \times \left(\frac{5 \times 4 \times 3 \times 2!}{2!}\right)$$

$$= 4 \times \left(\frac{5 \times 4 \times 3 \times 2!}{2!}\right)$$

$$= 240$$

Hence, option (a) is correct.

47. How many distinct matrices exist with all four entries taken from {1, 2}?

(a) 16 (b) 24
(c) 32 (d) 48

(**a**) Given digits are 1, 2.

Let matrix = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\because$ Each entries can filled with 2 ways.

Therefore, number of distinct matrices

$= 2 \times 2 \times 2 \times 2$

$= 16$

Hence, option (a) is correct.

48. If $i = \sqrt{-1}$, then how many values does i^{-2n} have for different $n \in \mathbb{Z}$?

(a) One (b) Two
(c) Four (d) Infinite

(**b**) Given that, $i = \sqrt{-1}$

To find $(i)^{-2n}$

Let $i = r(\cos\theta + i\sin\theta)$

$\Rightarrow \quad r = 1, \theta = \frac{\pi}{2}$

$$\therefore \quad i = \left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$$

$$\therefore (i)^{-2n} = \left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)^{-2n}$$

$$= \cos\left(\frac{-2n\pi}{2}\right) + i\sin\left(\frac{-2n\pi}{2}\right)$$

$$= \cos(n\pi) - i\sin(n\pi)$$

$$= (-1)^n = \begin{cases} -1; & \text{if } n \text{ is odd} \\ 1; & \text{if } n \text{ is even} \end{cases}$$

$\therefore$ Option (b) is correct.

49. If $x = \frac{a}{b-c}, y = \frac{b}{c-a}, z = \frac{c}{a-b}$, then what is the value of the following?

$$\begin{vmatrix} 1 & -x & x \\ 1 & 1 & -y \\ 1 & z & 1 \end{vmatrix}$$

(a) 0 (b) 1
(c) abc (d) $ab + bc + ca$

(**a**) Given, $x = \frac{a}{b-c}, y = \frac{b}{c-a}, z = \frac{c}{a-b}$

$$\therefore \begin{vmatrix} 1 & -x & x \\ 1 & 1 & -y \\ 1 & z & 1 \end{vmatrix}$$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$= \begin{vmatrix} 1 & -x & x \\ 0 & 1+x & -y-x \\ 0 & z+x & 1-x \end{vmatrix}$$

$$= (1+x)(1-x) - (-y-x)(z+x)$$

$$= 1 - x^2 + x^2 + (y+z)x + yz$$

$$= 1 + \left(\frac{b}{c-a} + \frac{c}{a-b}\right)\left(\frac{a}{b-c}\right) + \left(\frac{b}{c-a} \times \frac{c}{a-b}\right)$$

$$= 1 + \left(\frac{ab - b^2 + c^2 - ac}{(a-b)(c-a)}\right)\left(\frac{a}{b-c}\right) + \left(\frac{bc}{(c-a)(a-b)}\right)$$

$$= 1 + \frac{(b-c)(a-b-c)a}{(a-b)(c-a)(b-c)} + \frac{bc}{(c-a)(a-b)}$$

$$= \frac{(a-b)(c-a) + a^2 - ab - ac + bc}{(a-b)(c-a)}$$

$$= \frac{ac - a^2 - bc + ab + a^2 - ab - ac + bc}{(a-b)(c-a)}$$

$= 0$

Hence, option (a) is correct.

50. Consider the following in respect of the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

1. Inverse of A does not exist
2. $A^3 = A$
3. $3A = A^2$

Which of the above are correct ?

(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

(**c**) Given matrix, $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

$\because |A| = 1(1-1) - 1(1-1) + 1(1-1) = 0$

$\therefore A^{-1}$ doesn't exist.

Now, $A^2 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix} = 3A$

and $A^3 = \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 9 & 9 & 9 \\ 9 & 9 & 9 \\ 9 & 9 & 9 \end{bmatrix} \neq A$

Hence, option (c) is correct.

Directions (Q.Nos. 51 and 52)

Consider the following for the next two questions that follow.

A circle is passing through the points $(5, -8)$, $(-2, 9)$ and $(2, 1)$.

51. What are the coordinate of the centre of the circle?

(a) $(-2, -50)$ (b) $(-50, -20)$
(c) $(-24, -58)$ (d) $(-58, -24)$

(**d**) Given that, circle is passing through the points $(5, -8)$, $(-2, 9)$ and $(2, 1)$.

Let the equation of circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$ ...(i)

If Eq. (i) passes through $(5, -8)$

$\therefore 25 + 64 + 10g - 16f + c = 0$

$\Rightarrow 10g - 16f + c + 89 = 0$ ...(ii)

If Eq. (i) passes through $(-2, 9)$

$4 + 81 - 4g + 18f + c = 0$

$-4g + 18f + c + 85 = 0$ ...(iii)

If Eq. (i) passes through (2, 1)

$\Rightarrow 4 + 1 + 4g + 2f + c = 0$

$4g + 2f + c + 5 = 0$ ...(iv)

On solving Eqs. (ii), (iii) and (iv)

Eqs. (ii) – Eq. (iii)

$\Rightarrow 14g - 34f + 4 = 0$

$\Rightarrow 7g - 17f + 2 = 0$ ...(v)

Eq. (iv) – Eq. (iii)

$\Rightarrow 8g - 16f - 80 = 0$

$g - 2f - 10 = 0$ ...(vi)

Eq. (v) $-7 \times$ Eq. (vi)

$-3f + 72 = 0$

$f = 24$

From Eq. (vi)

$g = 2f + 10$

$g = 58$

From Eq. (iv) $c = -4g - 2f - 5$

$= -232 - 48 - 5$

$c = -285$

$\therefore$ Centre $= (-g, -f)$

$= (-58, -24)$

$\therefore$ Option (d) is correct.

52. If r is the radius of the circle, then which one of the following is correct?

(a) $r < 10$ (b) $10 < r < 30$
(c) $30 < r < 60$ (d) $r > 60$

(**d**) Since, the centre of the above circle

$= (-58, -24)$

$g = 58$, $f = 24$ and $c = -285$

$\therefore$ Radius $= \sqrt{g^2 + f^2 - c}$

$= \sqrt{(58)^2 + (24)^2 - (-285)}$

$= \sqrt{3364 + 576 + 285}$

$= \sqrt{4225}$

$r = 65$ unit.

$\therefore$ Option (d) is correct.

Directions (Q.Nos. 53 and 54)

Consider the following for the next two questions that follow.

The two vertices of an equilateral triangle are (0, 0) and (2, 2).

53. Consider the following statements.

1. The third vertex has atleast one irrational coordinate.
2. The area is irrational.

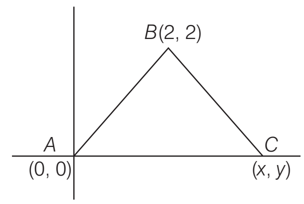
Which of the above statements is/are correct?

(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

(**c**) Let vertex of $A = (0, 0)$

$B = (2, 2)$

$C = (x, y)$



$\because \Delta ABC$ is equilateral triangle.

$\therefore AB = BC$

$\Rightarrow \sqrt{(2-0)^2 + (2-0)^2}$

$= \sqrt{(2-x)^2 + (2-y)^2}$

$\Rightarrow 8 = 4 + x^2 - 4x + 4 + y^2 - 4y$

$\Rightarrow x^2 + y^2 - 4x - 4y = 0$ ...(i)

and $AB = AC$

$\Rightarrow \sqrt{(2-0)^2 + (2-0)^2}$

$= \sqrt{(x-0)^2 + (y-0)^2}$

$\Rightarrow 8 = x^2 + y^2$ ...(ii)

and $AC = BC$

$\Rightarrow \sqrt{x^2 + y^2} = \sqrt{(x-2)^2 + (y-2)^2}$

$\Rightarrow x^2 + y^2 = x^2 + y^2 - 4x - 4y + 8$

$\Rightarrow x + y = 2$ ...(iii)

From Eqs. (ii) and (iii)

$8 = x^2 + (2-x)^2$

$x^2 + 4 + x^2 - 4x = 8$

$x^2 - 2x - 2 = 0$

$x = \frac{2 \pm \sqrt{4+8}}{2}$

$= \frac{1 \pm \sqrt{3}}{2}$

Hence, third vertex atleast one irrational coordinate.

$\Rightarrow$ Area will also be irrational.

Hence, option (c) is correct.

54. The difference of coordinates of the third vertex is

(a) 0 (b) $\sqrt{3}$
(c) $2\sqrt{2}$ (d) $2\sqrt{3}$

(**d**) Since, $x = \frac{1 \pm \sqrt{3}}{2}$

and $y = 2 - x$

$y = 2 - \frac{1 \pm \sqrt{3}}{2}$

$y = \frac{3 \pm \sqrt{3}}{2}$

If $x = \frac{1 + \sqrt{3}}{2}$, $y = \frac{3 - \sqrt{3}}{2}$

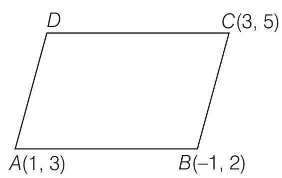
$\therefore |x - y| = 2\sqrt{3}$

Hence, option (d) is correct.

Directions (Q. Nos. 55 and 56) *Consider the following for the questions that follow.*
The coordinates of three consecutive vertices of a parallelogram ABCD are A(1, 3), B(– 1, 2) and C(3, 5).

55. What is the equation of the diagonal *BD*?
(a) $2x - 3y + 2 = 0$
(b) $3x - 2y + 5 = 0$
(c) $2x - 3y + 8 = 0$
(d) $3x - 2y - 5 = 0$

(***c***) Given, vertices of parallelogram are
$A = (1, 3), B = (-1, 2), C = (3, 5)$



ABCD is a parallelogram, then
Mid-point of AC = Mid-point of BD

$$\Rightarrow \left(\frac{1+3}{2}, \frac{3+5}{2}\right) = \left(\frac{-1+x}{2}, \frac{2+y}{2}\right)$$

$$\Rightarrow \quad \frac{-1+x}{2} = \frac{4}{2} \quad \Rightarrow x = 5$$

and $$\frac{2+y}{2} = \frac{8}{2} \quad \Rightarrow y = 6$$

$\therefore$ Point = (5, 6)
$\therefore$ Equation of *BD*,
where $B = (-1, 2)$ and $D = (5, 6)$

$$y - 2 = \frac{6-2}{5-(-1)}(x+1)$$

$$y - 2 = \frac{4}{6}(x+1)$$

$$6y - 12 = 4x + 4$$

$$\Rightarrow \quad 2x - 3y + 8 = 0$$

$\therefore$ Option (c) is correct.

56. What is the area of the parallelogram?
(a) 1 sq. unit (b) $\frac{3}{2}$ sq. units
(c) 2 sq. units (d) $\frac{5}{2}$ sq. units

(***c***) The vertices of parallelogram are $A(1, 3), B(-1, 2), C(3, 5)$ and $D(5, 6)$.
$\therefore$ Area = | Area of ΔABC+ Area of ΔACD |

$$= \frac{1}{2}|\,1(2-5) - 1(5-3) + 3(3-2)\,|$$

$$+ \frac{1}{2}|\,1(5-6) + 3(6-3) + 5(3-5)\,|$$

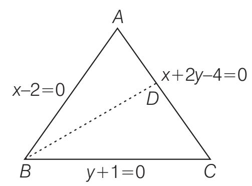
$$= \frac{1}{2}|-3-2+3+(-1)+9-10|$$

= 2 sq. units
$\therefore$ Option (c) is correct.

Directions (Q. Nos. 57 and 58) *Consider the following for the next two questions that follow.*
The equations of the sides AB, BC and CA of a triangle ABC are $x - 2 = 0$, $y + 1 = 0$ and $x + 2y - 4 = 0$ respectively.

57. What is the equation of the altitude through *B* on *AC*?
(a) $x - 3y + 1 = 0$ (b) $x - 3y + 4 = 0$
(c) $2x - y + 4 = 0$ (d) $2x - y - 5 = 0$

(***d***) Equation of $AB \Rightarrow x - 2 = 0$ ...(i)
Equation of $BC \Rightarrow y + 1 = 0$ ...(ii)
Equation of $AC \Rightarrow x + 2y - 4 = 0$ ...(iii)



On solving Eq. (i) and Eq. (ii), we get
$$x = 2, y = -1$$
$$\therefore \quad B = (2, -1)$$

$$\text{Slope of } AC = \frac{-\text{ coefficient of } x}{\text{coefficient of } y}$$

$$m_1 = -\frac{1}{2}$$

$$\therefore \text{Slope of altitude } BD = \frac{-1}{m_1} = \frac{-1}{-\frac{1}{2}} = 2$$

$\therefore$ Equation of altitude *BD* drawn from *B* on *AC* having slope 2.
$$y + 1 = 2(x - 2)$$
$$y + 1 = 2x - 4$$
$$\Rightarrow \quad 2x - y - 5 = 0$$
Hence, option (d) is correct.

58. What are the coordinates of circumcentre of the triangle?
(a) (4, 0) (b) (2, 1)
(c) (0, 4) (d) (2, – 1)

(***a***) Slope of line *AB*

$$\Rightarrow \frac{-\text{ coefficient of } x}{\text{coefficient of } y} = \frac{-1}{0} = \infty$$

$$\text{Slope of line } BC = -\frac{0}{1} = 0$$

$\therefore$ Angle between *AB* and *BC*

$$= \left|\frac{\infty - (0)}{1 + \infty \cdot (0)}\right|$$

$$\Rightarrow \quad \tan\theta = \infty \quad \left|\theta = \frac{\pi}{2}\right|$$

$\because \Delta ABC$ is right angled triangle.
$\therefore$ Circumcentre will lie on Hypotenuse *AC* i.e. $x + 2y - 4 = 0$ at mid point.
Equation of $AB : x - 2 = 0$ ...(i)
Equation of $AC : x + 2y - 4 = 0$ ...(ii)
Equation of $BC : y + 1 = 0$ . ..(iii)

On solving Eqs. (i) and (ii)
$$x = 2, y = 1$$
$$\therefore \quad A = (2, 1)$$
On solving Eqs. (ii) and (iii)
$$y = -1, x = 6$$
$$\therefore \quad c = (6, -1)$$
$\therefore$ Circumcentre will be mid-point of

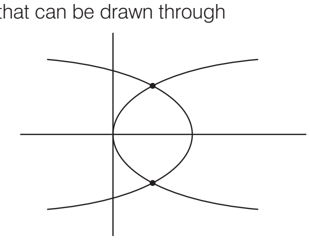
$$AC = \left(\frac{2+6}{2}, \frac{1-1}{2}\right) = (4, 0)$$

$\therefore$ Option (a) is correct.

Directions (Q. Nos. 59 and 60) *Consider the following for the next two questions that follow.*
The two ends of the latus rectum of a parabola are (– 2, 4) and (– 2, – 4).

59. What is the maximum number of parabolas that can be drawn through these two points as end points of latusrectum?
(a) Only one (b) Two
(c) Four (d) Infinite

(***b***) The maximum number of parabolas that can be drawn through

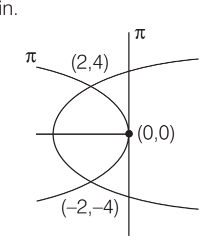


These two points as end points of latusrectum = two
$\therefore$ Option (b) is correct.

60. Consider the following statements in respect of such parabolas
1. One of the parabolas passes through the origin (0, 0).
2. The focus of one of the parabolas lies at (– 2, 0).

Which of the above statements is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

(***a***) Let parabola-1 passes through origin.



It's equation will be
$$y^2 = -4ax$$

Whose leading points of latusrectum will be $(-a, 2a)$ and $(-a, -2a)$

$\therefore \quad a = 2$

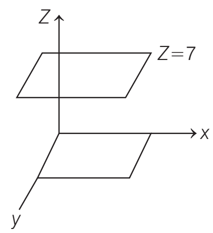
$\therefore \quad$ Focus $= (-2, 0)$

Hence, option (a) is correct.

61. The locus of a point $P(x, y, z)$ which moves in such a way that $z = 7$ is a

(a) line parallel to X-axis
(b) line parallel to Y-axis
(c) line parallel to Z-axis
(d) plane parallel to xy-plane

(*d*) Since, point moves in a plane $z = 7$ which will be parallel to xy-plane.



Hence, option (d) is correct.

62. Consider the following statements

1. A line in space can have infinitely many direction ratios.
2. It is possible for certain line that the sum of the squares of direction cosines can be equal to sum of its direction cosines.

Which of the above statements is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

(*c*) Since, we know that A line in space can have infinitely many direction ratio and also it is possible for certain line that the sum of the squares of direction cosine can be equal to sum of its direction cosines.

For example, (1, 0, 0) is the direction cosines for X-axis.

$\therefore l = 1, m = 0, n = 0$, then

$l^2 + m^2 + n^2 = 1^2 + 0^2 + 0^2$

Hence, option (c) is correct.

63. The xy-plane divides the line segment joining the points $(-1, 3, 4)$ and $(2, -5, 6)$.

(a) internally in the ratio 2 : 3
(b) internally in the ratio 3 : 2
(c) externally in the ratio 2 : 3
(d) externally in the ratio 2 : 1

(*c*) Since, we know that xy-plane divides the line segment joining the points (x_1, y_1, z_1) and (x_2, y_2, z_2) in the ratio $\left(-\frac{z_1}{z_2}\right)$.

Hence, $\quad -\frac{4}{6} = -\frac{2}{3}$

where (–) indicates externally division.

Hence, option (c) is correct.

64. The number of spheres of radius r touching the coordinate axes is

(a) 4 (b) 6
(c) 8 (d) infinite

(*c*) Since, we know that the number of spheres of radius r touching the coordinate axes is 8.

Hence, option (c) is correct.

65. $ABCDEFGH$ is a cuboid with base $ABCD$. Let $A(0, 0, 0)$, $B(12, 0, 0)$, $C(12, 6, 0)$ and $G(12, 6, 4)$ be the vertices. If α is the angle between AB and AG. β is the angle between AC and AG, then what is the value of $\cos 2\alpha + \cos 2\beta$?

(a) $\frac{40}{49}$ (b) $\frac{64}{49}$
(c) $\frac{120}{49}$ (d) $\frac{160}{49}$

(*b*) Given, $ABCDEFGH$ is a cuboid.

$\because$ Angle between AB and $AG = \alpha$

d.r's of $AB = (12 - 0, 0 - 0, 0 - 0)$
$= (12, 0, 0)$

d.r's of $AG = (12 - 0, 6 - 0, 4 - 0)$
$= (12, 6, 4)$

$$\therefore \quad \cos\alpha = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$= \frac{144 + 0 + 0}{\sqrt{12^2}\sqrt{12^2 + 6^2 + 4^2}}$$

$$\cos\alpha = \frac{144}{12 \times 14} = \frac{6}{7}$$

Now, d.r's of $AC = (12, 6, 0)$

d.r's of $\quad AG = (12, 6, 4)$

$$\therefore \quad \cos\beta = \frac{144 + 36}{\sqrt{180} \times 14}$$

$$= \frac{180}{\sqrt{180} \times 14} = \frac{\sqrt{180}}{14}$$

$\therefore \cos 2\alpha + \cos 2\beta$

$$= 2\cos^2\alpha - 1 + 2\cos^2\beta - 1$$

$$= 2\left(\left(\frac{6}{7}\right)^2 + \left(\frac{\sqrt{180}}{14}\right)^2\right) - 2$$

$$= 2\left(\frac{36}{49} + \frac{180}{196}\right) - 2$$

$$= \frac{72}{49} + \frac{90}{49} - 2$$

$$= \frac{162 - 98}{49} = \frac{64}{49}$$

Hence, option (b) is correct.

66. Let $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ be unit vectors such that $\mathbf{a} \times \mathbf{b}$ is perpendicular to $\mathbf{c}$. If θ is the angle between $\mathbf{a}$ and $\mathbf{b}$, then which of the following is/are correct?

1. $\mathbf{a} \times \mathbf{b} = \sin\theta\, \mathbf{c}$
2. $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = 0$

Select the correct answer using the code given below.

(a) only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

(*c*) Given, that $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ be unit vectors such that $\mathbf{a} \times \mathbf{b}$ is perpendicular to $\mathbf{c}$.

angle between $\mathbf{a}$ and $\mathbf{b} = \theta$

$\therefore \quad \mathbf{a} \times \mathbf{b} = |\mathbf{a}||\mathbf{b}|\sin\theta \cdot \mathbf{c}$

$\{\because \mathbf{a} \times \mathbf{b}$ is the vector perpendicular to $\mathbf{a}$ and $\mathbf{b}\}$

$= 1 \cdot 1 \cdot \sin\theta \cdot \mathbf{c}$

$= \sin\theta\, \mathbf{c}$

Since, $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are lying on the same plane.

$\therefore \quad \mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = 0$

Hence, option (c) is correct.

67. If $\mathbf{a} + 3\mathbf{b} = 3\hat{\mathbf{i}} - \hat{\mathbf{j}}$ and $2\mathbf{a} + \mathbf{b} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}}$, then what is the angle between $\mathbf{a}$ and $\mathbf{b}$?

(a) 0 (b) $\frac{\pi}{6}$
(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

(*d*) Given, $\mathbf{a} + 3\mathbf{b} = 3\hat{\mathbf{i}} - \hat{\mathbf{j}}$ ...(i)

and $\quad 2\mathbf{a} + \mathbf{b} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}}$ ...(ii)

Eq. (i) $\times$ 2 – Eq. (ii)

$(2\mathbf{a} + 6\mathbf{b}) - (2\mathbf{a} + \mathbf{b}) = 2(3\hat{\mathbf{i}} - \hat{\mathbf{j}}) - (\hat{\mathbf{i}} - 2\hat{\mathbf{j}})$

$5\mathbf{b} = 5\hat{\mathbf{i}}$

$\therefore \quad \mathbf{b} = \hat{\mathbf{i}}$

From Eq. (i)

$\mathbf{a} = (3\hat{\mathbf{i}} - \hat{\mathbf{j}}) - 3\mathbf{b}$

$\mathbf{a} = 3\hat{\mathbf{i}} - \hat{\mathbf{j}} - 3\hat{\mathbf{i}}$

$\mathbf{a} = -\hat{\mathbf{j}}$

$\therefore \quad \mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$

where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$.

