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INDEX

NETWORK THEORY

■ Unit and Dimension

Quantities	Unit	Dimension
Resistance	Ohm	$\left[ML^2T^{-3}A^{-2}\right]$
Resistivity	Ohm-meter	$\left[ML^3T^{-3}A^{-2}\right]$
Conductivity	mho/m or Siemens/m	$[M^{-1}L^{-3}T^3A^2]$
Voltage	Volt	$[ML^2T^{-3}A^{-1}]$
Current	Ampere	$[A]$
Electric Power	Watt	$[ML^2T^{-3}]$
Electric Energy	kWh	$[ML^2T^{-2}]$
Permittivity	Farad/meter	$[M^{-1}L^{-3}T^4A^2]$
Electric field intensity	V/m or N/C	$[MLT^3A^{-1}]$
Electric flux density	C/m^2	$[MLT^{-3}A^{-1}]$
Capacitance	Farad	$[M^{-1}L^{-2}T^4A^2]$
Inductance	Henry	$[ML^2T^{-2}A^{-2}]$
Permeability	Henry/meter	$[MLT^{-2}A^{-2}]$
Magnetic Flux density	Tesla or Weber/m^2	$[ML^{-1}T^{-2}A^{-1}]$
Magnetic field intensity	A/m or Oersted or N/Wb	$[MT^{-2}A^{-1}]$
mmf	AT or Gilbert	$[A]$
Reluctance	AT/Wb or per Henry	$[M^{-1}L^{-2}T^2A^2]$
Permeance	Wb/AT	$[ML^2T^{-2}A^{-2}]$
Luminous flux	lumen	$[ML^2T^{-3}]$
Illumination	Lux or lumen/ m^2	$[MT^{-3}]$

■ Resistor

Resistance (R) = $\frac{\rho\ell}{A}$