$(-\hat{\mathbf{j}}) \cdot (\hat{\mathbf{i}}) = 1 \cdot 1 \cdot \cos\theta$

$0 = \cos\theta$

$\therefore \quad \theta = \frac{\pi}{2}$

Hence, option (d) is correct.

68. If $(\mathbf{a} + \mathbf{b})$ is perpendicular to $\mathbf{a}$ and magnitude of $\mathbf{b}$ is twice that of $\mathbf{a}$, then what is the value of $(4\mathbf{a} + \mathbf{b}) \cdot \mathbf{b}$ equal to?

(a) 0 (b) 1
(c) $8|\mathbf{a}|^2$ (d) $8|\mathbf{b}|^2$

(a) Given, $\mathbf{a} + \mathbf{b}$ is perpendicular to $\mathbf{a}$.

$\therefore \quad (\mathbf{a} + \mathbf{b}) \cdot \mathbf{a} = 0$

$\Rightarrow \quad |\mathbf{a}|^2 + \mathbf{b} \cdot \mathbf{a} = 0$

$\mathbf{a} \cdot \mathbf{b} = -|\mathbf{a}|^2$

and $\quad |\mathbf{b}| = 2|\mathbf{a}|$

$\therefore \quad (4\mathbf{a} + \mathbf{b}) \cdot \mathbf{b} = 4\mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{b}$

$= 4(-|\mathbf{a}|^2) + |\mathbf{b}|^2$

$= -4|\mathbf{a}|^2 + (2|\mathbf{a}|)^2$

$= -4|\mathbf{a}|^2 + 4|\mathbf{a}|^2$

$= 0$

Hence, option (a) is correct.

69. Let $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ be three vectors such $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are coplanar. Which of the following is/are correct?

1. $(\mathbf{a} \times \mathbf{b}) \times \mathbf{c}$ is coplanar with $\mathbf{a}$ and $\mathbf{b}$
2. $(\mathbf{a} \times \mathbf{b}) \times \mathbf{c}$ is perpendicular to $\mathbf{a} \times \mathbf{b}$

Select the correct answer using the code given below.

(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

(c) Given that $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ are coplanar.

$\Rightarrow \quad \mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = 0 = \mathbf{b} \cdot (\mathbf{c} \times \mathbf{a}) = \mathbf{c} \cdot (\mathbf{a} \times \mathbf{b})$

$\Rightarrow \quad [\mathbf{a}\ \mathbf{b}\ \mathbf{c}] = [\mathbf{b}\ \mathbf{c}\ \mathbf{a}] = [\mathbf{c}\ \mathbf{a}\ \mathbf{b}] = 0$

$\because \quad (\mathbf{a} \times \mathbf{b}) \times \mathbf{c} = -\mathbf{c} \times (\mathbf{a} \times \mathbf{b})$

$= -[(\mathbf{c} \cdot \mathbf{b})\mathbf{a} - (\mathbf{c} \cdot \mathbf{a})\mathbf{b}]$

$= -[[\mathbf{c}\ \mathbf{b}\ \mathbf{a}] - [\mathbf{c}\ \mathbf{a}\ \mathbf{b}]]$

$= 2[\mathbf{c}\ \mathbf{a}\ \mathbf{b}] = 0$

Hence, $\{(\mathbf{a} \times \mathbf{b}) \times \mathbf{c}\} \cdot (\mathbf{a} \times \mathbf{b}) = 0$

$\Rightarrow (\mathbf{a} \times \mathbf{b}) \times \mathbf{c}$ is perpendicular to $\mathbf{a} \times \mathbf{b}$.

and coplanar with $\mathbf{a}$ and $\mathbf{b}$.

Hence, option (c) is correct.

70. If the position vectors of A and B are $(\sqrt{2}-1)\hat{\mathbf{i}} - \hat{\mathbf{j}}$ and $\hat{\mathbf{i}} + (\sqrt{2}+1)\hat{\mathbf{j}}$ respectively, then what is the magnitude of $\mathbf{AB}$?

(a) $2\sqrt{2}$ (b) $3\sqrt{2}$
(c) $2\sqrt{3}$ (d) $3\sqrt{3}$

(c) Given that, $\mathbf{OA} = (\sqrt{2}-1)\hat{\mathbf{i}} - \hat{\mathbf{j}}$

and $\quad \mathbf{OB} = \hat{\mathbf{i}} + (\sqrt{2}+1)\hat{\mathbf{j}}$

$\therefore \quad \mathbf{AB} = \mathbf{OB} - \mathbf{OA}$

$= (1 - \sqrt{2} + 1)\hat{\mathbf{i}} + (\sqrt{2} + 1 + 1)\hat{\mathbf{j}}$

$\mathbf{AB} = (2 - \sqrt{2})\hat{\mathbf{i}} + (\sqrt{2} + 2)\hat{\mathbf{j}}$

$\therefore |\mathbf{AB}| = \sqrt{(2-\sqrt{2})^2 + (2+\sqrt{2})^2}$

$= \sqrt{4 + 2 - 4\sqrt{2} + 4 + 2 + 4\sqrt{2}}$

$= \sqrt{12} = 2\sqrt{3}$

Hence, option (c) is correct.

71. If $y = (1+x)(1+x^2)(1+x^4)(1+x^8)(1+x^{16})$, then what is $\dfrac{dy}{dx}$ at $x = 0$ equal to?

(a) 0 (b) 1
(c) 2 (d) 4

(b) Given, $y = (1+x)(1+x^2)(1+x^4)(1+x^8)(1+x^{16})$

$\therefore \dfrac{dy}{dx} = (1+x)(1+x^2)(1+x^4)(1+x^8)\cdot(16x^{15})$
$+ (1+x)(1+x^2)(1+x^4)(8x^7)(1+x^{16})$
$+ (1+x)(1+x^2)(4x^3)(1+x^8)(1+x^{16})$
$+ (1+x)(2x)(1+x^4)(1+x^8)(1+x^{16})$
$+ (1)(1+x^2)(1+x^4)(1+x^8)(1+x^{16})$

$\therefore \left.\dfrac{dy}{dx}\right|_{x=0} = 0 + 0 + 0 + 0 + 1 = 1$

Hence, option (b) is correct.

72. If $y = \cos x \cdot \cos 4x \cdot \cos 8x$, then what is $\dfrac{1}{y}\dfrac{dy}{dx}$ at $x = \dfrac{\pi}{4}$ equal to?

(a) -1 (b) 0 (c) 1 (d) 3

(a) Given, $y = \cos x \cdot \cos 4x \cdot \cos 8x$

$\therefore \log y = \log \cos x + \log \cos 4x + \log \cos 8x$

On differentiating w.r.t 'x'.

$\dfrac{1}{y}\cdot\dfrac{dy}{dx} = \dfrac{1}{\cos x}(-\sin x) + \dfrac{1}{\cos 4x}(-4\sin 4x) + \dfrac{1}{\cos 8x}(-8\sin 8x)$

$= -\tan x - 4\tan 4x - 8\tan 8x$

$\therefore \left(\dfrac{1}{y}\dfrac{dy}{dx}\right)_{\text{at } x = \frac{\pi}{4}} = -\tan\dfrac{\pi}{4} - 4\tan\pi - 8\tan 2\pi$

$= -1 - 0 - 0 = -1$

Hence, option (a) is correct.

73. Let $f(x)$ be a polynomial function such that $fof(x) = x^4$. What is $f'(1)$ equal to?

(a) 0 (b) 1 (c) 2 (d) 4

(c) Given, $f(x)$ be a polynomial such that

$fof(x) = x^4$

To find $\quad f'(1) = ?$

$\because \quad fof(x) = x^4 \Rightarrow f(x) = x^2$

$\therefore \quad f'(x) = 2x$

$\Rightarrow \quad f'(1) = 2 \times 1 = 2$

Hence, option (c) is correct.

74. What is $\lim\limits_{n \to \infty} \dfrac{a^n + b^n}{a^n - b^n}$ where $a > b > 1$, equal to?

(a) -1 (b) 0
(c) 1
(d) Limit does not exist

(c) Given, $\lim\limits_{n \to \infty} \dfrac{a^n + b^n}{a^n - b^n}$ where $a > b > 1$

$\Rightarrow \quad \lim\limits_{n \to \infty} \dfrac{a^n\left[1 + \left(\dfrac{b}{a}\right)^n\right]}{a^n\left[1 - \left(\dfrac{b}{a}\right)\right]^n} \quad \left[\because \dfrac{b}{a} < 1\right]$

$\therefore \quad = \dfrac{1+0}{1-0} = 1$

$\therefore \quad \left(\dfrac{b}{a}\right)^\infty = 0$

Hence, option (c) is correct.

75. Let $f(x) = \begin{cases} 1 + \dfrac{x}{2k}, & 0 < x < 2 \\ kx, & 2 \le x < 4 \end{cases}$

If $\lim\limits_{x \to 2} f(x)$ exists, then what is the value of k?

(a) -2 (b) -1
(c) 0 (d) 1

(d) Let $f(x) \begin{cases} 1 + \dfrac{x}{2k}; & 0 < x < 2 \\ kx; & 2 \le x < 4 \end{cases}$

$\because \lim\limits_{x \to 2} f(x)$ exists

$\Rightarrow \quad \lim\limits_{x \to 2^-} f(x) = \lim\limits_{x \to 2^+} (kx)$

$\lim\limits_{n \to 2^-}\left(1 + \dfrac{x}{2k}\right) = \lim\limits_{x \to 2^+} (kx)$

$1 + \dfrac{2}{2k} = 2k$

$\dfrac{2}{2k} = 2k - 1$

$2 = 4k^2 - 2k$

$4k^2 - 2k - 2 = 0$

$2k^2 - k - 1 = 0$

$(2k+1)(k-1) = 0$

$\Rightarrow \quad k = 1$ or $k = -\dfrac{1}{2}$

Hence, option (d) is correct.

76. Consider the following statements in respect of $f(x) = |x| - 1$:

1. $f(x)$ is continuous at $x = 1$.
2. $f(x)$ is differentiable at $x = 0$.

Which of the above statements is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

(a) Given, $f(x) = |x| - 1$

Since, modulus function is continuous.

$\Rightarrow f(x)$ is continuous at $x = 1$

and $|x|$ is not differentiable if $x = 0$

$\therefore f(x) = |x| - 1$ is not differentiable at $x = 0$

Hence, statement (1) is correct and (2) is false.

Hence, option (a) is correct.

77. If $f(x)=\dfrac{[x]}{[x]}, x\neq 0,$

where [ ] denotes the greatest integer function, then what is the right-hand limit of $f(x)$ at $x=1$?

(a) – 1
(b) 0
(c) 1
(d) Right-hand limit of $f(x)$ at $x=1$ does not exist

(*c*) Given that, $f(x)=\dfrac{[x]}{|x|}, x\neq 0$

$$=\lim_{x\to 1^+}\frac{[x]}{|x|}$$

$x=1+h$, where $h\to 0$

$$\therefore \quad \lim_{h\to 0}=\frac{[1+h]}{[1+h]}=\frac{1}{|1+0|}=1$$

Hence, option (c) is correct.

78. Consider the following statements in respect of the function.

$f(x)=\sin\left(\dfrac{1}{x^2}\right), x\neq 0.$

1. It is continuous at $x=0$, if $f(0)=0$.
2. It is continuous at $x=\dfrac{2}{\sqrt{x}}$.

Which of the above statements is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

(*b*) Given that, $f(x)=\sin\left(\dfrac{1}{x^2}\right), x\neq 0$

At $x=0$,

LHL $\lim\limits_{x\to 0^-}\sin\left(\dfrac{1}{x^2}\right)$

= value in between – 1 and + 1

RHL $\lim\limits_{x\to 0^+}\sin\left(\dfrac{1}{x^2}\right)$

= value in between – 1 and + 1

$\therefore$ Limit doesn't exist $\Rightarrow f(x)$ is not continuous at $x=0$.

At $x=\dfrac{2}{\sqrt{\pi}}$,

$$\lim_{x\to\frac{2}{\sqrt{\pi}}}\sin\left(\frac{1}{x^2}\right)=\sin\frac{\pi}{4}=\frac{1}{\sqrt{2}}$$

Also $\quad f\left(\dfrac{2}{\sqrt{\pi}}\right)=\sin\left(\dfrac{1}{\left(\frac{2}{\sqrt{\pi}}\right)^2}\right)=\dfrac{1}{\sqrt{2}}$

$\therefore f(x)$ is continuous at $x=\dfrac{2}{\sqrt{\pi}}$

Hence, option (b) is correct.

79. What is the range of the function $f(x)=1-\sin x$ defined on entire real line?

(a) (0, 2) (b) [0, 2]
(c) (– 1, 1) (d) [– 1, 1]

(*b*) Given that, $f(x)=1-\sin x$

Since, the range of $\sin x$ is $[-1, 1]$.

$$-1\le \sin x\le 1$$
$$-1\le -\sin x\le 1$$
$$1-1\le 1-\sin x\le 1+1$$
$$0\le 1-\sin x\le 2$$

$\therefore$ Range = [0, 2]

Hence, option (b) is correct.

80. What is the slope of the tangent of $y=\cos^{-1}(\cos x)$ at $x=-\dfrac{\pi}{4}$?

(a) – 1 (b) 0
(c) 1 (d) 2

(*a*) Given that, $y=\cos^{-1}(\cos x)$

Since range of $\cos^{-1}x$ is $[0, \pi]$.

$\therefore \quad y=\cos^{-1}(\cos x)=-x,$

if $\quad x\in(-\pi, 0)$

$\because \quad x=-\dfrac{\pi}{4}$

$\therefore \quad y=-x$

$\Rightarrow \quad \dfrac{dy}{dx}=-1$

$\therefore$ Slope of tangent = – 1

Hence, option (a) is correct.

81. What is the integral of $f(x)=1+x^2+x^4$ with respect to x^2?

(a) $x+\dfrac{x^3}{3}+\dfrac{x^5}{5}+C$

(b) $\dfrac{x^3}{3}+\dfrac{x^5}{5}+C$

(c) $x^2+\dfrac{x^4}{4}+\dfrac{x^6}{6}+C$

(d) $x^2+\dfrac{x^4}{2}+\dfrac{x^6}{3}+C$

(*d*) Given function, $f(x)=1+x^2+x^4$

$\therefore$ Integral of $f(x)$ w.r.t x^2.

$$=\int(1+x^2+x^4)\cdot 2x\,dx$$
$$=\int(2x+2x^3+2x^5)dx$$
$$=x^2+\frac{x^4}{2}+\frac{x^6}{3}+C$$

Hence, option (d) is correct.

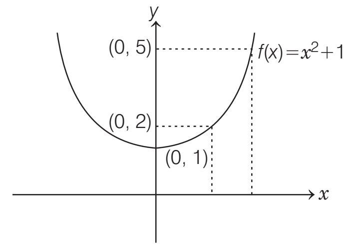
82. Consider the following statements in respect of the function $f(x)=x^2+1$ in the interval (1, 2).

1. The maximum value of the function is 5.
2. The minimum value of the function is 2.

Which of the above statements is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

(*c*) Given function,



$f(x)=x^2+1 \quad$ in (1, 2)

$\Rightarrow \quad y=x^2+1$

$x^2=(y-1)$

Which is the equation of parabola with vertex (0, 1).

At $\quad x=1$

$f(1)=1^2+1=2$

$f(2)=2^2+1=5$

Hence, maximum value of the function in (1, 2) is 5 and minimum value is 2.

Hence, option (c) is correct.

83. If $f(x)$ satisfies $f(1)=f(4)$, then what is $\int_1^4 f'(x)\,dx$ equal to?

(a) – 1 (b) 0
(c) 1 (d) 2

(*b*) $\quad f(1)=f(4)$

$$\therefore \int_1^4 f'(x)dx=[f(x)]_1^4=f(4)-f(1)$$
$$=f(1)-f(1)=0$$

84. What is $\int_0^{\frac{\pi}{2}} e^{\ln(\cos x)}dx$ equal to?

(a) – 1 (b) 0
(c) 1 (d) 2

(*c*) Let $I=\displaystyle\int_0^{\frac{\pi}{2}} e^{\ln(\cos x)}dx=\int_0^{\frac{\pi}{2}}(\cos x)dx$

$$=[\sin x]_0^{\frac{\pi}{2}}=\sin\frac{\pi}{2}-\sin 0=1$$

Hence, option (c) is correct.

85. If $\int\sqrt{1-\sin 2x}\,dx=A$ $\sin x+B\cos x+C$, where $0<x<\dfrac{\pi}{4}$, then which one of the following is correct?

(a) $A+B=0$ (b) $A+B-2=0$
(c) $A+B+2=0$ (d) $A+B-1=0$

(*b*) Given that,

$\int\sqrt{1-\sin 2x}\,dx=A\sin x+B\cos x+C,$

where $0\le x\le\dfrac{\pi}{4}$.

Let

$I=\int\sqrt{\cos^2 x+\sin^2 x-2\sin x\cdot\cos x}\;dx$

$I=\int\sqrt{(\cos x-\sin x)^2}\;dx$

$\left\{\because \cos x>\sin x \text{ when } 0<x<\dfrac{\pi}{4}\right\}$

$$I = \int (\cos x - \sin x)\,dx$$
$$I = \sin x + \cos x + C$$
$$= A \sin x + B \cos x + C$$
$\therefore \quad A = 1, B = 1$

$\therefore A + B - 2 = 1 + 1 - 2 = 0$

Hence, option (b) is correct.

86. What is the order of the differential equation of all ellipses whose axes are along the coordinate axes?

(a) 1 (b) 2
(c) 3 (d) 4

(***b***) Since, the equation of ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$\therefore$ There are 2 variable a and b.

$\therefore$ Order of the differential equation = 2

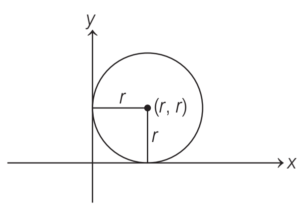
Hence option (d) is correct.

87. What is the degree of the differential equation of all circles touching both the coordinate axes in the first quadrant?

(a) 1 (b) 2
(c) 3 (d) 4

(***b***) If r be the radius of circle.

Since, the circle touching both the coordinate axes in the first quadrant.



$\therefore$ Centre = (r, r) and radius = r

$\therefore$ Equation of circle

$$(x - r)^2 + (y - r)^2 = r^2$$
$$x^2 + y^2 - 2xr - 2yr + r^2 = 0 \quad \ldots(i)$$
$$2x + 2yy' - 2r - 2ry' = 0$$
$$r(1 + y') = x + yy'$$
$$r = \frac{x + yy'}{1 + y'}$$

Putting the value of r in Eq. (i)

$$x^2 + y^2 - 2x\frac{(x + yy')}{1 + y'} - 2y\frac{(x + yy')}{1 + y'} + \left(\frac{x + yy'}{1 + y'}\right)^2 = 0$$
$$(1 + y')^2 x^2 + (1 + y')^2 y^2 - 2x(x + yy')(1 + y') - 2y(x + yy')(1 + y') + (x + yy')^2 = 0$$
$$(1 + y')^2 (x^2 + y^2) - 2(x - y)(x + yy')(1 + y') + (x + yy')^2 = 0$$

Hence, the degree of the differential equation is 2.

88. What is the differential equation of $y = A - \frac{B}{x}$?

(a) $xy_2 + y_1 = 0$ (b) $xy_2 + 2y_1 = 0$
(c) $xy_2 - 2y_1 = 0$ (d) $2xy_2 + y_1 = 0$

(***b***) Given, $y = A - \frac{B}{x}$

On differentiating w.r.t 'x'

$$\frac{dy}{dx} = 0 - B\left(-\frac{1}{x^2}\right) = \frac{B}{x^2}$$
$$x^2\frac{dy}{dx} = B$$

On differentiating again w.r.t 'x'.

$$x^2 \cdot \frac{d^2y}{dx^2} + \frac{dy}{dx}\cdot(2x) = 0$$
$$\Rightarrow \quad x(xy_2 + 2y_1) = 0$$
$$\Rightarrow \quad xy_2 + 2y_1 = 0$$

Hence, option (b) is correct.

89. What is $\int_0^{\pi} \ln\left(\tan\frac{x}{2}\right)dx$ equal to?

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) 2

(***a***) Let $I = \int_0^{\pi} \ln\left(\tan\frac{x}{2}\right)dx \quad \ldots(i)$

$$I = \int_0^{\pi} \ln\left(\cot\left(\frac{\pi - x}{2}\right)\right)dx$$
$$I = \int_0^{\pi} \ln\left(\cot\left(\frac{x}{2}\right)\right)dx \quad \ldots(ii)$$

Adding Eq. (i) and Eq. (ii)

$$2I = \int_0^{\pi} \left\{\ln\left(\tan\frac{x}{2}\right) + \ln\left(\cot\frac{x}{2}\right)\right\}dx$$
$$= \int_0^{\pi} \ln\left(\tan\frac{x}{2}\cdot\cot\frac{x}{2}\right)dx = \int_0^{\pi} \ln(1)\,dx$$
$$2I = 0$$
$$\therefore \quad I = 0$$

Hence, option (a) is correct.

90. Where does the tangent to the curve $y = e^x$ at the point $(0, 1)$ meet X-axis?

(a) (1, 0) (b) (– 1, 0)
(c) (2, 0) (d) $\left(-\frac{1}{2}, 0\right)$

(***b***) Given curve, $y = e^x$

$$\therefore \quad \frac{dy}{dx} = e^x$$
$$\left(\frac{dy}{dx}\right)_{at(0,\,1)} = e^0 = 1$$

$\therefore$ Equation of tangent at (0, 1).

$$y - 1 = \left(\frac{dy}{dx}\right)_{at(0,\,1)}(x - 0)$$
$$y - 1 = x$$

Since, (– 1, 0) satisfies above equation.

Hence, option (b) is correct.

91. Consider the following statements in respect of the function $f(x) = x + \frac{1}{x}\cdot$

1. The local maximum value of $f(x)$ is less than its local minimum value.
2. The local maximum value of $f(x)$ occurs at $x = 1$.

Which of the above statements is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

(***a***) Given, $f(x) = x + \frac{1}{x}$

$$f'(x) = 1 - \frac{1}{x^2} \text{ and } f''(x) = \frac{2}{x^3}$$

For critical points $f'(x) = 0$

$$1 - \frac{1}{x^2} = 0$$
$$x = \pm 1$$

At $x = 1, \quad f''(x) = 2 > 0$

$\Rightarrow f(x)$ is minimum at $x = 1$

$\Rightarrow f(1) = 2$

At $\quad x = -1\, f''(x) = -2 < 0$

$\Rightarrow f(x)$ is maximum at $x = -1$

$\Rightarrow f(-1) = -2$

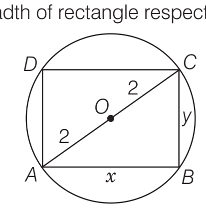
Hence, statement (1) is correct and (2) is false.

$\therefore$ Option (a) is correct.

92. What is the maximum area of a rectangle that can be inscribed in a circle of radius 2 units?

(a) 4 sq. units (b) 6 sq. units
(c) 8 sq. units (d) 16 sq. units

(***c***) Let x and y be the length and breadth of rectangle respectively.



In ΔABC,

$$x^2 + y^2 = 16$$
$$\Rightarrow \quad y = \sqrt{16 - x^2}$$

$\therefore$ Area of rectangle, $A = xy$

$$A = x\sqrt{16 - x^2}$$
$$\frac{dA}{dx} = \sqrt{16 - x^2} + \frac{x}{2\sqrt{16 - x^2}}(-2x)$$
$$= \frac{16 - x^2 - x^2}{\sqrt{16 - x^2}} = \frac{16 - 2x^2}{\sqrt{16 - x^2}}$$

For maximum A,

$$\frac{dA}{dx} = 0$$

$\Rightarrow \dfrac{16-2x^2}{\sqrt{16-x^2}} = 0 \quad \Rightarrow \quad 16-2x^2 = 0$

$\Rightarrow \quad x^2 = 8 \quad \Rightarrow \quad x = \pm 2\sqrt{2}$

Now, $\dfrac{d^2A}{dx^2} = \dfrac{-4x.\sqrt{16-x^2} - (16-2x^2)\dfrac{1}{2\sqrt{16-x^2}}(-2x)}{16-x^2}$

$= \dfrac{-4x(16-x^2)+x(16-2x^2)}{(16-x^2)^{3/2}}$

$= \dfrac{-3x(16-x^2)}{(16-x^2)^{3/2}}$

$\left(\dfrac{d^2A}{dx^2}\right)_{\text{at } x=2\sqrt{2}} = \dfrac{-3(2\sqrt{2})(16-8)}{(16-8)^{3/2}}$ (Negative)

$\therefore \quad y = \sqrt{16-(2\sqrt{2})^2} = 2\sqrt{2}$

Area $= 2\sqrt{2} \times 2\sqrt{2} = 8$ sq. units

Hence, A is maximum at $x = 2\sqrt{2}$.

93. What is $\displaystyle\int \frac{dx}{x(x^2+1)}$ equal to?

(a) $\dfrac{1}{2}\ln\left(\dfrac{x^2}{x^2+1}\right) + C$

(b) $\ln\left(\dfrac{x^2}{x^2+1}\right) + C$

(c) $\dfrac{3}{2}\ln\left(\dfrac{x^2}{x^2+1}\right) + C$

(d) $\dfrac{1}{2}\ln\left(\dfrac{x^2+1}{x^2}\right) + C$

(*a*) Let $\quad I = \displaystyle\int \frac{dx}{x(x^2+1)}$

$\because \dfrac{1}{x(x^2+1)} = \dfrac{A}{x} + \dfrac{Bx+C}{x^2+1}$

(by using partial fraction)

$\dfrac{1}{x(x^2+1)} = \dfrac{Ax^2 + A + Bx^2 + Cx}{x(x^2+1)}$

$1 = (A+B)x^2 + Cx + A$

$\Rightarrow \quad A + B = 0, C = 0, A = 1$

$\therefore \quad B = -A = -1$

$\therefore I = \displaystyle\int \frac{dx}{x(x^2+1)} = \int\left(\frac{1}{x} - \frac{x}{x^2+1}\right)dx$

$= \ln x - \dfrac{1}{2}\ln(x^2+1) + C$

$= \dfrac{1}{2}(2\ln x) - \dfrac{1}{2}\ln(x^2+1) + C$

$= \dfrac{1}{2}(\ln x^2 - \ln(x^2+1)) + C$

$= \dfrac{1}{2}\ln\left(\dfrac{x^2}{x^2+1}\right) + C$

Hence, option (a) is correct.

94. What is the derivative of e^{e^x} with respect to e^x?

(a) e^{e^x} (b) e^x

(c) $e^{e^x}e^x$ (d) ee^x

(*a*) Let $y_1 = e^{e^x}$ and $y_2 = e^x$

$\therefore \quad \dfrac{dy_1}{dx} = e^{e^x}\cdot e^x, \dfrac{dy_2}{dx} = e^x$

$\Rightarrow \quad \dfrac{dy_1}{dy_2} = \dfrac{e^{e^x}\cdot e^x}{e^x} = e^{e^x}$

$\therefore$ Option (a) is correct.

95. What is the condition that $f(x) = x^3 + x^2 + kx$ has no local extremum?

(a) $4k < 1$ (b) $3k > 1$

(c) $3k < 1$ (d) $3k \le 1$

(*b*) Given that, $f(x) = x^3 + x^2 + kx$

$\because f(x)$ has no local extremum.

$\Rightarrow \quad f'(x) \ne 0$

$\Rightarrow \quad 3x^2 + 2x + k \ne 0$

for no extremum, $D < 0$

$\Rightarrow \quad (2)^2 - 4(3)(k) < 0$

$\Rightarrow \quad 4 - 12k < 0$

$3k > 1$

$\therefore$ Option (b) is correct.

96. If $f(x) = 2^x$, then what is $\displaystyle\int_2^{10}\frac{f'(x)}{f(x)}dx$ equal to?

(a) 4ln2 (b) ln4

(c) ln5 (d) 8ln2

(*d*) Given, $f(x) = 2^x$

$\therefore \quad \displaystyle\int_2^{10}\frac{f'(x)}{f(x)}dx = [\ln f(x)]_2^{10} = [\ln 2^x]_2^{10}$

$= [x \ln 2]_2^{10}$

$= 10\ln 2 - 2\ln 2$

$= 8\ln 2$

$\therefore$ Option (d) is correct.

97. If $\displaystyle\int_{-2}^{0} f(x)\,dx = k$, then $\displaystyle\int_{-2}^{0}|f(x)|\,dx$ is

(a) less than k

(b) greater than k

(c) less than or equal to k

(d) greater than or equal to k

(*d*) Given, $\displaystyle\int_{-2}^{0} f(x)dx = k$

To find $\displaystyle\int_{-2}^{0}|f(x)|dx$

Let $f(x) = x$

$\therefore \quad \displaystyle\int_{-2}^{0} x\,dx = \left[\frac{x^2}{2}\right]_{-2}^{0} = -2 = k$

$\therefore \quad \displaystyle\int_{-2}^{0}|x|dx = -\int_{-2}^{0} x\,dx$

$= -(-2) = 2 \ge k$

$\therefore$ Option (d) is correct.

98. If the function $f(x) = x^2 - kx$ is monotonically increasing in the interval $(1, \infty)$, then which one of the following is correct?

(a) $k < 2$ (b) $2 < k < 3$

(c) $3 < k < 4$ (d) $k > 4$

(*a*) Let the function $f(x) = x^2 - kx$ is monotonically increasing in $(1, \infty)$.

$\Rightarrow \quad f'(x) \ge 0$

$\Rightarrow \quad 2x - k \ge 0$

$\Rightarrow k \le 2x$ in $(1, \infty)$ at lower value

at $\quad x = 1$

$k < 2$

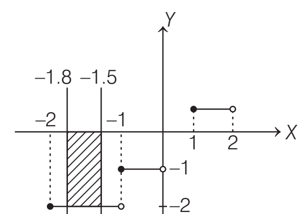
Hence, option (a) is correct.

99. What is the area bounded by $y = [x]$, where $[\cdot]$ is the greatest integer function, the X-axis and the lines $x = -1.5$ and $x = -1.8$?

(a) 0.3 sq. unit (b) 0.4 sq. unit

(c) 0.6 sq. unit (d) 0.8 sq. unit

(*c*) Given, $y = [x]$



$\therefore \quad$ Area $= \displaystyle\int_{-1.8}^{-1.5}[x]\,dx = \int_{-1.8}^{-1.5}(-2)\,dx$

$= -2(x)_{-1.8}^{-1.5}$

$= -2(-1.5 + 1.8) = -0.6$

$\therefore \quad$ Area $= 0.6$ sq. unit

Hence, option (c) is correct.

100. The tangent to the curve $x^2 = y$ at $(1, 1)$ makes an angle θ the with the positive direction of X-axis. Which one of the following is correct?

(a) $\theta < \dfrac{\pi}{6}$ (b) $\dfrac{\pi}{6} < \theta < \dfrac{\pi}{4}$

(c) $\dfrac{\pi}{4} < \theta < \dfrac{\pi}{3}$ (d) $\dfrac{\pi}{3} < \theta < \dfrac{\pi}{2}$

(*d*) Given, curve $y = x^2$

$\dfrac{dy}{dx} = 2x$

$\left.\dfrac{dy}{dx}\right|_{\text{at}(1,\,1)} = 2 \times 1 = 2$

$\Rightarrow \quad \tan\theta = 2$

$\because \tan\dfrac{\pi}{3} = \sqrt{3} = 1.732$ and $\tan\dfrac{\pi}{2} = \infty$

$\therefore \quad \dfrac{\pi}{3} < \theta < \dfrac{\pi}{2}$

Hence, option (d) is correct.

101. Consider the following relations for two events E and F.

1. $P(E \cap F) \geq P(E) + P(F) - 1$
2. $P(E \cup F) = P(E) + P(F) + P(E \cap F)$
3. $P(E \cup F) \leq P(E) + P(F)$

Which of the above relations is/are correct?

(a) 1 only (b) 3 only
(c) 1 and 3 only (d) 1, 2 and 3

(c) Let E and F be two events.

Then, $P(E \cup F) = P(E) + P(F) - P(E \cap F)$ ...(i)

or $P(E \cup F) \leq P(E) + P(F)$

$P(E \cup F) \leq 1$

$-P(E \cap F) \geq -1$

$\Rightarrow P(E) + P(F) - P(E \cup F) \geq P(E) + P(F) - 1$

$\Rightarrow P(E \cap F) \geq P(E) + P(F) - 1$

Hence, option (c) is correct.

102. If $P(A / B) < P(A)$, then which one of the following is correct?

(a) $P(B|A) < P(B)$ (b) $P(B|A) > P(B)$
(c) $P(B|A) = P(B)$ (d) $P(B|A) > P(A)$

(a) If $P\left(\frac{A}{B}\right) < P(A)$

$\because \quad P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} < P(A)$

$\Rightarrow \frac{P(A \cap B)}{P(A)} < P(B)$

$\Rightarrow \quad P\left(\frac{B}{A}\right) < P(B)$

Hence, option (a) is correct.

103. When the measure of central tendency is available in the form of mean, which one of the following is the most reliable and accurate measure of variability?

(a) Range
(b) Mean deviation
(c) Standard deviation
(d) Quartile deviation

(c) When the measure of central tendency is available in the form of mean then, we know that Standard Deviation is the most reliable and accurate measure of variability.

Hence, option (c) is correct.

104. A problem is given to three students A, B and C, whose probabilities of solving the problem independently are $\frac{1}{2}, \frac{3}{4}$ and p, respectively. If the probability that the problem can be solved is $\frac{29}{32}$, then what is the value of p ?

(a) $\frac{2}{5}$ (b) $\frac{2}{3}$
(c) $\frac{1}{3}$ (d) $\frac{1}{4}$

(d) Given, $P(A) = \frac{1}{2}, P(B) = \frac{3}{4}, P(C) = p$

$\because$ Probability that the problem can not be solved $= P(\overline{A}) \cdot P(\overline{B}) \cdot P(\overline{C})$

$$= \left(1 - \frac{1}{2}\right)\left(1 - \frac{3}{4}\right)(1 - p)$$

$$= \frac{1}{2} \times \frac{1}{4}(1 - p)$$

$$= \frac{1 - p}{8}$$

$\therefore$ Probability that the problem can be solved

= 1 – Probability that the problem cannot be solved

$$\frac{29}{32} = 1 - \frac{(1 - p)}{8}$$

$$\frac{1 - p}{8} = 1 - \frac{29}{32}$$

$$\frac{1 - p}{8} = \frac{3}{32}$$

$$1 - p = \frac{3}{4}$$

$$\therefore \quad p = \frac{1}{4}$$

Hence, option (d) is correct.

105. In a cricket match a batsman hits a six 8 times out of 60 balls he plays. What is the probability that on a ball played he does not hit a six?

(a) $\frac{2}{3}$ (b) $\frac{1}{15}$ (c) $\frac{2}{15}$ (d) $\frac{13}{15}$

(d) Since, the batsman hits a six 8 times out of 60 balls.

The batsman could not hit sixes in (60-8) balls.

$\therefore$ Probability that on a ball played he does not hit six $= \frac{52}{60}$

$$p = \frac{13}{15}$$

Hence, option (d) is correct.

Directions (Q. Nos. 106 and 107)

Consider the following for the questions that follow.

Two regression lines are given as $3x - 4y + 8 = 0$ and $4x - 3y - 1 = 0$

106. Consider the following statements.

1. The regression line of y on x is $y = \frac{3}{4}x + 2$
2. The regression line of x on y is $x = \frac{3}{4}y + \frac{1}{4}$

Which of the above statements is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

(c) Two regression lines are

$3x - 4y + 8 = 0$, $4x - 3y - 1 = 0$

for finding the regression line of y on x

$$3x - 4y + 8 = 0$$

$$4y = 3x + 8$$

$$y = \left(\frac{3}{4}\right)x + 2 \quad \text{...(i)}$$

and the regression line of x on y:

$$4x - 3y - 1 = 0$$

$$4x = 3y + 1$$

$$x = \frac{3}{4}y + \frac{1}{4} \quad \text{...(ii)}$$

Hence, option (c) is correct.

107. Consider the following statements.

1. The coefficient of correlations r is $\frac{3}{4}$
2. The means of x and y are 3 and 4 respectively.

Which of the above statements is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

(a) Since, regression line of y on x

$$\Rightarrow \quad y = \frac{3}{4}x + 2$$

$$\therefore \quad b_{xy} = \frac{3}{4}$$

and regression line of x on y

$$\Rightarrow \quad x = \frac{3}{4}y + \frac{1}{4}$$

$$\therefore \quad b_{yx} = \frac{3}{4}$$

$\therefore$ Coefficient of correlations

$$r = \sqrt{b_{xy} \times b_{yx}} = \sqrt{\frac{3}{4} \times \frac{3}{4}}$$

$$r = \frac{3}{4}$$

Means of x and y are nothing but the solution of regression lines

$$3x - 4y + 8 = 0$$

and $$4x - 3y - 1 = 0$$

$$3x - 4y = -8 \quad \text{...(i)}$$

$$4x - 3y = 1 \quad \text{...(ii)}$$

Eq. (i) × 4 – Eq. (ii) × 3

$$12x - 16y = -32$$

$$12x - 9y = 3$$

$$7y = 35 \Rightarrow y = 5$$

$$\therefore \quad 4x = 1 + 3 \times 5$$

$$x = 4$$

$\therefore$ Statement (2) is wrong.

Hence, option (a) is correct.

Directions (Q. Nos. 108 and 109) *Consider the following for the questions that follow.*
The marks obtained by 60 students in a certain subject out of 75 are given below.

Marks	Number of students
15-20	4
20-25	5
25-30	11
30-35	6
35-40	5
40-45	8
45-50	9
50-55	6
55-60	4
60-65	2

108. What is the median?
(a) 35 (b) 38
(c) 39 (d) 40

(*c*)

Marks	Frequency	Cumulative frequency
15-20	4	4
20-25	5	9
25-30	11	20
30-35	6	26 = Cf
35-40	5	31
40-45	8	39
45-50	9	48
50-55	6	54
55-60	4	58
60-65	2	60
	$N = 60$	

$\therefore \frac{N}{2} = 30 \Rightarrow$ model class will be 35 – 40.

$\therefore$ lower limit $(l) = 35$

$h = 40 - 35 = 5$

$$\therefore \quad \text{Median} = l + \frac{\frac{N}{2} - C.f}{f} \times h$$

$$= 35 + \frac{30 - 26}{5} \times 5 = 39$$

Hence, option (c) is correct.

109. What is the mode?
(a) 27.27 (b) 27.73
(c) 27.93 (d) 28.27

Marks	Frequency
15-20	4
20-25	$5 \to f_0$
25-30	$11 \to f_1$
30-35	$6 \to f_2$
35-40	5
40-45	8
45-50	9
50-55	6
55-60	4
60-65	2

(*b*) Highest frequency is given for class 25-30.

$\therefore$ Model class will be 25-30.

$\therefore \quad l = 25, h = 5$

$\therefore \quad f_1 = 11, f_0 = 5, f_2 = 6$

$$\therefore \text{ Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$= 25 + \frac{11 - 5}{22 - 5 - 6} \times 5$$

$$= 25 + \frac{6}{11} \times 5 = \frac{275 + 30}{11}$$

$$= 27.73$$

Hence, option (b) is correct.

110. What is the mean of natural numbers contained in the interval [15, 64]?
(a) 36.8 (b) 38.3
(c) 39.5 (d) 40.3

(*c*) Mean of natural numbers contained in [15, 64].

$$= \frac{15 + 16 + 17 + \dots + 64}{50}$$

$$= \frac{\sum_{n=1}^{64} n - \sum_{r=1}^{14} r}{50}$$

$$= \frac{\frac{64 \times 65}{2} - \frac{14 \times 15}{2}}{50}$$

$$= \frac{2080 - 105}{50} = 39.5$$

Hence, option (c) is correct.

111. For the set of numbers $x, x, x + 2, x + 3, x + 10$ where x is a natural number, which of the following is/are correct?
1. Mean > Mode
2. Median> Mean
Select the correct answer using the code given below.
(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

(*a*) Given data $x, x, x + 2, x + 3, x + 10$ where $x \in N$.

$$\therefore \text{Mean} = \frac{x + x + x + 2 + x + 3 + x + 10}{5}$$

$$= \frac{5x + 15}{5} = x + 3$$

$\therefore$ Mode $= x$

$$\text{Median} = \left(\frac{5+1}{2}\right)^{\text{th}} \text{ term} = 3^{\text{rd}} \text{ term}$$

$$= x + 2$$

$\therefore$ Mean > Mode and Median < Mean

Hence, correct option is (a).

112. The mean of 10 observations is 5.5. If each observation is multiplied by 4 and subtracted from 44, then what is the new mean?
(a) 20 (b) 22
(c) 34 (d) 44

(*b*) Given that, the mean of 10 observation is 5.5.

$$\because \quad \text{Mean} = \frac{x_1 + x_2 + \dots + x_{10}}{10}$$

$$5.5 = \frac{x_1 + x_2 + \dots + x_{10}}{10}$$

$$\therefore \quad \sum_{i=1}^{10} x_i = 55$$

Also, given that new observations are obtained by multiplying by 4 and subtracting from 44.

Hence, new mean $= 44 - 4 \times 5.5$

$= 44 - 22 = 22$

Hence, correct option is (b).

113. If g is the geometric mean of 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, then which one of the following is correct?
(a) $8 < g < 16$ (b) $16 < g < 32$
(c) $32 < g < 64$ (d) $g > 64$

(*c*) Given that, geometric mean of 2, 4, 8, 16, 32, 64, 126, 256, 512, 1024 is g.

$$\therefore \quad g = \sqrt[10]{2 \times 4 \times 8 \times 16 \times 32 \times 64 \times 128 \times 256 \times 512 \times 1024}$$

$$= (2^{1+2+3+\dots+10})^{\frac{1}{10}}$$

$$g = (2^{55})^{\frac{1}{10}} \Rightarrow g = (2)^{\frac{11}{2}}$$

$$\therefore \quad 2^5 < g < 2^6 \Rightarrow 32 < g < 64$$

Hence, option (c) is correct.

114. If the harmonic mean of 60 and x is 48, then what is the value of x?
(a) 32 (b) 36
(c) 40 (d) 44

(*c*) Given, harmonic mean of 60 and x is 48.

$$\because \quad H = \frac{2ab}{a + b}$$

$$48 = \frac{2 \times 60 \times x}{60 + x}$$

$$2880 + 48x = 120x$$

$$72x = 2880$$

$$x = 40$$

Hence, option (c) is correct.

115. What is the mean deviation of first 10 even natural numbers?

(a) 5 (b) 5.5
(c) 10 (d) 10.5

(**a**) Mean deviation of first 10 even natural numbers

Since, mean $(\bar{x}) = \dfrac{2+4+6+\ldots 20}{10}$

$= \dfrac{2(10 \times 11)}{20} = 11$

$\therefore$ Mean deviation

$= \dfrac{|2-11| + |4-11| + |6-11| + \ldots + |20-11|}{10}$

$= \dfrac{9+7+5+3+1+1+3+5+7+9}{10}$

$= 5$

Hence, option (a) is correct.

116. If $\sum_{i=1}^{10} x_i = 110$ and $\sum_{i=1}^{10} x_i^2 = 1540$, then what is the variance?

(a) 22 (b) 33
(c) 44 (d) 55

(**b**) Given, $\sum_{i=1}^{10} x_i = 110$

and $\sum_{i=1}^{10} x_i^2 = 1540$

$\because$ Variance $= \dfrac{\Sigma x_i^2}{n} - \left(\dfrac{\Sigma x_i}{n}\right)^2$

$= \dfrac{1540}{10} - \left(\dfrac{110}{10}\right)^2$

$= 154 - 121$

$= 33$

Hence, option (b) is correct.

117. 3-digit numbers are formed using the digits 1, 3, 7 without repetition of digits. A number is randomly selected. What is the probability that the number is divisible by 3?

(a) 0 (b) $\frac{1}{3}$
(c) $\frac{1}{4}$ (d) $\frac{1}{8}$

(**a**) Let 3-digit numbers using the digits without repetition 1, 3, 7 are 3!.

Since, the sum of the digits

$= 1 + 3 + 7 = 11$

which is not divisible by 3.

$\therefore P$ (number of divisible by 3) $= \dfrac{0}{3!}$

$= 0$

Hence, option (a) is correct.

118. What is the probability that the roots of the equation $x^2 + x + n = 0$ are real, where $n \in N$ and $n < 4$?

(a) 0 (b) $\frac{1}{4}$
(c) $\frac{1}{3}$ (d) $\frac{1}{2}$

(**a**) Given, equation $x^2 + x + n = 0$,

where $n \in N, n < 4$

$\therefore \quad n \in \{1, 2, 3\}$

Since, above equation is quadratic.

So, for each value of n, we have two roots.

$\therefore$ Total number of roots = 6

When $n = 1$

$x^2 + x + 1 = 0$

$x = \dfrac{-1 \pm \sqrt{1-4}}{2} = \dfrac{-1 \pm \sqrt{3}i}{2}$

when $n = 2$,

$x^2 + x + 2 = 0$

$\Rightarrow \quad x = \dfrac{-1 \pm \sqrt{1-8}}{2} = \dfrac{-1 \pm \sqrt{7}i}{2}$

and $n = 3$, $x^2 + x + 3 = 0$

$\Rightarrow \quad x = \dfrac{- \pm \sqrt{11}i}{2}$

There are no real roots.

$\therefore P$ (roots are real) $= \dfrac{0}{6} = 0$

Hence, option (a) is correct.

119. If A and B are two events such that $P(\text{not } A) = \dfrac{7}{10}$, $P(\text{not } B) = \dfrac{3}{10}$ and $P(A \mid B) = \dfrac{3}{14}$, then what is $P(B \mid A)$ equal to?

(a) $\frac{11}{14}$ (b) $\frac{9}{14}$
(c) $\frac{1}{4}$ (d) $\frac{1}{2}$

(**d**) Given, $P(\text{not } A) = \dfrac{7}{10}, P(\text{not } B) = \dfrac{3}{10}$

$P\left(\dfrac{A}{B}\right) = \dfrac{3}{14}$

$\therefore \quad P\left(\dfrac{B}{A}\right) = \dfrac{P(A \cap B)}{P(A)} \quad \ldots(\text{i})$

$\because \quad P(\text{not } A) = \dfrac{7}{10}$

$\therefore \quad P(A) = 1 - \dfrac{7}{10} = \dfrac{3}{10}$

$P(\text{not } B) = \dfrac{3}{10}$

$\therefore \quad P(B) = 1 - \dfrac{3}{10} = \dfrac{7}{10}$

$P\left(\dfrac{A}{B}\right) = \dfrac{P(A \cap B)}{P(B)}$

$\dfrac{3}{14} = \dfrac{P(A \cap B)}{\frac{7}{10}}$

$\therefore \quad P(A \cap B) = \dfrac{3}{20}$

Eq. (i) $\Rightarrow P\left(\dfrac{B}{A}\right) = \dfrac{\frac{3}{20}}{\frac{3}{10}} = \dfrac{1}{2}$

Hence, option (d) is correct.

120. Seven white balls and three black balls are randomly placed in a row. What is the probability that no two black balls are placed adjacently?

(a) $\frac{7}{15}$ (b) $\frac{8}{15}$
(c) $\frac{11}{15}$ (d) $\frac{13}{15}$

(**a**) There are 10 balls among which 7 are white and 3 are black.

$\therefore$ Number of ways to arrange 10 balls

$= 10!$

If we put the balls in such a way that no two black balls are placed adjacently.

$\therefore$ Number of arrangements $= 7! \times {}^8P_3$

$\therefore \quad P = \dfrac{7! \times 8!}{5! \times 10!}$

$= \dfrac{6 \times 7}{9 \times 10} = \dfrac{7}{15}$

Hence, option (a) is correct.

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SOLVED PAPER 2021 (I)

PAPER I : Mathematics

1. The smallest positive integer n for which

$$\left(\frac{1-i}{1+i}\right)^{n^2} = 1$$

where $i = \sqrt{-1}$, is

(a) 2 (b) 4 (c) 6 (d) 8

(a) $\left(\frac{1-i}{1+i}\right)^{n^2} = 1$, where $i = \sqrt{-1}$

$$\left(\frac{1-i}{1+i} \times \frac{1-i}{1-i}\right)^{n^2} = 1$$

$$\left(\frac{1+i^2-2i}{1-i^2}\right)^{n^2} = 1$$

$$\left(\frac{1-1-2i}{1+1}\right)^{n^2} = 1$$

$\Rightarrow \quad (-i)^{n^2} = (-i)^4$

$\Rightarrow \quad n^2 = 4$

$n = 2$

Hence, option (a) is correct.

2. The value of x, satisfying the equation $\log_{\cos x} \sin x = 1$, where $0 < x < \frac{\pi}{2}$, is

(a) $\frac{\pi}{12}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

(c) $\log_{\cos x} \sin x = 1$, where $0 < x < \frac{\pi}{2}$

$\Rightarrow (\cos x)^1 = \sin x \Rightarrow \cos x = \sin x$

$\Rightarrow \tan x = 1 \Rightarrow \tan x = \tan \pi/4$

$\Rightarrow x = \pi/4$

Hence, option (c) is correct.

3. If Δ is the value of the determinant

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

then what is the value of the following determinant?

$$\begin{vmatrix} pa_1 & b_1 & qc_1 \\ pa_2 & b_2 & qc_2 \\ pa_3 & b_3 & qc_3 \end{vmatrix}$$

$(p \neq 0 \text{ or } 1, q \neq 0 \text{ or } 1)$

(a) $p\Delta$ (b) $q\Delta$

(c) $(p+q)\Delta$ (d) $pq\Delta$

(d) Given, $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \Delta$

$$\therefore \quad \begin{vmatrix} pa_1 & b_1 & qc_1 \\ pa_2 & b_2 & qc_2 \\ pa_3 & b_3 & qc_3 \end{vmatrix}$$

$$= p \cdot q \cdot \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$= pq\Delta$

Hence, option (d) is correct.

4. If $C_0, C_1, C_2, \ldots, C_n$ are the coefficients in the expansion of $(1+x)^n$, then what is the value of $C_1 + C_2 + C_3 + \ldots + C_n$?

(a) 2^n (b) $2^n - 1$

(c) 2^{n-1} (d) $2^n - 2$

(b) $\because (1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \ldots + {}^nC_n x^n$

and we know that

${}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n = 2^n$

$\therefore {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n = 2^n - {}^nC_0$

$= 2^n - 1$

Hence, option (b) is correct.

5. If $a+b+c=4$ and $ab+bc+ca=0$, then what is the value of the following determinant?

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

(a) 32 (b) -64 (c) -128 (d) 64

(b) Let

$$\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

$$= \begin{vmatrix} a+b+c & b & c \\ a+b+c & c & a \\ a+b+c & a & b \end{vmatrix}$$

(by $C_1 \to C_1 + C_2 + C_3$)

$$= (a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix}$$

(To take common $a+b+c$ from C_1)

$$= (a+b+c)\begin{vmatrix} 0 & b-c & c-a \\ 0 & c-a & a-b \\ 1 & a & b \end{vmatrix}$$

(by $R_1 \to R_1 - R_2, R_2 \to R_2 - R_3$)

$= (a+b+c)\,[(b-c)(a-b) - (c-a)^2]$

$= (a+b+c)\,(ab - b^2 - ca + bc - c^2 - a^2 + 2ca)$

$= -(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$= -(a+b+c)[(a+b+c)^2-3(ab+bc+ca)]$

$= -(4)[16-0] = -64.$

6. The number of integer values of k, for which the equation $2\sin x = 2k+1$ has a solution, is

(a) zero (b) one
(c) two (d) four

(c) Given,

$$2\sin x = 2k+1$$

$\because$ $-1 \le \sin x \le 1 \Rightarrow -2 \le 2\sin x \le 2$

$-2-1 \le 2\sin x - 1 \le 2-1$

$-3 \le 2k \le 1$

$\frac{-3}{2} \le k \le \frac{1}{2} \Rightarrow -1.5 \le k \le 0.5$

$\therefore$ Integer values of $k = -1, 0$

Hence, option (c) is correct.

7. If $a_1, a_2, a_3, \ldots, a_9$ are in GP, then what is the value of the following determinant?

$$\begin{vmatrix} \ln a_1 & \ln a_2 & \ln a_3 \\ \ln a_4 & \ln a_5 & \ln a_6 \\ \ln a_7 & \ln a_8 & \ln a_9 \end{vmatrix}$$

(a) 0 (b) 1
(c) 2 (d) 4

(a) Let first term and common ratio of GP are a and r respectively.

$$\therefore \begin{vmatrix} \log a_1 & \log a_2 & \log a_3 \\ \log a_4 & \log a_5 & \log a_6 \\ \log a_7 & \log a_8 & \log a_9 \end{vmatrix} = \begin{vmatrix} \log a & \log ar & \log ar^2 \\ \log ar^3 & \log ar^4 & \log ar^5 \\ \log ar^6 & \log ar^7 & \log ar^8 \end{vmatrix}$$

$$= \begin{vmatrix} \log a & \log a + \log r & \log a + 2\log r \\ \log a + 3\log r & \log a + 4\log r & \log a + 5\log r \\ \log a + 6\log r & \log a + 7\log r & \log a + 8\log r \end{vmatrix}$$

$[\because \log mn = \log m + \log n]$

$$= \begin{vmatrix} \log a & \log r & \log r \\ \log a + 3\log r & \log r & \log r \\ \log a + 6\log r & \log r & \log r \end{vmatrix}$$

(by $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_2$)

$= 0$ $[\because C_2 = C_3]$

8. If the roots of the quadratic equation $x^2 + 2x + k = 0$ are real, then

(a) $k < 0$ (b) $k \le 0$
(c) $k < 1$ (d) $k \le 1$

(d) Given quadratic equation,

$$x^2 + 2x + k = 0 \quad \ldots \text{(i)}$$

Since, roots are real

$\Rightarrow$ $D \ge 0 \Rightarrow b^2 - 4ac \ge 0$

$(2)^2 - 4(1)(k) \ge 0 \Rightarrow 4 \ge 4k \Rightarrow k \le 1$

Hence, option (d) is correct.

9. If $n = 100!$, then what is the value of the following?

$$\frac{1}{\log_2 n} + \frac{1}{\log_3 n} + \frac{1}{\log_4 n} + \ldots + \frac{1}{\log_{100} n}$$

(a) 0 (b) 1 (c) 2 (d) 3

(b) $\frac{1}{\log_2 n} + \frac{1}{\log_3 n} + \ldots + \frac{1}{\log_{100} n}$

$= \log_n 2 + \log_n 3 + \log_n 4 + \ldots + \log_n 100$

$= \log_n (2 \cdot 3 \cdot 4 \cdot 5 \ldots 100)$

$= \log_{100!}(100!)$ $[\because n = 100!]$

$= 1$ $[\because \log_a^a = 1]$

Hence, option (b) is correct.

10. If $z = 1 + i$, where $i = \sqrt{-1}$, then what is the modulus of $z + \frac{2}{z}$?

(a) 1 (b) 2 (c) 3 (d) 4

(b) $z = 1 + i$, where $i = \sqrt{-1}$

$$\left| z + \frac{2}{z} \right| = \left| (1+i) + \frac{2}{(1+i)} \right| = \left| (1+i) + \frac{2}{(1+i)} \times \frac{(1-i)}{(1-i)} \right|$$

$$= \left| (1+i) + \frac{2(1-i)}{2} \right| = |1 + i + 1 - i| = |2| = 2$$

Hence, option (b) is correct.

11. If A and B are two matrices such that AB is of order $n \times n$, then which one of the following is correct?

(a) A and B should be square matrices of same order.
(b) Either A or B should be a square matrix.
(c) Both A and B should be of same order.
(d) Orders of A and B need not be the same.

(d) Given that, order of matrix $AB = n \times n$

If we take $A_{n \times p}$ and $B_{p \times n}$, then AB will be of order $n \times n$.

So, orders of A and B need not be the same, is correct.

Hence, option (d) is correct.

12. How many matrices of different orders are possible with elements comprising all prime numbers less than 30?

(a) 2 (b) 3 (c) 4 (d) 6

(c) $\because$ Prime numbers less than 30 = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29}

$\Rightarrow$ Number of elements = 10

$\therefore$ Possible order of matrices with 10 elements = $10 \times 1, 1 \times 10, 2 \times 5, 5 \times 2$

$\therefore$ Nubmer of matrices of different order = 4

Hence, option (c) is correct.

13. Let, $A = \begin{vmatrix} p & q \\ r & s \end{vmatrix}$

where p, q, r and s are any four different prime numbers less than 20. What is the maximum value of the determinant?

(a) 215 (b) 311
(c) 317 (d) 323

(c) $A = \begin{vmatrix} p & q \\ r & s \end{vmatrix}$, prime numbers less than 20

$= \{2, 3, 5, 7, 11, 13, 17, 19\} \Rightarrow A = ps - rq$

For maximum values of A, p and s must be maximum and r and q must be minimum.

Then, $p = 17, s = 19, r = 2, q = 3$

$\therefore$ $A = 17 \times 19 - 2 \times 3$

$= 323 - 6 = 317$

Hence, option (c) is correct.

14. If A and B are square matrices of order 2 such that $\det(AB) = \det(BA)$, then which one of the following is correct?
(a) A must be a unit matrix
(b) B must be a unit matrix
(c) Both A and B must be unit matrices
(d) A and B need not be unit matrices

(d) $A_{2\times 2}$ and $B_{2\times 2}$ are two matrices

and $|AB| = |BA| \Rightarrow |A||B| = |B||A|$

Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} -2 & 1 \\ 3/2 & -1/2 \end{bmatrix}$ then, $|AB| = |BA|$

Hence, we can say A and B need not be the unit matrices.
Hence, option (d) is correct.

15. What is $\cot 2x \cot 4x - \cot 4x \cot 6x - \cot 6x \cot 2x$ equal to?
(a) -1 (b) 0 (c) 1 (d) 2

(c) $\because \cot 6x = \cot(2x + 4x)$

$$\cot 6x = \frac{\cot 2x \cdot \cot 4x - 1}{\cot 2x + \cot 4x} \quad \left[\because \cot(A+B) = \frac{\cot A \cot B - 1}{\cot A + \cot B}\right]$$

$\Rightarrow \cot 6x \cdot \cot 2x + \cot 6x \cdot \cot 4x = \cot 2x \cdot \cot 4x - 1$
$\therefore \cot 2x \cdot \cot 4x - \cot 4x \cdot \cot 6x - \cot 6x \cdot \cot 2x = 1$
Hence, option (c) is correct.

16. If $\tan x = -\frac{3}{4}$ and x is in the second quadrant, then what is the value of $\sin x \cdot \cos x$?
(a) $\frac{6}{25}$ (b) $\frac{12}{25}$ (c) $-\frac{6}{25}$ (d) $-\frac{12}{25}$

(d) Given,
$\tan x = \frac{-3}{4}$ and x is in the 2nd quadrant.

Let perpendicular be $3k$ and base be $4k$, then

Hypotenuse $= \sqrt{(3k)^2 + (4k)^2} = 5k$

$\sin x = \frac{3}{5}$ and $\cos x = \frac{-4}{5}$

$\therefore \sin x \cdot \cos x = \frac{3}{5} \times \left(\frac{-4}{5}\right) = \frac{-12}{25}$

Hence, option (d) is correct.

17. What is the value of the following?
$$\text{cosec}\left(\frac{7\pi}{6}\right)\sec\left(\frac{5\pi}{3}\right)$$
(a) $\frac{4}{3}$ (b) 4 (c) -4 (d) $-\frac{4}{\sqrt{3}}$

(c) $\text{cosec}\left(\frac{7\pi}{6}\right)\cdot\sec\left(\frac{5\pi}{3}\right) = \text{cosec}\left(\pi + \frac{\pi}{6}\right)\cdot\sec\left(2\pi - \frac{\pi}{3}\right)$

$= -\text{cosec}\frac{\pi}{6}\cdot\sec\frac{\pi}{3} = -2 \times 2 = -4$

Hence, option (c) is correct.

18. If the determinant
$$\begin{vmatrix} x & 1 & 3 \\ 0 & 0 & 1 \\ 1 & x & 4 \end{vmatrix} = 0$$
then what is x equal to?
(a) -2 or 2 (b) -3 or 3 (c) -1 or 1 (d) 3 or 4

(c) Given, $\begin{vmatrix} x & 1 & 3 \\ 0 & 0 & 1 \\ 1 & x & 4 \end{vmatrix} = 0$ … (i)

$-1(x^2 - 1) = 0 \Rightarrow x^2 - 1 = 0 \Rightarrow x^2 = 1$
$\Rightarrow \quad x = \pm 1 \Rightarrow 1 - x^2 = 0$
$\therefore \quad x^2 = 1$
$x = +1, -1$
Hence, option (c) is correct.

19. What is the value of the following?
$\tan 31° \tan 33° \tan 35° \ldots \tan 57° \tan 59°$
(a) -1 (b) 0 (c) 1 (d) 2

(c) $\tan 31° \cdot \tan 33° \cdot \tan 35° \ldots \tan 57° \cdot \tan 59°$
$= \tan 31° \cdot \tan 33° \cdot \tan 35° \ldots x \tan 45° x \ldots \tan(90° - 33°) \cdot \tan(90° - 31°)$
$= \tan 31° \cdot \tan 33° \cdot \tan 35° \ldots \cot 35° \cdot \cot 33° \cdot \cot 31°$
$= (\tan 31° \cdot \cot 31°) \cdot (\tan 33° \cdot \cot 33°) \cdot (\tan 35° \cdot \cot 35°) \ldots$
$= 1 \cdot 1 \cdot 1 \ldots = 1$
Hence, option (c) is correct.

20. If $f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & x(x+1) \\ 3x(x-1) & 2(x-1)(x-2) & x(x+1)(x-1) \end{vmatrix}$
then what is $f(-1) + f(0) + f(1)$ equal to?
(a) 0 (b) 1
(c) 100 (d) -100

(a) $f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & x(x+1) \\ 3x(x-1) & 2(x-1)(x-2) & x(x+1)(x-1) \end{vmatrix}$

$f(-1) = \begin{vmatrix} 1 & -1 & 0 \\ -2 & 2 & 0 \\ 6 & 12 & 0 \end{vmatrix} = 0 \Rightarrow f(0) = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 4 & 0 \end{vmatrix} = 0$

$f(1) = \begin{vmatrix} 1 & 1 & 2 \\ 2 & 0 & 2 \\ 0 & 0 & 0 \end{vmatrix} = 0$

$\therefore f(-1) + f(0) + f(1) = 0 + 0 + 0 = 0$
Hence, option (a) is correct.

21. The equation $\sin^{-1} x - \cos^{-1} x = \frac{\pi}{6}$ has
(a) no solution (b) unique solution
(c) two solutions (d) infinite number of solutions

(b) $\because \sin^{-1} x - \cos^{-1} x = \frac{\pi}{6}$ … (i)

and we know that $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ … (ii)

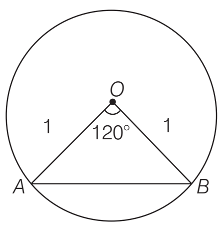
Adding Eqs. (i) and (ii), we get
$2\sin^{-1} x = \frac{\pi}{6} + \frac{\pi}{2} \Rightarrow 2\sin^{-1} x = \frac{2\pi}{3}$
$\Rightarrow \quad \sin^{-1} x = \frac{\pi}{3}$
$x = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$
Hence, the given equation has a unique solution.
Hence, option (b) is correct.

22. What is the value of the following?
$(\sin 24^\circ + \cos 66^\circ)(\sin 24^\circ - \cos 66^\circ)$
(a) -1 (b) 0 (c) 1 (d) 2

(b) $(\sin 24^\circ + \cos 66^\circ)(\sin 24^\circ - \cos 66^\circ)$
$= (\sin 24^\circ + \cos 66^\circ)\{\sin(90^\circ - 66^\circ) - \cos 66^\circ\}$ $[\because \sin(90^\circ - \theta) = \cos\theta]$
$= (\sin 24^\circ + \cos 66^\circ)(\cos 66^\circ - \cos 66^\circ)$
$= (\sin 24^\circ + \cos 66^\circ)(0) = 0$
Hence, option (b) is correct.

23. A chord subtends an angle 120° at the centre of a unit circle. What is the length of the chord?
(a) $\sqrt{2} - 1$ units (b) $\sqrt{3} - 1$ units
(c) $\sqrt{2}$ units (d) $\sqrt{3}$ units

(d) Given, radius of the circle = 1 unit



$\angle AOB = 120^\circ$
By using cosine rule,
$$\cos 120^\circ = \frac{OA^2 + OB^2 - AB^2}{2 \cdot OA \cdot OB} \quad \dots \text{(i)}$$
Let $AB = x$ unit, $OA = 1$ unit, $OB = 1$ unit
From Eq. (i),
$$\frac{-1}{2} = \frac{1 + 1 - x^2}{2 \cdot 1 \cdot 1} \Rightarrow -1 = 2 - x^2$$
$\Rightarrow x^2 = 3 \Rightarrow x = \sqrt{3}$ unit
Hence, option (d) is correct.

24. What is $(1 + \cot\theta - \operatorname{cosec}\theta)(1 + \tan\theta + \sec\theta)$ equal to?
(a) 1 (b) 2 (c) 3 (d) 4

(b) $(1 + \cot\theta - \operatorname{cosec}\theta)(1 + \tan\theta + \sec\theta)$
$$= \left(1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)\left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)$$
$$= \left(\frac{\sin\theta + \cos\theta - 1}{\sin\theta}\right)\left(\frac{\sin\theta + \cos\theta + 1}{\cos\theta}\right)$$
$$= \frac{(\sin\theta + \cos\theta)^2 - 1^2}{\sin\theta \cdot \cos\theta}$$
$$= \frac{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1}{\sin\theta\cos\theta}$$
$$= \frac{1 + 2\sin\theta \cdot \cos\theta - 1}{\sin\theta \cdot \cos\theta} = 2$$
Hence, option (b) is correct.

25. What is $\dfrac{1 + \tan^2\theta}{1 + \cot^2\theta} - \left(\dfrac{1 - \tan\theta}{1 - \cot\theta}\right)^2$ equal to?
(a) 0 (b) 1
(c) $2\tan\theta$ (d) $2\cot\theta$

(a) $\dfrac{1 + \tan^2\theta}{1 + \cot^2\theta} - \left(\dfrac{1 - \tan\theta}{1 - \cot\theta}\right)^2$
$$= \frac{1 + \tan^2\theta}{1 + \dfrac{1}{\tan^2\theta}} - \left(\frac{1 - \tan\theta}{1 - \dfrac{1}{\tan\theta}}\right)^2$$
$$= \tan^2\theta\left(\frac{1 + \tan^2\theta}{\tan^2\theta + 1}\right) - \left(\frac{\tan\theta(1 - \tan\theta)}{\tan\theta - 1}\right)^2$$
$= \tan^2\theta - \tan^2\theta = 0$
Hence, option (a) is correct.

26. What is the interior angle of a regular octagon of side length 2 cm?
(a) $\dfrac{\pi}{2}$ (b) $\dfrac{3\pi}{4}$ (c) $\dfrac{3\pi}{5}$ (d) $\dfrac{3\pi}{8}$

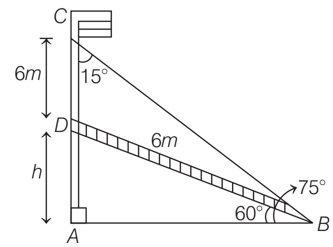
(b) Given, length of side of regular octagon = 2 cm
$\because$ Sum of interior angles of octagon
$= (8 - 2) \times 180^\circ = 6 \times 180^\circ$
[$\because$ sum of interior angles of polygon $= (n - 2) \times 180^\circ$]
$\therefore$ Interior angle $= \dfrac{6 \times 180^\circ}{8}$
$= 135^\circ = \dfrac{3\pi}{4}$
Hence, option (b) is correct.

27. If $7\sin\theta + 24\cos\theta = 25$, then what is the value of $(\sin\theta + \cos\theta)$?
(a) 1 (b) $\dfrac{26}{25}$ (c) $\dfrac{6}{5}$ (d) $\dfrac{31}{25}$

(d) Given, $7\sin\theta + 24\cos\theta = 25$
Since, we know that if
$a\sin\theta + b\cos\theta = c$
then $b\sin\theta - a\cos\theta = \sqrt{a^2 + b^2 - c^2}$
$\because \quad 7\sin\theta + 24\cos\theta = 25 \quad \dots$ (i)
$\therefore 24\sin\theta - 7\cos\theta = \sqrt{7^2 + 24^2 - 25^2}$
$24\sin\theta - 7\cos\theta = 0 \quad \dots$ (ii)
Eq. (i) $\times$ 7 + Eq. (ii) $\times$ 24
$49\sin\theta + 168\cos\theta = 175$
$576\sin\theta - 168\cos\theta = 0$
$625\sin\theta = 175$
$$\sin\theta = \frac{175}{625} = \frac{7}{25}$$
$$\therefore \cos\theta = \sqrt{1 - \left(\frac{7}{25}\right)^2} = \frac{24}{25}$$
$$\therefore \sin\theta + \cos\theta = \frac{7}{25} + \frac{24}{25} = \frac{31}{25}$$
Hence, option (d) is correct.

28. A ladder 6 m long reaches a point 6 m below the top of a vertical flagstaff. From the foot of the ladder, the elevation of the top of the flagstaff is 75°. What is the height of the flagstaff ?
(a) 12 m (b) 9 m
(c) $(6 + \sqrt{3})$ m (d) $(6 + 3\sqrt{3})$ m

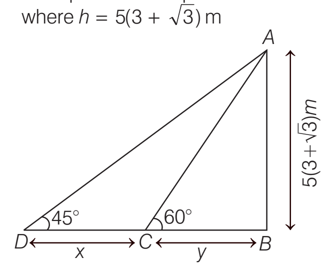
(d) Let AC be a vertical flagstaff.
$\therefore$ $CD = 6$ m, $BD = 6$ m
$\angle CBD = 75^\circ$



Let $AD = h$ meter
In ΔABC
$90 + 75 + \angle C = 180^\circ$ [$\because$ sum of interior angle of triangle is 180°]
$\angle C = 15^\circ$
In ΔBCD,
$BD = CD \Rightarrow \angle BCD = \angle CBD = 15^\circ$
$\therefore$ $\angle ABD = 75^\circ - 15^\circ = 60^\circ$
In ΔABD, $\sin 60^\circ = \dfrac{h}{6} \Rightarrow \dfrac{\sqrt{3}}{2} = \dfrac{h}{6}$
$h = 3\sqrt{3}$ m
$\therefore$ Height of the flagstaff $= (h + 6)$ m
$= (3\sqrt{3} + 6)$ m
Hence, option (d) is correct.

29. The shadow of a tower is found to be x metre longer, when the angle of elevation of the sun changes from 60° to 45°. If the height of the tower is $5(3 + \sqrt{3})$ m, then what is x equal to?
(a) 8 m (b) 10 m
(c) 12 m (d) 15 m

(b) In the given diagram,
AB represents the position of tower, where $h = 5(3 + \sqrt{3})$ m



$CD = x$ m
In ΔABC,
$$\tan 60^\circ = \frac{5(3 + \sqrt{3})}{BC} \Rightarrow \sqrt{3} = \frac{5(3 + \sqrt{3})}{BC}$$

$\therefore \quad BC = 5(\sqrt{3}+1)\,m$

In ΔABD,

$$\tan 45^\circ = \frac{5(3+\sqrt{3})}{BD}$$

$$\Rightarrow \quad 1 = \frac{5(3+\sqrt{3})}{BD}$$

$\therefore \quad BD = 5(3+\sqrt{3})\,m$

Since, $x = BD - BC$

$x = 5(3+\sqrt{3}) - 5(\sqrt{3}+1)$

$x = 5(3+\sqrt{3}-\sqrt{3}-1)$

$x = 10\,m$

Hence, option (b) is correct.

30. If $3\cos\theta = 4\sin\theta$, then what is the value of $\tan(45^\circ+\theta)$?

(a) 10 (b) 7 (c) $\frac{7}{2}$ (d) $\frac{7}{4}$

(b) If $3\cos\theta = 4\sin\theta$

$$\Rightarrow \quad \frac{3}{4} = \frac{\sin\theta}{\cos\theta} \Rightarrow \tan\theta = \frac{3}{4}$$

$$\therefore \tan(45^\circ+\theta) = \frac{\tan 45^\circ + \tan\theta}{1-\tan 45^\circ\cdot\tan\theta}$$

$$= \frac{1+\frac{3}{4}}{1-1\times\frac{3}{4}} = \frac{4+3}{4-3} = 7$$

Hence, option (b) is correct.

31. $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$ holds, when

(a) $x \in R$

(b) $x \in R - (-1, 1)$ only

(c) $x \in R - \{0\}$ only

(d) $x \in R - [-1, 1]$ only

(a) Since, $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$

for all $x \in R$.

Hence, option (a) is correct.

32. If $\tan A = \frac{1}{7}$, then what is $\cos 2A$ equal to?

(a) $\frac{24}{25}$ (b) $\frac{18}{25}$ (c) $\frac{12}{25}$ (d) $\frac{6}{25}$

(a) $\tan A = \frac{1}{7}$

$$\therefore \cos 2A = \frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-(1/7)^2}{1+(1/7)^2}$$

$$= \frac{49-1}{49+1} = \frac{48}{50}$$

$$\cos 2A = \frac{24}{25}$$

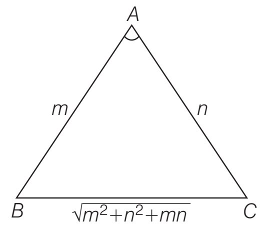
Hence, option (a) is correct.

33. The sides of a triangle are m, n and $\sqrt{m^2+n^2+mn}$. What is the sum of the acute angles of the triangle?

(a) 45° (b) 60° (c) 75° (d) 90°

(b) Let $AB = m$, $AC = n$

$$BC = \sqrt{m^2+n^2+mn}$$



By using cosine rule,

$$\cos A = \frac{AB^2+AC^2-BC^2}{2AB\cdot AC}$$

$$\Rightarrow \cos A = \frac{m^2+n^2-m^2-n^2-mn}{2mn}$$

$$\Rightarrow \cos A = \frac{-1}{2} \quad \Rightarrow \quad A = 120^\circ$$

$\therefore \angle B + \angle C = 180 - \angle A$

[$\because$ sum of interior angle is 180°]

$= 180^\circ - 120^\circ$

$\angle B + \angle C = 60^\circ$

Hence, option (b) is correct.

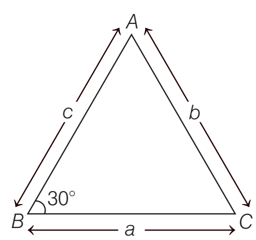
34. What is the area of the triangle ABC with sides $a = 10$cm, $c = 4$cm and angle $B = 30^\circ$?

(a) $16\,cm^2$ (b) $12\,cm^2$ (c) $10\,cm^2$ (d) $8\,cm^2$

(c) Given, $a = 10$ cm

$c = 4$ cm

$\angle B = 30^\circ$



$\because$ Area of triangle $= \frac{1}{2}ac\sin(\angle B)$

$$= \frac{1}{2}\times 10\times 4\times\sin 30^\circ = \frac{1}{2}\times 40\times\frac{1}{2}$$

$= 10$ sq cm

Hence, option (c) is correct.

35. Consider the following statements

1. $A = \{1, 3, 5\}$ and $B = \{2, 4, 7\}$ are equivalent sets.
2. $A = \{1, 5, 9\}$ and $B = \{1, 5, 5, 9, 9\}$ are equal sets

Which of the above statements is/are correct?

(a) 1 only (b) 2 only (c) Both 1 and 2 (d) Neither 1 nor 2

(c) $A = \{1, 3, 5\}$ and $B = \{2, 4, 7\}$

Since, number of elements are same in both the sets.

$\Rightarrow$ A and B are equivalent sets.

If $A = \{1, 5, 9\}$, $B = \{1, 5, 5, 9, 9\}$

Which is nothing but $B = \{1, 5, 9\}$

Since, elements are same in A and B

$\Rightarrow$ A and B are equal sets

Hence, option (c) is correct.

36. Consider the following statements

1. The null set is a subset of every set.
2. Every set is a subset of itself.
3. If a set has 10 elements, then its power set will have 1024 elements.

Which of the above statements are correct?

(a) 1 and 2 only (b) 2 and 3 only (c) 1 and 3 only (d) 1, 2 and 3

(d) Since we know that null set is a subset of every set and every set is a subset of itself.

If $\quad n(A) = 10$

$\therefore n(P(A)) = 2^{10} = 1024$

$\therefore$ all the given statements are true.

Hence, option (d) is correct.

37. Let R be a relation defined as xRy if and only if $2x+3y = 20$, where $x, y \in N$. How many elements of the form (x, y) are there in R?

(a) 2 (b) 3 (c) 4 (d) 6

(b) $\because xRy \Leftrightarrow 2x+3y = 20$

where, $x, y \in \mathbb{N}$

$$\because \quad y = \frac{20-2x}{3}$$

All ordered pair which satisfies the given relations are (1, 6), (4, 4), (7, 2).

$\therefore \quad R = \{(1, 6), (4, 4), (7, 2)\}$

$\therefore \quad n(R) = 3$

Hence, option (b) is correct.

38. Consider the following statements

1. A function $f: \mathbb{Z} \to \mathbb{Z}$, defined by $f(x) = x+1$, is one-one as well as onto.
2. A function $f: \mathbb{N} \to \mathbb{N}$, defined by $f(x) = x+1$, is one-one but not onto.

Which of the above statement(s) is/are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

(c) **Statement I**

$f : \mathbb{Z} \to z$

$f(x) = x + 1$

Let $f(x_1) = f(x_2)$

$\Rightarrow x_1 + 1 = x_2 + 1$

$\Rightarrow x_1 = x_2$

$\Rightarrow f$ is one-one in $\mathbb{Z}$.

and every element of co-domain has its pre-image in domain.

$\Rightarrow f$ is onto.

Statement II

$f : \mathbb{N} \to \mathbb{N}$

$f(x) = x + 1$

Let $f(x_1) = f(x_2)$

$x_1 + 1 = x_2 + 1$

$\Rightarrow x_1 = x_2$

$\Rightarrow f$ is one-one in $\mathbb{N}$.

But there is no element in $\mathbb{N}$ such that

$f(x) = 1$

Hence, f is not onto on $\mathbb{N}$.

Given statements are correct.

Hence, option (c) is correct.

39. Consider the following in respect of a complex number z.

1. $\overline{(z^{-1})} = (\bar{z})^{-1}$
2. $zz^{-1} = |z|^2$

Which of the above is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

(a) Let $z = x + iy$

$\bar{z} = x - iy$

$(\bar{z})^{-1} = \dfrac{1}{x - iy} = \dfrac{x + iy}{x^2 + y^2}$

Also, $z^{-1} = \dfrac{1}{x + iy} = \dfrac{x - iy}{x^2 + y^2}$

$\overline{(z^{-1})} = \dfrac{x + iy}{x^2 + y^2} = (\bar{z})^{-1}$

$\therefore$ Statement 1 is correct.

$|z| = \sqrt{x^2 + y^2}$

$\Rightarrow |z|^2 = x^2 + y^2$

But $zz^{-1} = (x + iy)\dfrac{(x - iy)}{x^2 + y^2}$

$= \dfrac{x^2 + y^2}{x^2 + y^2} = 1 \neq |z|^2$

$\therefore$ Statement 2 is wrong.

Hence, option (a) is correct.

40. Consider the following statements in respect of an arbitrary complex number z.

1. The difference of z and its conjugate is an imaginary number.
2. The sum of z and its conjugate is a real number.

Which of the above statement(s) is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

(c) Let $z = x + iy$

$\bar{z} = x - iy$

$\therefore z - \bar{z} = x + iy - x + iy = 2iy$ which is an imaginary number.

$\Rightarrow$ Statement-1 is correct.

Also, $z + \bar{z} = x + iy + x - iy = 2x$ which is real.

$\Rightarrow$ Statement-2 is correct.

Hence, option (c) is correct.

41. What is the modulus of the complex number $i^{2n+1}(-i)^{2n-1}$, where $n \in N$ and $i = \sqrt{-1}$?

(a) -1 (b) 1 (c) $\sqrt{2}$ (d) 2

(b) Let $z = i^{2n+1}(-i)^{2n-1}$, where $n \in \mathbb{N}$

$= (i)^{2n}(i)\,(-i)^{2n}(-i)^{-1}$

$= (i^{2n})(-1)^{2n}.(i^{2n})\left(\dfrac{i}{-i}\right)$

$= (i^{4n})(-1) = (i^4)^n.(-1)$

$= -1 = -1 + 0i$

$\therefore |z| = 1$

Hence, option (b) is correct.

42. If α and β are the roots of the equation $4x^2 + 2x - 1 = 0$, then which one of the following is correct?

(a) $\beta = -2\alpha^2 - 2\alpha$ (b) $\beta = 4\alpha^2 - 3\alpha$
(c) $\beta = \alpha^2 - 3\alpha$ (d) $\beta = -2\alpha^2 + 2\alpha$

(a) Given quadratic equation

$4x^2 + 2x - 1 = 0$ ... (i)

If α, β are the roots of Eq. (i), then these value will satisfy the given equation.

$4\alpha^2 + 2\alpha - 1 = 0$ ... (ii)

and $4\beta^2 + 2\beta - 1 = 0$ ... (iii)

From Eq. (i),

Sum of roots $= \dfrac{-2}{4}$

$\alpha + \beta = \dfrac{-1}{2}$

$\beta = \dfrac{-1}{2} - \alpha$

On putting the value of β in Eq. (iii),

$4\left(\dfrac{-1}{2} - \alpha\right)^2 + 2\beta - 1 = 0$

$4\left(\dfrac{1}{4} + \alpha^2 + \alpha\right) - 1 = -2\beta$

$1 + 4\alpha^2 + 4\alpha - 1 = -2\beta$

$\Rightarrow \beta = \dfrac{4(\alpha^2 + \alpha)}{-2}$

$\beta = -2\alpha^2 - 2\alpha$

Hence, option (a) is correct.

43. If one root of $5x^2 + 26x + k = 0$ is reciprocal of the other, then what is the value of k?

(a) 2 (b) 3 (c) 5 (d) 8

(c) Given quadratic equation

$5x^2 + 26x + k = 0$ ... (i)

Let α and β be the roots.

According to question, $\beta = \dfrac{1}{\alpha}$

$\because$ Product of roots $= \dfrac{k}{5}$

$\alpha \cdot \beta = \dfrac{k}{5}$

$\Rightarrow \alpha \cdot \dfrac{1}{\alpha} = \dfrac{k}{5} \Rightarrow 1 = \dfrac{k}{5}$

$\Rightarrow k = 5$

Hence, option (c) is correct.

44. In how many ways can a team of 5 players be selected from 8 players so as not to include a particular player?

(a) 42 (b) 35
(c) 21 (d) 20

(c) Given that there are 8 players among which one particular player is there.

Hence, number of ways to select 5 players $= {}^{8-1}C_5$

$= {}^7C_5 = \dfrac{7 \times 6}{1 \times 2} = 21$

Hence, option (c) is correct.

45. What is the coefficient of the middle term in the expansion of $(1 + 4x + 4x^2)^5$?

(a) 8064 (b) 4032
(c) 2016 (d) 1008

(a) $(1 + 4x + 4x^2)^5$

$= \{(1 + 2x)^2\}^5 = (1 + 2x)^{10}$

$\therefore$ Total number of term in the expansion of $(1 + 2x)^{10} = 10 + 1 = 11$

$\therefore$ Middle term $= \left(\dfrac{11 + 1}{2}\right)$th term

$= 6$th term

$T_6 = T_{5+1} = {}^{10}C_5\,(2x)^5$

$= {}^{10}C_5 \times 2^5 \times x^5$

$\therefore$ Coefficient of middle term $= {}^{10}C_5 \cdot 2^5$

$= \dfrac{10 \times 9 \times 8 \times 7 \times 6}{1 \times 2 \times 3 \times 4 \times 5} \times 2^5 = 8064$

Hence, option (a) is correct.

46. What is $C(n,1)+C(n,2)+...+C(n,n)$ equal to?

(a) $2+2^2+2^3+...+2^n$
(b) $1+2+2^2+2^3+...+2^n$
(c) $1+2+2^2+2^3+...+2^{n-1}$
(d) $2+2^2+2^3+...+2^{n-1}$

(c) $C(n,1)+C(n,2)+\ldots+C(n,n)$

$= {}^nC_1+{}^nC_2+{}^nC_3+\ldots+{}^nC_n$

$\{\because {}^nC_0+{}^nC_1+{}^nC_2+\ldots+{}^nC_n=2^n\}$

$=2^n-{}^nC_0=2^n-1$

Now, we shall solve the option to check whether sum is 2^n-1 or not.

Let's take

$S=1+2+2^2+2^3+\ldots+2^{n-1}$ which forms a GP.

where $a=1$

$r=\frac{2}{1}=2>1$

$\because \quad S=\frac{a(r^n-1)}{r-1}$

$\therefore \quad S=\frac{1(2^n-1)}{2-1}=2^n-1$

Hence, $2^n-1={}^nC_1+{}^nC_2+\ldots+{}^nC_n$

$\therefore$ Option (c) is correct.

47. What is the sum of the coefficients of first and last terms in the expansion of $(1+x)^{2n}$, where n is a natural number?

(a) 1 (b) 2
(c) n (d) $2n$

(b) Expand $(1+x)^{2n}$ by using binomial expansion

$={}^{2n}C_0x^0+{}^{2n}C_1x^1+{}^{2n}C_2x^2+\ldots+{}^{2n}C_{2n}x^{2n}$

$\therefore$ The coefficient of first and last term of the expansion

$={}^{2n}C_0+{}^{2n}C_{2n}$

$=1+1=2$

Hence, option (b) is correct.

48. If the first term of an AP is 2 and the sum of the first five terms is equal to one-fourth of the sum of the next five terms, then what is the sum of the first ten terms?

(a) – 500 (b) – 250
(c) 500 (d) 250

(b) Given, first term of an AP $(a)=2$

and $a_1+a_2+a_3+a_4+a_5=\frac{1}{4}(a_6+a_7+a_8+a_9+a_{10})$, where $a_n=a+(n-1)d$

$\Rightarrow \frac{5}{2}[2a+(5-1)d]$

$=\frac{1}{4}\times\frac{5}{2}[2a_6+(5-1)d]$

[$\because$ sum of n terms of AP, $S_n=\frac{n}{2}[2a+(n-1)d]$]

$4(2\times 2+4d)=2a_6+4d$

$16+16d=2a_6+4d$

$16+16d=2(a+5d)+4d$

$16+16d=2a+14d$

$16+16d=2\times 2+14d$

$2d=-12 \Rightarrow d=-6$

$\therefore \quad S_{10}=\frac{10}{2}[2a+(10-1)d]$

$=5[2\times 2+9(-6)]=5[4-54]$

$S_{10}=-250$

Hence, option (b) is correct.

49. Consider the following statements

1. If each term of a GP is multiplied by same non-zero number, then the resulting sequence is also a GP.
2. If each term of a GP is divided by same non-zero number, then the resulting sequence is also a GP.

Which of the above statements is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

(c) Let us take a GP.

$a, ar, ar^2, \ldots$ is in GP.

$\Rightarrow ak, akr, akr^2, \ldots$ will also be in GP where, k is non-zero number.

If $k=\frac{1}{m}, m\neq 0$

$\Rightarrow \frac{a}{m}, \frac{a}{m}r, \frac{a}{m}r^2, \ldots$ will also be in GP.

Hence, both statements are correct.

50. How many 5-digit prime numbers can be formed using the digits 1, 2, 3, 4, 5 if the repetition of digits is not allowed?

(a) 5 (b) 4 (c) 3 (d) 0

(d) Given digits are 1, 2, 3, 4, 5

Since, the sum of digits $=1+2+3+4+5=15$ is divisible by 3.

$\Rightarrow$ Every 5 digit number formed by the given digits will be divisible by 3.

$\Rightarrow$ There is no prime number.

Hence, option (d) is correct.

51. If $f(x+1)=x^2-3x+2$, then what is $f(x)$ equal to?

(a) x^2-5x+4 (b) x^2-5x+6
(c) x^2+3x+3 (d) x^2-3x+1

(b) If $f(x+1)=x^2-3x+2$

Let $x+1=y$

$\Rightarrow \quad x=y-1$ or $x\to x-1$

$\therefore f(x)=(x-1)^2-3(x-1)+2$

$=x^2+1-2x-3x+3+2$

$=x^2-5x+6$

Hence, option (b) is correct.

52. If $x^2, x, -8$ are in AP, then which one of the following is correct?

(a) $x\in\{-2\}$ (b) $x\in\{4\}$
(c) $x\in\{-2,4\}$ (d) $x\in\{-4,2\}$

(c) If $x^2, x, -8$ are in AP, then

$2x=x^2-8$

$\Rightarrow \quad x^2-2x-8=0$

$\Rightarrow (x-4)(x+2)=0$

$x\in\{-2,4\}$

Hence, option (c) is correct.

53. The third term of a GP is 3. What is the product of its first five terms?

(a) 81
(b) 243
(c) 729
(d) Cannot be determined due to insufficient data

(b) Given

$a_3=3$

$\because \quad a_3=ar^2$ in GP [$\because a_n=ar^{n-1}$ in GP]

$ar^2=3 \quad \ldots$ (i)

To find $a_1\cdot a_2\cdot a_3\cdot a_4\cdot a_5$

$=a(ar)(ar^2)(ar^3)(ar^4)$

$=a^5r^{10}=(ar^2)^5=3^5=243$

Hence, option (b) is correct.

54. The element in the ith row and the j th column of a determinant of third order is equal to $2(i+j)$. What is the value of the determinant?

(a) 0 (b) 2 (c) 4 (d) 6

(a) Given,

$a_{ij}=2(i+j)$

$\therefore a_{11}=2(1+1)=4, a_{21}=2(2+1)=6$

$a_{12}=2(1+2)=6, a_{22}=2(2+2)=8$

$a_{13}=2(1+3)=8, a_{23}=2(2+3)=10$

$a_{31}=2(3+1)=8, a_{32}=2(3+2)=10$,

$a_{33}=2(3+3)=12$

$$\Delta=\begin{vmatrix}4&6&8\\6&8&10\\8&10&12\end{vmatrix}=2\cdot2\cdot2\begin{vmatrix}2&3&4\\3&4&5\\4&5&6\end{vmatrix}$$

$=8[2(24-25)-3(18-20)+4(15-16)]$

$\Delta=8[-2+6-4]$

$\Delta=0$

Hence, option (a) is correct.

55. With the numbers 2, 4, 6, 8, all the possible determinants with these four different elements are constructed. What is the sum of the values of all such determinants?

(a) 128 (b) 64 (c) 32 (d) 0

(d) Given numbers are 2, 4, 6, 8.

$\therefore$ We can form determinant of order 2.

Number of determinats

$= 4 \times 3 \times 2 \times 1 = 24$

Let's observe some determinants

$\begin{vmatrix} 2 & 6 \\ 8 & 4 \end{vmatrix} = 8 - 48 = -40, \begin{vmatrix} 6 & 2 \\ 4 & 8 \end{vmatrix} = 40$

$\begin{vmatrix} 2 & 8 \\ 6 & 4 \end{vmatrix} = 8 - 48 = -40, \begin{vmatrix} 6 & 4 \\ 2 & 8 \end{vmatrix} = 40$

$\begin{vmatrix} 4 & 8 \\ 6 & 2 \end{vmatrix} = 8 - 48 = -40, \begin{vmatrix} 8 & 4 \\ 2 & 6 \end{vmatrix} = 40$

$\begin{vmatrix} 4 & 6 \\ 8 & 2 \end{vmatrix} = 8 - 48 = -40, \begin{vmatrix} 8 & 2 \\ 4 & 6 \end{vmatrix} = 40$

Hence, we can see that we are getting a pattern where each determinant value will be neutralised by other value.

Hence, sum of the values of all determinants = 0

Hence, option (d) is correct.

56. What is the radius of the circle $4x^2 + 4y^2 - 20x + 12y - 15 = 0$?

(a) 14 units (b) 10.5 units
(c) 7 units (d) 3.5 units

(d) Given equation of circle

$4x^2 + 4y^2 - 20x + 12y - 15 = 0$

$\Rightarrow x^2 + y^2 - 5x + 3y - \frac{15}{4} = 0$

On comparing with
$x^2 + y^2 + 2gx + 2fy + c = 0$

$g = \frac{-5}{2}, f = \frac{3}{2}, c = \frac{-15}{4}$

$\therefore$ Radius $= \sqrt{g^2 + f^2 - c}$

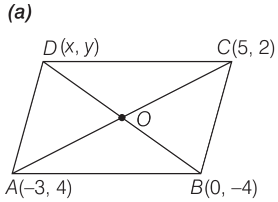
$= \sqrt{\frac{25}{4} + \frac{9}{4} + \frac{15}{4}} = \frac{7}{2} = 3.5$ unit

Hence, option (d) is correct.

57. A parallelogram has three consecutive vertices $(-3, 4), (0, -4)$ and $(5, 2)$.The fourth vertex is

(a) (2, 10) (b) (2, 9)
(c) (3, 9) (d) (4, 10)

(a)



Let the fourth vertex be $D(x, y)$

Diagonals of a parallelogram bisect each other.

O is mid-point of AC.

$\Rightarrow$ Coordinate of O $\left(\frac{-3+5}{2}, \frac{4+2}{2}\right)$ or (1, 3)

O is mid-point of BD.

$\Rightarrow$ Coordinate of O is $\left(\frac{x+0}{2}, \frac{y-4}{2}\right)$ or $\left(\frac{x}{2}, \frac{y-4}{2}\right)$

Therefore, compare the coordinate of O

$\Rightarrow \frac{x}{2} = 1 \Rightarrow x = 2$

and $\frac{y-4}{2} = 3 \Rightarrow y = 10$

Hence, the fourth vertex is (2, 10).

58. If the lines $y + px = 1$ and $y - qx = 2$ are perpendicular, then which one of the following is correct?

(a) $pq + 1 = 0$ (b) $p + q + 1 = 0$
(c) $pq - 1 = 0$ (d) $p - q + 1 = 0$

(c) Given $y + px = 1$ ... (i)

$y - qx = 2$ ... (ii)

Eqs. (i) and (ii) are perpendicular

$\Rightarrow m_1 \cdot m_2 = -1$ where m_1 and m_2 are the slope of Eqs. (i) and (ii)

and $m = \frac{-\text{ coefficient of } x}{\text{coefficient of } y}$

$\Rightarrow \frac{-p}{1} \times \frac{-(-q)}{1} = -1$

$\Rightarrow -pq = -1 \Rightarrow pq - 1 = 0$

Hence, option (c) is correct.

59. If A, B and C are in AP, then the straight line $Ax + 2By + C = 0$ will always pass through a fixed point. The fixed point is

(a) (0, 0) (b) (– 1, 1)
(c) (1, – 2) (d) (1, – 1)

(d) Given A, B, C are in AP.

$\Rightarrow 2B = A + C$

$\Rightarrow A - 2B + C = 0$ ... (i)

On comparing $A - 2B + C = 0$ with the given line $Ax + 2By + C = 0$, we get $x = 1$, $y = -1$

Hence, line $Ax + 2By + C = 0$ will pass through (1, – 1)

Hence, option (d) is correct.

60. If the image of the point $(-4, 2)$ by a line mirror is $(4, -2)$, then what is the equation of the line mirror?

(a) $y = x$ (b) $y = 2x$
(c) $4y = x$ (d) $y = 4x$

(b) Let $A = (-4, 2)$

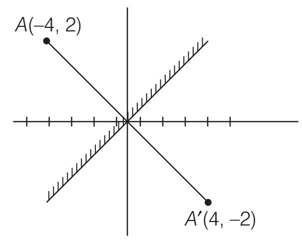


image point
$A' = (4, -2)$

$\therefore$ Mid-point of

$AA' = \left(\frac{-4+4}{2}, \frac{2+(-2)}{2}\right) = (0, 0)$

Slope of $AA' = \frac{-2-2}{4-(-4)}$

$= \frac{-4}{8} = \frac{-1}{2}$

Since, AA' and mirror line are perpendicular.

Slope of line mirror

$= \frac{-1}{\text{Slope of } AA'} = \frac{-1}{-1/2} = 2$

Equation of a line is $y - y_1 = m(x - x_1)$

$\therefore$ Equation of a line mirror is

$y - 0 = 2(x - 0)$

$\Rightarrow y = 2x$

Hence, option (b) is correct.

61. Consider the following statements in respect of the points $(p, p - 3)$, $(q + 3, q)$ and $(6, 3)$

1. The points lie on a straight line.
2. The points always lie in the first quadrant only for any value of p and q.

Which of the above statement(s) is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

(a) Given points are $A(p, p-3)$, $B(q+3, q)$ and $C(6, 3)$

As, Points lies on a straight line,

so slope of AB = slope of BC

$\frac{q - p + 3}{q + 3 - p} = \frac{3 - q}{6 - q - 3}$

$\left[\because \text{slope of a line} = \frac{y_2 - y_1}{x_2 - x_1}\right]$

$1 = 1$

$\therefore$ Statement 1 is correct.

But it's not necessary that the collinear points lie in the first quadrant only.

$\therefore$ Statement 2 is wrong.

Hence, option (a) is correct.

62. What is the acute angle between the lines $x - 2 = 0$ and $\sqrt{3}x - y - 2 = 0$?

(a) 0° (b) 30° (c) 45° (d) 60°

(b) $l_1 : x - 2 = 0$

$l_2 : \sqrt{3}x - y - 2 = 0$

$\therefore$ Slope of line l_1, $m_1 = \dfrac{-\text{Coefficient of } x}{\text{Coefficient of } y}$

$= \dfrac{-1}{0} = \infty$

The line l_1 is parallel to Y-axis or perpendicular to X-axis.

$\therefore$ Slope of line, l_2, $m_2 = \dfrac{-\sqrt{3}}{-1} = \sqrt{3}$

The line l_2 makes an angle 60° from positive X-axis.

$\therefore$ Angle between l_1 and

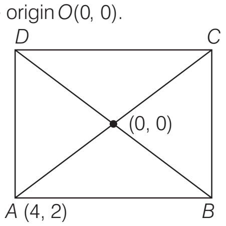
$l_2 = 90^\circ - 60^\circ = 30^\circ$

Hence, option (b) is correct.

63. The point of intersection of diagonals of a square $ABCD$ is at the origin and one of its vertices is at $A(4, 2)$. What is the equation of the diagonal BD?

(a) $2x + y = 0$ (b) $2x - y = 0$
(c) $x + 2y = 0$ (d) $x - 2y = 0$

(a) Since, diagonal BD passes through the origin $O(0, 0)$.



$\because$ Slope of $OA = \dfrac{0-2}{0-4} = \dfrac{1}{2}$

$\because OA$ and OB are perpendicular to each other

$\therefore$ slope of $OB = \dfrac{-1}{\text{slope of } OA} = \dfrac{-1}{1/2}$

$= -2$

$\therefore$ Eqs. of BD having slope -2 and passes through (0, 0)

$y - 0 = -2\,[x - 0]$

[$\because$ Equation of a line $\Rightarrow y - y_1 = m\,(x - x_1)$]

$\Rightarrow \quad 2x + y = 0$

Hence, option (a) is correct.

64. If any point on a hyperbola is $(3\tan\theta, 2\sec\theta)$, then what is the eccentricity of the hyperbola?

(a) $\dfrac{3}{2}$ (b) $\dfrac{5}{2}$ (c) $\dfrac{\sqrt{11}}{2}$ (d) $\dfrac{\sqrt{13}}{2}$

(d) Given point is $(3\tan\theta, 2\sec\theta)$

$\Rightarrow x = 3\tan\theta,\ y = 2\sec\theta$

$\dfrac{x}{3} = \tan\theta,\ \dfrac{y}{2} = \sec\theta$

$\because \sec^2\theta - \tan^2\theta = 1$

$\dfrac{y^2}{4} - \dfrac{x^2}{9} = 1$ which represents conjugate Hyperbola.

$\Rightarrow a^2 = 9,\ b^2 = 4$

$\therefore \quad e = \sqrt{1 + \dfrac{a^2}{b^2}}$

$= \sqrt{1 + \dfrac{9}{4}} = \sqrt{\dfrac{13}{4}}$

$e = \dfrac{\sqrt{13}}{2}$

Hence, option (d) is correct.

65. Consider the following with regard to eccentricity (e) of a conic section

1. $e = 0$ for circle
2. $e = 1$ for parabola
3. $e < 1$ for ellipse

Which of the above are correct?

(a) 1 and 2 (b) 2 and 3
(c) 1 and 3 (d) 1, 2 and 3

(d) Since, we know that circle has eccentricity 0

and parabola has eccentricity 1

and ellipse has eccentricity $e < 1$

and hyperbola has eccentricity $e > 1$.

Hence, option (d) is correct.

66. What is the angle between the two lines having direction ratios $\langle 6, 3, 6\rangle$ and $\langle 3, 3, 0\rangle$?

(a) $\dfrac{\pi}{6}$ (b) $\dfrac{\pi}{4}$
(c) $\dfrac{\pi}{3}$ (d) $\dfrac{\pi}{2}$

(b) Direction ratios of line $l_1 = <6, 3, 6>$

$a_1 = 6,\ b_1 = 3,\ c_1 = 6$

Direction ratios of line $l_2 = <3, 3, 0>$

$\Rightarrow a_2 = 3,\ b_2 = 3,\ c_2 = 0$

$\because \cos\theta = \dfrac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\,\sqrt{a_2^2 + b_2^2 + c_2^2}}$

$= \dfrac{6\times 3 + 3\times 3 + 6\times 0}{\sqrt{6^2 + 3^2 + 6^2}\cdot\sqrt{3^2 + 3^2 + 0}}$

$\Rightarrow \cos\theta = \dfrac{27}{9\times 3\sqrt{2}}$

$\Rightarrow \cos\theta = \dfrac{1}{\sqrt{2}} = \cos\dfrac{\pi}{4}$

$\therefore \quad \theta = \dfrac{\pi}{4}$

Hence, option (b) is correct.

67. If l, m, n are the direction cosines of the line $x - 1 = 2(y + 3) = 1 - z$, then what is $l^4 + m^4 + n^4$ equal to?

(a) 1 (b) $\dfrac{11}{27}$ (c) $\dfrac{13}{27}$ (d) 4

(b) Given line is

$x - 1 = 2(y + 3) = 1 - z$

$\Rightarrow \dfrac{x-1}{2} = \dfrac{y+3}{1} = \dfrac{1-z}{2}$

$\Rightarrow \dfrac{x-1}{2} = \dfrac{y-(-3)}{1} = \dfrac{z-1}{-2}$

$\therefore$ Direction ratios are $<2, 1, -2>$

$\therefore$ Direction cosines are

$< \dfrac{2}{\sqrt{2^2 + 1^2 + (-2)^2}}, \dfrac{1}{\sqrt{2^2 + 1^2 + (-2)^2}}, \dfrac{-2}{\sqrt{2^2 + 1^2 + (-2)^2}}$

$\therefore l = \dfrac{2}{3},\ m = \dfrac{1}{3},\ n = \dfrac{-2}{3}$

$\therefore l^4 + m^4 + n^4 = \left(\dfrac{2}{3}\right)^4 + \left(\dfrac{1}{3}\right)^4 + \left(\dfrac{-2}{3}\right)^4$

$= \dfrac{16 + 1 + 16}{81} = \dfrac{33}{81} = \dfrac{11}{27}$

Hence, option (b) is correct.

68. What is the projection of the line segment joining $A(1, 7, -5)$ and $B(-3, 4, -2)$ on Y-axis?

(a) 5 (b) 4 (c) 3 (d) 2

(c) $A = (1, 7, -5)$ and $B = (-3, 4, -2)$

$\therefore$ Direction ratios of

$AB = <(-3-1), (4-7), (-2+5)>$

$= <-4, -3, 3>$

$\Rightarrow a = -4,\ b = -3,\ c = 3$

Direction cosines of Y-axis $= <0, 1, 0>$

$l = 0,\ m = 1,\ n = 0$

$\therefore$ Projection of AB on Y-axis

$= |\,al + bm + cn\,|$

$= |-4\times 0 + (-3)\times 1 + 3\times 0| = 3$

Hence, option (c) is correct.

69. What is the number of possible values of k for which the line joining the points $(k, 1, 3)$ and $(1, -2, k+1)$ also passes through the point $(15, 2, -4)$?

(a) Zero (b) One (c) Two (d) Infinite

(c) Let $A = (k, 1, 3)$, $B = (1, -2, k+1)$ and $C = (15, 2, -4)$

Since, line AB passes through C also.

Hence, points A, B and C are collinear.

$\therefore \begin{vmatrix} k & 1 & 3 \\ 1 & -2 & k+1 \\ 15 & 2 & -4 \end{vmatrix} = 0$

$k(8 - 2k - 2) - 1(-4 - 15k - 15) + 3(2 + 30) = 0$

$6k - 2k^2 + 19 + 15k + 96 = 0$

$2k^2 - 21k - 115 = 0$ which is quadratic equation.

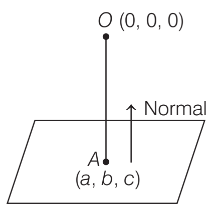
$\Rightarrow k$ has two values.

Hence, option (c) is correct.

70. The foot of the perpendicular drawn from the origin to the plane $x + y + z = 3$ is

(a) (0, 1, 2)
(b) (0, 0, 3)
(c) (1, 1, 1)
(d) (– 1, 1, 3)

(c) Let the foot of the perpendicular drawn from the origin to the plane $x + y + z = 3$ be (a, b, c).



Direction ratios of the plane $= < 1, 1, 1 >$

$\therefore$ Direction ratios of OA and normal will be in the same ratio.

$$\therefore \quad \frac{a-0}{1} = \frac{b-0}{1} = \frac{c-0}{1}$$

$\Rightarrow a = 1, b = 1, c = 1$

$\therefore \; A = (1, 1, 1)$

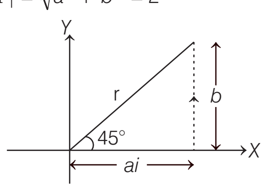
Hence, option (c) is correct.

71. A vector $\mathbf{r} = a\hat{\mathbf{i}} + b\hat{\mathbf{j}}$ is equally inclined to both x and y axes. If the magnitude of the vector is 2 units, then what are the values of a and b respectively?

(a) $\frac{1}{2}, \frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$
(c) $\sqrt{2}, \sqrt{2}$ (d) 2, 2

(c) $\mathbf{r} = a\hat{\mathbf{i}} + b\hat{\mathbf{j}}$

$|\mathbf{r}| = \sqrt{a^2 + b^2} = 2$



Since, $\mathbf{r}$ is equally inclined from X-axis and Y-axis.

Hence, $\mathbf{r}$ makes 45° from the X-axis.

$\therefore a = |\mathbf{r}| \cos 45°$ and $b = |\mathbf{r}| \sin 45°$

$a = 2 \times \frac{1}{\sqrt{2}}$, and $b = 2 \times \frac{1}{\sqrt{2}}$

$a = \sqrt{2}$ and $b = \sqrt{2}$

Hence, option (c) is correct.

72. Consider the following statements in respect of a vector $\mathbf{c} = \mathbf{a} + \mathbf{b}$, where $|\mathbf{a}| = |\mathbf{b}| \neq 0$

1. $\mathbf{c}$ is perpendicular to $(\mathbf{a} - \mathbf{b})$.
2. $\mathbf{c}$ is perpendicular to $(\mathbf{a} \times \mathbf{b})$.

Which of the above statements is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

(c) $\mathbf{c} = \mathbf{a} + \mathbf{b}$ where $\mathbf{a} = \mathbf{b} \neq 0$

Consider, $\mathbf{c} \cdot (\mathbf{a} - \mathbf{b}) = (\mathbf{a} + \mathbf{b}) \cdot (\mathbf{a} - \mathbf{b})$

$= |\mathbf{a}|^2 - |\mathbf{b}|^2 = |\mathbf{b}|^2 - |\mathbf{b}|^2 = 0$

$\Rightarrow \mathbf{c}$ is perpendicular to $(\mathbf{a} - \mathbf{b})$.

Also, $\mathbf{c} \cdot (\mathbf{a} \times \mathbf{b}) = (\mathbf{a} + \mathbf{b}) \cdot (\mathbf{a} \times \mathbf{b})$

$= \mathbf{a} \cdot (\mathbf{a} \times \mathbf{b}) + \mathbf{b} \cdot (\mathbf{a} \times \mathbf{b})$

$= 0 + 0 = 0$

$\Rightarrow \mathbf{c}$ is perpendicular to $(\mathbf{a} \times \mathbf{b})$

Hence, option (c) is correct.

73. If $\mathbf{a}$ and $\mathbf{b}$ are two vectors such that $|\mathbf{a} + \mathbf{b}| = |\mathbf{a} - \mathbf{b}| = 4$, then which one of the following is correct?

(a) $\mathbf{a}$ and $\mathbf{b}$ must be unit vectors
(b) $\mathbf{a}$ must be parallel to $\mathbf{b}$
(c) $\mathbf{a}$ must be perpendicular to $\mathbf{b}$
(d) $\mathbf{a}$ must be equal to $\mathbf{b}$

(c) Given, $|\mathbf{a} + \mathbf{b}| = |\mathbf{a} - \mathbf{b}| = 4$

$\Rightarrow |\mathbf{a} + \mathbf{b}|^2 = |\mathbf{a} - \mathbf{b}|^2$

$|\mathbf{a}|^2 + |\mathbf{b}|^2 + 2\mathbf{a} \cdot \mathbf{b}$

$= |\mathbf{a}|^2 + |\mathbf{b}|^2 - 2\mathbf{a} \cdot \mathbf{b}$

$\Rightarrow 4\,\mathbf{a} \cdot \mathbf{b} = 0 \Rightarrow \mathbf{a} \cdot \mathbf{b} = 0$

$\Rightarrow \mathbf{a}$ must be perpendicular to $\mathbf{b}$.

Hence, option (c) is correct.

74. If $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are coplanar, then what is $(2\mathbf{a} \times 3\mathbf{b}) \cdot 4\mathbf{c} + (5\mathbf{b} \times 3\mathbf{c}) \cdot 6\mathbf{a}$ equal to?

(a) 114 (b) 66 (c) 0 (d) – 66

(c) Given that, $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are coplanar

$\Rightarrow \quad [\mathbf{a}\;\mathbf{b}\;\mathbf{c}] = 0 \quad \ldots \text{(i)}$

$\therefore (2\mathbf{a} \times 3\mathbf{b}) \cdot 4\mathbf{c} + (5\mathbf{b} \times 3\mathbf{c}) \cdot 6\mathbf{a}$

$= 2 \cdot 3 \cdot 4[\mathbf{a}\;\mathbf{b}\;\mathbf{c}] + 5 \cdot 3 \cdot 6[\mathbf{b}\;\mathbf{c}\;\mathbf{a}]$

$= 24[\mathbf{a}\;\mathbf{b}\;\mathbf{c}] + 90[\mathbf{a}\;\mathbf{b}\;\mathbf{c}] \; \{\because [\mathbf{a}\;\mathbf{b}\;\mathbf{c}] = [\mathbf{b}\;\mathbf{c}\;\mathbf{a}]\}$

$= 24 \times 0 + 90 \times 0 = 0$

Hence, option (c) is correct.

75. Consider the following statements

1. The cross product of two unit vectors is always a unit vector.
2. The dot product of two unit vectors is always unity.
3. The magnitude of sum of two unit vectors is always greater than the magnitude of their difference.

Which of the above statements are not correct?

(a) 1 and 2 (b) 2 and 3
(c) 1 and 3 (d) 1, 2 and 3

(d) **Statement I**

Let $\mathbf{a}$ and $\mathbf{b}$ are unit vectors

i.e. $|\mathbf{a}| = |\mathbf{b}| = 1$

$\mathbf{a} \times \mathbf{b} = |\mathbf{a}||\mathbf{b}| \sin\theta\, \hat{n}$

$|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin\theta$

$= \sin\theta \in [-1, 1]$

Therefore, statement I is incorrect.

Statement II

Let $\mathbf{a}$ and $\mathbf{b}$ are unit vectors

i.e. $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta$

$= \cos\theta \in [-1, 1]$

Therefore, statement II is also incorrect.

Statement III

Let $\mathbf{a} = \hat{i}$ and $\mathbf{b} = \hat{j}$

$\Rightarrow \quad |\mathbf{a}| = 1, |\mathbf{b}| = 1$

$|\mathbf{a} + \mathbf{b}| = |\hat{i} + \hat{j}| = \sqrt{2}$

$|\mathbf{a} - \mathbf{b}| = |\hat{i} - \hat{j}| = \sqrt{2}$

76. If $\lim\limits_{x \to a} \frac{a^x - x^a}{x^a - a^a} = -1$ then what is the value of a?

(a) – 1 (b) 0 (c) 1 (d) 2

(c) $\lim\limits_{x \to a} \frac{a^x - x^a}{x^a - a^a} = -1$

$$\Rightarrow \quad \lim_{x \to a} \frac{a^x - x^a}{x^a - a^a} = -1 \left(\frac{0}{0} \text{ form}\right)$$

By using L' Hospital rule,

$$\lim_{x \to a} \frac{a^x \log_e a - a x^{a-1}}{a x^{a-1} - 0} = -1$$

$$\Rightarrow \quad \frac{a^a \log_e a - a \cdot a^{a-1}}{a \cdot a^{a-1}} = -1$$

$$\Rightarrow \quad \frac{a^a(\log_e a - 1)}{a^a} = -1$$

$\Rightarrow \quad \log_e a = -1 + 1$

$\Rightarrow \quad \log_e a = 0$

$\therefore \quad a = e^0 = 1$

$\therefore \quad a = 1$

Hence, option (c) is correct.

77. A particle starts from origin with a velocity (in m/s) given by the equation $\frac{dx}{dt} = x + 1$. The time (in second) taken by the particle to traverse a distance of 24 m is

(a) $\ln 24$ (b) $\ln 5$
(c) $2 \ln 5$ (d) $2 \ln 4$

(c) $\frac{dx}{dt} = x + 1$

$$\Rightarrow \quad \frac{dx}{x+1} = dt$$

On integrating both sides

$$\int \frac{dx}{x+1} = \int dt$$

$$\ln(x+1) = t + c \quad \dots (i)$$

Since, at $t = 0$, distance $(x) = 0$

$\therefore \ln(0+1) = 0 + c$

$\boxed{0 = c}$

$\therefore \ln(x+1) = t$

At $x = 24$ m

$t = \ln(24+1) = \ln 25 = \ln 5^2$

$t = 2\ln 5$

Hence, option (c) is correct.

78. What is $\int_0^a \frac{f(a-x)}{f(x)+f(a-x)}\,dx$ equal to?

(a) a (b) $2a$

(c) 0 (d) $\frac{a}{2}$

(d) Let $I = \int_0^a \frac{f(a-x)}{f(x)+f(a-x)}\,dx \quad \dots (i)$

$x \to a - x$

$$I = \int_0^a \frac{f(a-a+x)}{f(a-x)+f(a-a+x)}\,dx$$

$$I = \int_0^a \frac{f(x)}{f(a-x)+f(x)}\,dx \quad \dots (ii)$$

Adding Eqs. (i) and (ii), we get

$$2I = \int_0^a \frac{f(a-x)+f(x)}{f(a-x)+f(x)} \cdot dx$$

$$2I = \int_0^a 1 \cdot dx$$

$$2I = x\big|_0^a$$

$$2I = a - 0$$

$$I = \frac{a}{2}$$

Hence, option (d) is correct.

79. What is $\lim_{x \to -1} \frac{x^3 + x^2}{x^2 + 3x + 2}$ equal to?

(a) 0 (b) 1 (c) 2 (d) 3

(b) Given, $\lim_{x \to -1} = \frac{x^3 + x^2}{x^2 + 3x + 2} \left(\frac{0}{0} \text{ from}\right)$

By using L' Hospital rule

$$\lim_{x \to -1} = \frac{3x^2 + 2x}{2x + 3}$$

$$= \frac{3(-1)^2 + 2(-1)}{2(-1) + 3} = \frac{3-2}{-2+3} = 1$$

Hence, option (b) is correct.

80. If $\int_0^a [f(x) + f(-x)]dx = \int_{-a}^a g(x)dx$ then what is $g(x)$ equal to?

(a) $f(x)$ (b) $f(-x) + f(x)$

(c) $-f(x)$ (d) None of these

(a) Given that,

$\int_0^a [f(x) + f(-x)]\,dx = \int_{-a}^a g(x)\,dx$

If $g(x) = f(x)$

R.H.S. Let $I = \int_{-a}^a g(x)\,dx$

$\Rightarrow \quad I = \int_{-a}^a f(x)\,dx \quad \dots (i)$

$\left[\because \int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx\right]$

$I = \int_{-a}^a f(-x)\,dx \quad \dots (ii)$

Adding Eqs. (i) and (ii), we get

$2I = \int_{-a}^a [f(x) + f(-x)]\,dx$ (even function)

$\Rightarrow 2I = 2\int_0^a [f(x) + f(-x)]\,dx$

$I = \int_0^a [f(x) + f(-x)]dx =$ L.H.S

$\Rightarrow g(x) = f(x)$ and

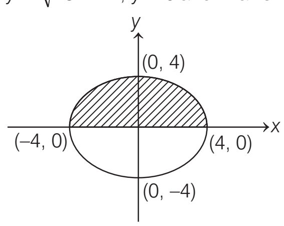
Hence, option (a) is correct.

81. What is the area bounded by $y = \sqrt{16 - x^2}$, $y \geq 0$ and the X-axis?

(a) 16π sq. units (b) 8π sq. units

(c) 4π sq. units (d) 2π sq. units

(b) Shaded portion in the diagram represents the area bounded by $y = \sqrt{16 - x^2}$, $y \geq 0$ and X-axis.



Put $y = 0$, then $16 - x^2 = 0$

$\Rightarrow x = \pm 4$

$\therefore$ Required area $= \int_{-4}^4 \sqrt{16 - x^2}\,dx$

$= 2\int_0^4 \sqrt{16 - x^2}\,dx$

$= 2\left[\frac{x}{2}\sqrt{16 - x^2} + \frac{16}{2}\sin^{-1}\frac{x}{4}\right]_0^4$

$= 2\,[0 + 8\sin^{-1} 1] = 2 \times \frac{8\pi}{2}$

$= 8\pi$ sq units

Hence, option (b) is correct.

82. The curve $y = -x^3 + 3x^2 + 2x - 27$ has the maximum slope at

(a) $x = -1$ (b) $x = 0$

(c) $x = 1$ d) $x = 2$

(c) Given that, $y = -x^3 + 3x^2 + 2x - 27$

Slope $= \frac{dy}{dx} = -3x^2 + 6x + 2$

$\therefore f'(x) = -3x^2 + 6x + 2$

For maxima/minima of $f'(x)$.

$\frac{d}{dx}[f'(x)] = -6x + 6 = 0$

$\Rightarrow 6x = 6 \Rightarrow x = 1$

At $x = 1$, $\frac{d^2}{dx^2} f'(x) = -6 < 0$

$\therefore$ At $x = 1$, $f'(x)$ is maximum.

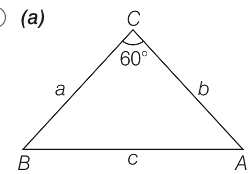
Hence, option (c) is correct.

83. A 24 cm long wire is bent to form a triangle with one of the angles as 60°. What is the altitude of the triangle having the greatest possible area?

(a) $4\sqrt{3}$ cm (b) $2\sqrt{3}$ cm

(c) 6 cm (d) 3 cm

(a)



Given, $a + b + c = 24$

$\Rightarrow \quad c = 24 - (a + b)$

Again $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

$\Rightarrow \cos 60° = \frac{a^2 + b^2 - c^2}{2ab}$

$\Rightarrow \quad \frac{1}{2} = \frac{a^2 + b^2 - c^2}{2ab}$

$\Rightarrow \quad ab = a^2 + b^2 - c^2$

$\Rightarrow \quad ab = a^2 + b^2 - [24 - (a + b)]^2$

$\Rightarrow \quad ab = a^2 + b^2 - 576 - (a + b)^2 + 48\,(a + b)$

$\Rightarrow ab = a^2 + b^2 - 576 - a^2 - b^2 - 2ab + 48\,(a + b)$

$\Rightarrow 3ab - 48(a + b) = -576$

$\Rightarrow \quad ab - 16(a + b) = -192$

$\Rightarrow \quad ab - 16a = 16b - 192$

$\Rightarrow \quad a(b - 16) = 16(b - 12)$

$\Rightarrow \quad a = \frac{16(b-12)}{b-16}$

Again $ar(\Delta ABC)$, $A = \frac{1}{2} ab \sin C$

$= \frac{1}{2} \times \frac{16(b-12)b}{b-16} \times \sin 60°$

$= \frac{1}{2} \times \frac{16(b-12)b}{b-16} \times \frac{\sqrt{3}}{2}$

$= \frac{4\sqrt{3}(b^2 - 12b)}{b-16}$

$\therefore \frac{dA}{db} = 4\sqrt{3} \left[\frac{(2b-12)(b-16) - (b^2 - 12b) \cdot 1}{b-16}\right]$

Maximum value for A

$\frac{dA}{db} = 0$

$\Rightarrow \frac{4\sqrt{3}}{b-16}[2b^2 - 32b - 12b + 192 - b^2 + 12b] = 0$

$\Rightarrow b^2 - 32b + 192 = 0$

$\Rightarrow (b-24)(b-8) = 0$

$\Rightarrow \quad b = 24, 8$

when, $b = 24$

$$a = \frac{16(24-12)}{24-16} = \frac{16 \times 12}{8} = 24$$

and $c = 24 - (24+24) = -24$

It is impossible,

when, $b = 8$

$$a = \frac{16(8-12)}{8-16} = \frac{16(-4)}{(-8)} = 8$$

and $c = 24 - (8+8) = 8$

$\therefore$ So triangle will be equilateral

$\therefore$ Height $= \frac{\sqrt{3}}{2}$ (side)

$= \frac{\sqrt{3}}{2} \times 8 = 4\sqrt{3}$ cm

84. If $f(x) = e^{|x|}$, then which one of the following is correct?

(a) $f'(0) = 1$ (b) $f'(0) = -1$
(c) $f'(0) = 0$ (d) $f'(0)$ does not exist

(d) Given that, $f(x) = e^{|x|}$

$$\Rightarrow f(x) = \begin{cases} e^x & ; x \geq 0 \\ e^{-x} & ; x < 0 \end{cases}$$

LHD at $x = 0$,

$$f'(0^-) = \lim_{h \to 0^-} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \to 0^-} \frac{e^{-h} - e^0}{h}$$

(by using L' Hospital rule)

$$= \lim_{h \to 0^-} -\frac{e^{-h}}{1} = -1$$

RHD at $x = 0$,

$$f'(0^+) = \lim_{h \to 0^+} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \to 0^+} \frac{e^h - e^0}{h}$$

$$= \lim_{h \to 0^+} \frac{e^h}{1} = e^0 = 1$$

$\because$ LHD $\neq$ RHD

$\therefore f'(x)$ does not exist at $x = 0$

Hence, option (d) is correct.

85. What is $\int \frac{dx}{\sec x + \tan x}$ equal to?

(a) $\ln(\sec x) + \ln|\sec x + \tan x| + c$
(b) $\ln(\sec x) - \ln|\sec x + \tan x| + c$
(c) $\sec x \tan x - \ln|\sec x - \tan x| + c$
(d) $\ln|\sec x + \tan x| - \ln|\sec x| + c$

(d) Let $I = \int \frac{dx}{\sec x + \tan x}$

$$I = \int \frac{1}{(\sec x + \tan x)} \times \frac{(\sec x - \tan x)}{(\sec x - \tan x)} dx$$

$$I = \int \frac{(\sec x - \tan x)}{\sec^2 x - \tan^2 x} dx$$

$$I = \int \frac{(\sec x - \tan x)}{1} dx$$

$$= \int \sec x \, dx - \int \tan x \, dx$$

$I = \log|\sec x + \tan x| - \log|\sec x| + C$

Hence, option (d) is correct.

86. What is $\int \frac{dx}{\sec^2(\tan^{-1} x)}$ equal to?

(a) $\sin^{-1} x + c$ (b) $\tan^{-1} x + c$
(c) $\sec^{-1} x + c$ (d) $\cos^{-1} x + c$

(b) Let $I = \int \frac{dx}{\sec^2(\tan^{-1} x)}$

$$I = \int \frac{dx}{1 + \tan^2(\tan^{-1} x)}$$

$[\because \sec^2 x = 1 + \tan^2 x]$

$$= \int \frac{dx}{1 + x^2}$$

$I = \tan^{-1} x + C$

Hence, option (b) is correct.

87. If $x + y = 20$ and $P = xy$, then what is the maximum value of P?

(a) 100 (b) 96
(c) 84 (d) 50

(a) Given, $x + y = 20$

$\Rightarrow y = 20 - x$

$P = xy$

$P = x(20 - x)$

$P = 20x - x^2$

$\therefore \frac{dP}{dx} = 20 - 2x$

For maxima/minima, $\frac{dP}{dx} = 0$

$20 - 2x = 0$

$\Rightarrow \quad x = 10$

At $x = 10$, $\frac{d^2P}{dx^2} = -2 < 0$

$\therefore P$ is maximum at $x = 10$

$\Rightarrow y = 20 - 10 = 10$

$\therefore$ Maximum value of $P = xy$

$= 10 \times 10 = 100$

Hence, option (a) is correct.

88. What is the derivative of $\sin(\ln x) + \cos(\ln x)$ with respect to x at $x = e$?

(a) $\frac{\cos 1 - \sin 1}{e}$ (b) $\frac{\sin 1 - \cos 1}{e}$
(c) $\frac{\cos 1 + \sin 1}{e}$ (d) 0

(a) Let $y = \sin(\ln x) + \cos(\ln x)$

$$\therefore \frac{dy}{dx} = \cos(\ln x) \cdot \frac{1}{x} + \left(-\sin(\ln x) \cdot \frac{1}{x}\right)$$

$$= \frac{1}{x}[\cos(\ln x) - \sin(\ln x)]$$

At $x = e$,

$$\frac{dy}{dx} = \frac{1}{e}[\cos(\ln e) - \sin(\ln e)]$$

$$= \frac{1}{e}[\cos 1 - \sin 1]$$

$[\because \ln e = 1]$

Hence, option (a) is correct.

89. If $x = e^t \cos t$ and $y = e^t \sin t$, then what is $\frac{dx}{dy}$ at $t = 0$ equal to?

(a) 0 (b) 1 (c) $2e$ (d) -1

(b) Given that, $x = e^t \cos t$, $y = e^t \sin t$

$$\therefore \frac{dx}{dt} = e^t \frac{d}{dt}\cos t + \cos t \frac{d}{dt} e^t$$

$$= e^t(-\sin t) + \cos t \cdot e^t$$

$$\frac{dy}{dt} = e^t \frac{d}{dt}\sin t + \sin t \cdot \frac{d}{dt} e^t$$

$$\frac{dy}{dt} = e^t \cos t + e^t \sin t$$

$$\therefore \frac{dx}{dy} = \frac{dx/dt}{dy/dt}$$

$$\frac{dx}{dy} = \frac{e^t(\cos t - \sin t)}{e^t(\cos t + \sin t)}$$

At $t = 0$,

$$\therefore \frac{dx}{dy} = \frac{\cos 0^\circ - \sin 0^\circ}{\cos 0^\circ + \sin 0^\circ} = \frac{1-0}{1+0}$$

$$\left(\frac{dx}{dy}\right)_{t=0} = 1$$

Hence, option (b) is correct.

90. What is the maximum value of $\sin 2x \cdot \cos 2x$?

(a) $\frac{1}{2}$ (b) 1 (c) 2 (d) 4

(a) Let $y = \sin 2x \cdot \cos 2x$

$$y = \frac{1}{2}[2\sin 2x \cdot \cos 2x]$$

$$y = \frac{1}{2}\sin 4x$$

Since, we know that

$$-1 \leq \sin 4x \leq 1 \Rightarrow \frac{-1}{2} \leq \frac{1}{2}\sin 4x \leq \frac{1}{2}$$

$\therefore$ Maximum value $= \frac{1}{2}$

Hence, option (a) is correct.

91. What is the derivative of e^x with respect to x^e?

(a) $\frac{xe^x}{ex^e}$ (b) $\frac{e^x}{x^e}$ (c) $\frac{xe^x}{x^e}$ (d) $\frac{e^x}{ex^e}$

(a) Let $y_1 = e^x$ and $y_2 = x^e$

$$\therefore \frac{dy_1}{dx} = e^x, \frac{dy_2}{dx} = ex^{e-1}$$

$\therefore \quad \dfrac{dy_1}{dy_2} = \dfrac{e^x}{ex^{e-1}} = \dfrac{xe^x}{ex^e}$

Hence, option (a) is correct.

92. If a differentiable function $f(x)$ satisfies $\lim\limits_{x \to -1} \dfrac{f(x)+1}{x^2-1} = -\dfrac{3}{2}$, then what is $\lim\limits_{x \to -1} f(x)$ equal to?

(a) $-\dfrac{3}{2}$ (b) -1

(c) 0 (d) 1

(b) Given, $\lim\limits_{x \to -1} \dfrac{f(x)+1}{x^2-1} = \dfrac{-3}{2}$

$\because \lim\limits_{x \to -1} \dfrac{f(x)+1}{x^2-1}$ has denominator 0 at $x = -1$

$\Rightarrow \lim\limits_{x \to -1} f(x) + 1 = 0$

$\Rightarrow \lim\limits_{x \to -1} f(x) = -1$

Hence, option (b) is correct.

93. If the function

$$f(x) = \begin{cases} a + bx, & x < 1 \\ 5, & x = 1 \\ b - ax, & x > 1 \end{cases}$$

is continuous, then what is the value of $(a+b)$?

(a) 5 (b) 10

(c) 15 (d) 20

(a) Given that, $f(x) = \begin{cases} a + bx & ; & x < 1 \\ 5 & ; & x = 1 \\ b - ax & ; & x > 1 \end{cases}$

$\because f(x)$ is continuous.

$\Rightarrow f(x)$ will be continuous at $x = 1$

$\lim\limits_{x \to 1^-} f(x) = f(1) = \lim\limits_{x \to 1^+} f(x)$

$\lim\limits_{x \to 1^-} (a + bx) = 5 = \lim\limits_{x \to 1^+} (b - ax)$

$a + b = 5 = b - a$

$\Rightarrow \quad a + b = 5$

Hence, option (a) is correct.

94. Consider the following statements in respect of the function $f(x) = \sin x$

1. $f(x)$ increases in the interval $(0, \pi)$.
2. $f(x)$ decreases in the interval $\left(\dfrac{5\pi}{2}, 3\pi\right)$.

Which of the above statement is/are correct?

(a) 1 only

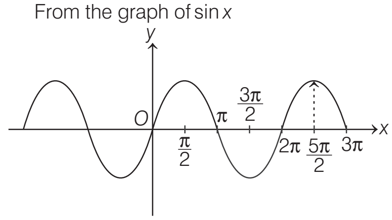
(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

(b) Given, $f(x) = \sin x$

From the graph of $\sin x$



We can see that $f(x)$ increases in $\left[0, \dfrac{\pi}{2}\right]$ and decreases in $\left[\dfrac{\pi}{2}, \pi\right]$ and $\left(\dfrac{5\pi}{2}, 3\pi\right)$.

$\Rightarrow$ Statement-1 is wrong and Statement-2 is correct.

Hence, option (b) is correct.

95. What is the domain of the function $f(x) = 3^x$?

(a) $(-\infty, \infty)$ (b) $(0, \infty)$

(c) $[0, \infty)$ (d) $(-\infty, \infty) - \{0\}$

(a) Given, $f(x) = 3^x$

$\because$ We know that, domain of exponential function is $(-\infty, \infty)$.

$\therefore$ Domain of $3^x = (-\infty, \infty)$

Hence, option (a) is correct.

96. If the general solution of a differential equation is $y^2 + 2cy - cx + c^2 = 0$, where c is an arbitrary constant, then what is the order of the differential equation?

(a) 1 (b) 2 (c) 3 (d) 4

(a) Given that, $y^2 + 2cy - cx + c^2 = 0$

Since, the above equation contains only one variable constant.

Hence, order of the differential equation = 1

Hence, option (a) is correct.

97. What is the degree of the following differential equation?

$$x = \sqrt{1 + \frac{d^2y}{dx^2}}$$

(a) 1

(b) 2

(c) 3

(d) Degree is not defined

(a) Let $x = \sqrt{1 + \dfrac{d^2y}{dx^2}}$

$\Rightarrow x^2 = 1 + \dfrac{d^2y}{dx^2} \Rightarrow \left(\dfrac{d^2y}{dx^2}\right)^1 = x^2 - 1$

$\therefore$ Degree = exponent of highest order derivative = 1

Hence, option (a) is correct.

98. Which one of the following differential equations has the general solution $y = ae^x + be^{-x}$?

(a) $\dfrac{d^2y}{dx^2} + y = 0$ (b) $\dfrac{d^2y}{dx^2} - y = 0$

(c) $\dfrac{d^2y}{dx^2} + y = 1$ (d) $\dfrac{dy}{dx} - y = 0$

(b) Given, $y = ae^x + be^{-x}$

$\therefore \quad \dfrac{dy}{dx} = ae^x - be^{-x}$

$\dfrac{d^2y}{dx^2} = ae^x + be^{-x} = y$

$\Rightarrow \dfrac{d^2y}{dx^2} - y = 0$

Hence, option (b) is correct.

99. What is the solution of the following differential equation?

$$\ln\left(\frac{dy}{dx}\right) + y = x$$

(a) $e^x + e^y = c$ (b) $e^{x+y} = c$

(c) $e^x - e^y = c$ (d) $e^{x-y} = c$

(c) Given, $\ln\left(\dfrac{dy}{dx}\right) + y = x$

$\Rightarrow \quad \ln\left(\dfrac{dy}{dx}\right) = x - y$

$\Rightarrow \quad \dfrac{dy}{dx} = e^{x-y}$

$\Rightarrow \quad \dfrac{dy}{dx} = \dfrac{e^x}{e^y}$

$\Rightarrow \quad e^y dy = e^x dx$

On integrating both sides,

$\int e^y dy = \int e^x dx$

$e^y + c = e^x \Rightarrow e^x - e^y = c$

Hence, option (c) is correct.

100. What is $\int e^{(2\ln x + \ln x^2)} dx$ equal to?

(a) $\dfrac{x^4}{4} + C$ (b) $\dfrac{x^3}{3} + C$

(c) $\dfrac{2x^5}{5} + C$ (d) $\dfrac{x^5}{5} + C$

(d) Let $I = \int e^{(2\ln x + \ln x^2)} dx$

$= \int e^{(\ln x^2 + \ln x^2)} dx$

$= \int e^{2\ln x^2} dx = \int e^{\ln (x^2)^2} dx = \int x^4 dx$

$I = \dfrac{x^5}{5} + C$

Hence, option (d) is correct.

101. Consider the following measures of central tendency for a set of N numbers

1. Arithmetic mean
2. Geometric mean

Which of the above uses/use all the data?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

(c) Since, we know that the measures of central tendency are Mean, Median and Mode.
Where Arithmetic Mean and Geometric mean are the type mean.
Hence, option (c) is correct.

102. The numbers of Science, Arts and Commerce graduates working in a company are 30, 70 and 50 respectively. If these figures are represented by a pie chart, then what is the angle corresponding to Science graduates?
(a) 36° (b) 72°
(c) 120° (d) 168°

(b) The ratio of Science, Arts and Commerce graduates
$= 30:70:50 = 3:7:5$
∴ Angle corresponding to Science graduates $= \frac{3}{3+7+5} \times 360^\circ$
$= \frac{3}{15} \times 360^\circ = 72^\circ$
Hence, option (b) is correct.

103. For a histogram based on a frequency distribution with unequal class intervals, the frequency of a class should be proportional to
(a) the height of the rectangle
(b) the area of the rectangle
(c) the width of the rectangle
(d) the perimeter of the rectangle

(b) Since, we know that for a histogram, based on a frequency distribution with equal intervals, the frequency of a class is proportional to height of the rectangle and for a histogram based on frequency distribution with unequal intervals, the frequency of a class is proportional to Area of the rectangle.
Hence, option (b) is correct.

104. The coefficient of correlation is independent of
(a) change of scale only
(b) change of origin only
(c) both change of scale and change of origin
(d) neither change of scale nor change of origin

(c) Since, we know that coefficient of correlation is independent of both change of scale and change of origin.
Hence, option (c) is correct.

105. The following table gives the frequency distribution of number of peas per pea pod of 198 pods

Number of peas	1	2	3	4	5	6	7
Frequency	4	33	76	50	26	8	1

What is the median of this distribution?
(a) 3 (b) 4 (c) 5 (d) 6

(a)

Number of Peas	Frequency	Cumulative frequency
1	4	4
2	33	37
3	76	113
4	50	163
5	26	189
6	8	197
7	1	198
	$\Sigma f = 198$	

$\because N = 198, \frac{N}{2} = \frac{198}{2} = 99$

$$\text{Median} = \frac{\frac{N}{2}\text{th term} + \left(\frac{N}{2}+1\right)\text{th term}}{2}$$
$$= \frac{99\text{th term} + 100\text{th term}}{2}$$
$$= \frac{3+3}{2} = 3$$
∴ Median = 3
Hence, option (a) is correct.

106. If M is the mean of n observations $x_1 - k, x_2 - k, x_3 - k, \ldots, x_n - k$, where k is any real number, then what is the mean of $x_1, x_2, x_3, \ldots, x_n$?
(a) M (b) $M + k$
(c) $M - k$ (d) kM

(b) Given that,
Mean of $x_1 - k, x_2 - k, x_3 - k, \ldots, x_n - k$
$$\therefore M = \frac{(x_1 - k) + (x_2 - k) + \ldots + (x_n - k)}{n}$$
$$M = \frac{(x_1 + x_2 + \ldots + x_n)}{n} - \frac{nk}{n}$$
$$M + k = \frac{x_1 + x_2 + \ldots + x_n}{n}$$
∴ Mean of $x_1, x_2, x_3, \ldots, x_n = M + k$
Hence, option (b) is correct.

107. What is the sum of deviations of the variate values 73, 85, 92, 105, 120 from their mean?
(a) – 2 (b) – 1
(c) 0 (d) 5

(c) Mean of 73, 85, 92, 105, 120
$$\bar{x} = \frac{73 + 85 + 92 + 105 + 120}{5}$$
$$= \frac{475}{5}$$
$\bar{x} = 95$
∴ Sum of deviations from their mean
$= (73 - 95) + (85 - 95) + (92 - 95) + (105 - 95) + (120 - 95)$
$= -22 - 10 - 3 + 10 + 25 = 0$
Hence, option (c) is correct.

108. Let x be the HM and y be the GM of two positive numbers m and n. If $5x = 4y$, then which one of the following is correct?
(a) $5m = 4n$ (b) $2m = n$
(c) $4m = 5n$ (d) $m = 4n$

(d) Given, two positive numbers are m and n.
∴ H.M. of m and $n = \frac{2mn}{m+n}$
$$x = \frac{2mn}{m+n} \quad \ldots \text{(i)}$$
G.M. of m and $n = \sqrt{mn}$
$$y = \sqrt{mn} \quad \ldots \text{(ii)}$$
$\because 5x = 4y$
$$5\left(\frac{2mn}{m+n}\right) = 4\sqrt{mn}$$
Squaring both sides, we get
$$\left(\frac{5mn}{m+n}\right)^2 = (2\sqrt{mn})^2$$
$$\Rightarrow \frac{25m^2n^2}{m^2 + n^2 + 2mn} = 4mn$$
$$\Rightarrow 25mn = 4m^2 + 4n^2 + 8mn \quad [\because m \neq 0, n \neq 0]$$
$$\Rightarrow 4m^2 + 4n^2 - 17mn = 0$$
$$\Rightarrow 4m^2 - 16mn - mn + 4n^2 = 0$$
$$\Rightarrow 4m(m - 4n) - n(m - 4n) = 0$$
$$\Rightarrow (m - 4n)(4m - n) = 0$$
$$\Rightarrow m = 4n \text{ or } n = 4m$$
Hence, option (d) is correct.

109. If the mean of a frequency distribution is 100 and the coefficient of variation is 45%, then what is the value of the variance?
(a) 2025 (b) 450
(c) 45 (d) 4.5

(a) Since, we know that
Coefficient of variation
$$(CV) = \frac{\sigma}{x} \times 100 \quad \ldots \text{(i)}$$
Where σ is standard deviation and $\bar{x}$ is mean.

Given, $\bar{x} = 100$ and $CV = 45\%$

From Eqs. (i), $45 = \frac{\sigma}{100} \times 100$

$\Rightarrow \quad \sigma = 45$

$\therefore$ Variance $= \sigma^2 = (45)^2 = 2025$

Hence, option (a) is correct.

110. Let two events A and B be such that $P(A) = L$ and $P(B) = M$. Which one of the following is correct?

(a) $P(A|B) < \frac{L+M-1}{M}$

(b) $P(A|B) > \frac{L+M-1}{M}$

(c) $P(A|B) \geq \frac{L+M-1}{M}$

(d) $P(A|B) = \frac{L+M-1}{M}$

(c) Given, $P(A) = L$, $P(B) = M$

$\because P(A \mid B) = \frac{P(A \cap B)}{P(B)}$

$P(A \mid B) = \frac{P(A) + P(B) - P(A \cup B)}{P(B)}$

$P(A \mid B) = \frac{L + M - P(A \cup B)}{P(B)}$

$\therefore \; P(A \cup B) = L + M - P(B)P(A \mid B)$

$\because \; 0 \leq P(A \cup B) \leq 1$

$\Rightarrow L + M - P(B) \cdot P(A \mid B) \leq 1$

$\Rightarrow P(B) \cdot P(A \mid B) \geq L + M - 1$

$P(A \mid B) \geq \frac{L+M-1}{M} \quad [\because P(B) = M]$

Hence, option (c) is correct.

111. For which of the following sets of numbers do the mean, median and mode have the same value?

(a) 12, 12, 12, 12, 24
(b) 6, 18, 18, 18, 30
(c) 6, 6, 12, 30, 36
(d) 6, 6, 6, 12, 30

(b) For option (a), Mean

$= \frac{12 + 12 + 12 + 12 + 24}{5}$

$= 14 \cdot 4 \neq$ mode (12)

For option (b), Mean

$= \frac{6 + 18 + 18 + 18 + 30}{5} = 18$

Mode = 18, Median = 18

Hence, for the data 6, 18, 18, 18, 30,

Mean = Mode = Median = 18

Hence, option (b) is correct.

112. The mean of 12 observations is 75. If two observations are discarded, then the mean of the remaining observations is 65. What is the mean of the discarded observations?

(a) 250
(b) 125
(c) 120
(d) Cannot be determined due to insufficient data

(b) Given, mean of 12 observations = 75 and M.

$\therefore \quad \bar{x} = \frac{\sum_{i=1}^{12} x_i}{12} \Rightarrow 75 = \frac{\sum_{i=1}^{12} x_i}{12}$

$\Rightarrow \quad \sum_{i=1}^{12} x_i = 900 \quad \ldots \text{(i)}$

Let observations x_{11} and x_{12} is discarded

then mean $= \frac{\sum_{i=1}^{10} x_i}{10} = 65$

$\therefore \quad \sum_{i=1}^{10} x_i = 10 \times 65 = 650 \quad \ldots \text{(ii)}$

From Eqs. (i)

$\sum_{i=1}^{12} x_i = 900$

$\Rightarrow \sum_{i=1}^{10} x_i + x_{11} + x_{12} = 900$

$\Rightarrow 650 + x_{11} + x_{12} = 900$

$\Rightarrow \quad x_{11} + x_{12} = 250$

$\therefore$ Mean of x_{11} and $x_{12} = \frac{250}{2} = 125$

Hence, option (b) is correct.

113. If k is one of the roots of the equation $x(x+1)+1=0$, then what is its other root?

(a) 1 (b) $-k$
(c) k^2 (d) $-k^2$

(c) Given, quadratic equation

$x(x + 1) + 1 = 0$

$x^2 + x + 1 = 0 \quad \ldots \text{(i)}$

Since, we know that ω, ω^2 are the roots of Equation when

$\omega = \frac{-1 + \sqrt{3}i}{2}$ and $\omega^2 = \frac{-1 - \sqrt{3}i}{2}$

$\Rightarrow$ If one of the roots of Eqs. (i) is k, then other root will be k^2.

Hence, option (c) is correct.

114. The geometric mean of a set of observations is computed as 10. The geometric mean obtained when each observation x_i is replaced by $3x_i^4$ is

(a) 810 (b) 900
(c) 30000 (d) 81000

(c) Given that, geometric mean of a set of observations = 10

Since, we know that if

Geometric mean of $x_1, x_2, x_3, \ldots x_n$ is G.

$\Rightarrow$ Geometric mean of $x_1^2, x_2^2, x_3^2, \ldots x_n^2$ is G^2

Geometric mean of $fx_1^2, fx_2^2, \ldots, fx_n^2$ is fG^2.

$\therefore$ Required geometric mean $= 3(10)^4$

$= 3 \times 10000 = 30000$

Hence, option (c) is correct.

115. If $P(A \cup B) = \frac{5}{6}$, $P(A \cap B) = \frac{1}{3}$ and $P(\bar{A}) = \frac{1}{2}$, then which of the following is/are correct?

1. A and B are independent events.
2. A and B are mutually exclusive events.

Select the correct answer using the code given below.

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

(a) Given, $P(A \cup B) = \frac{5}{6}$, $P(A \cap B) = \frac{1}{3}$

$P(\bar{A}) = \frac{1}{2}$

$\Rightarrow \quad P(A) = 1 - \frac{1}{2} = \frac{1}{2}$

$\therefore \; P(B) = P(A \cup B) - P(A) + P(A \cap B)$

$= \frac{5}{6} - \frac{1}{2} + \frac{1}{3} = \frac{4}{6} = \frac{2}{3}$

$\because$ If A and B are independents, then $P(A \cap B) = P(A) \cdot P(B)$

$\because P(A \cap B) = \frac{1}{3}$

and $P(A) \cdot P(B) = \frac{1}{2} \times \frac{2}{3} = \frac{1}{3}$

$\Rightarrow$ Statement-1 is correct.

If A and B are mutually exclusive, then

$P(A \cup B) = P(A) + P(B)$

$\because \quad P(A \cup B) = \frac{5}{6}$

$\therefore \quad P(A) + P(B) = \frac{1}{2} + \frac{2}{3} = \frac{7}{6}$

$\Rightarrow$ Statement-2 is wrong.

Hence, option (a) is correct.

116. The average of a set of 15 observations is recorded, but later it is found that for one observation, the digit in the tens place was wrongly recorded as 8 instead of 3. After correcting the observation, the average is

(a) reduced by $\frac{1}{3}$ (b) increased by $\frac{10}{3}$
(c) reduced by $\frac{10}{3}$ (d) reduced by 50

(c) Let unit digit for wrongly recorded observation = b

When tens digit is 8, then number

$= 10 \times 8 + b = 80 + b$

When tens digit is 3, then number

$= 10 \times 3 + b = 30 + b$

$\Rightarrow$ One observation is recorded $((80 + b) - (30 + b))$ more while calculating average.

Hence, after correcting the observation, the average will be reduced by

$$= \frac{\{(80 + b) - (30 + b)\}}{15}$$

$$= \frac{50}{15} = \frac{10}{3}$$

Hence, option (c) is correct.

117. A coin is tossed twice. If E and F denote occurrence of head on first toss and second toss respectively, then what is $P(E \cup F)$ equal to?

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$
(c) $\frac{3}{4}$ (d) $\frac{1}{3}$

(c) Given that, a coin is tossed twice.

$\therefore \quad S = \{HH, HT, TH, TT\}$

Given, E be the event of occurrence of head on first toss and F be the event of occurrence of head on second toss.

$\therefore \quad E = \{HH, HT\}$

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{2}{4} = \frac{1}{2}$

$F = \{TH, HH\}, P(F) = \frac{2}{4} = \frac{1}{2}$

$\because \quad E \cap F = \{HH\}$

$\therefore P(E \cap F) = \frac{1}{4}$

$\therefore P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$= \frac{1}{2} + \frac{1}{2} - \frac{1}{4}$

$= 1 - \frac{1}{4} = \frac{3}{4}$

Hence, option (c) is correct.

118. In a binomial distribution, the mean is $\frac{2}{3}$ and variance is $\frac{5}{9}$. What is the probability that random variable $X = 2$?

(a) $\frac{5}{36}$ (b) $\frac{25}{36}$
(c) $\frac{25}{54}$ (d) $\frac{25}{216}$

(d) For a binomial distribution mean $= np$ and variance $= npq$

Where p is probability of success and q is the probability of unsuccess and n is number of observations.

$\therefore$ Given, $np = \frac{2}{3}, npq = \frac{5}{9}$

$\Rightarrow \quad \frac{2}{3} \cdot q = \frac{5}{9}$

$q = \frac{5}{6}$

$\therefore \quad p = 1 - q = 1 - \frac{5}{6}$

$= \frac{1}{6}$

$\because \quad np = \frac{2}{3}$

$\Rightarrow \quad n\left(\frac{1}{6}\right) = \frac{2}{3}$

$\Rightarrow \quad n = 4$

$\because p(X = x) = {}^nC_x p^x q^{n-x}$

$\therefore P(X = 2) = {}^4C_2 p^2 q^{4-2}$

$= 6 \times \left(\frac{1}{6}\right)^2 \times \left(\frac{5}{6}\right)^2 = \frac{25}{216}$

Hence, option (d) is correct.

119. If the mode of the scores 10, 12, 13, 15, 15, 13, 12, 10, x is 15, then what is the value of x?

(a) 10 (b) 12
(c) 13 (d) 15

(d) Given, observations are

10, 12, 13, 15, 15, 13, 12, 10, x

$\because$ Mode = 15

Scores	Frequency
10	2
12	2
13	2
15	2

Since, frequency of all other numbers is same as frequency of 15.

But mode is the number of highest frequency.

$\therefore \quad x$ should be 15

Hence, option (d) is correct.

120. If A and B are two events such that $P(A) = \frac{3}{4}$ and $P(B) = \frac{5}{8}$, then consider the following statements

1. The minimum value of $P(A \cup B)$ is $\frac{3}{4}$.
2. The maximum value of $P(A \cap B)$ is $\frac{5}{8}$.

Which of the above statements is/are correct?

(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

(c) Given, $P(A) = \frac{3}{4}$

$P(B) = \frac{5}{8}$

$\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$

and $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

Here $P(A \cup B)$ will be minimum if $P(A \cap B)$ is maximum and *vice-versa*.

Since, minimum value of $P(A \cap B)$ is zero and maximum value of $P(A \cap B)$ is minimum $(P(A), P(B))$.

$\Rightarrow$ maximum $P(A \cap B)$

$= \text{minimum}\left(\frac{3}{4}, \frac{5}{8}\right)$

$= \frac{5}{8}$

$\Rightarrow$ Statement-2 is correct.

Also, minimum value of $P(A \cup B)$ is maximum $(P(A), P(B))$

$\therefore$ Minimum value of

$P(A \cup B) = \text{maximum}\left(\frac{3}{4}, \frac{5}{8}\right)$

$= \frac{3}{4}$

$\Rightarrow$ Statement-1 is correct.

Hence, correct option is (c).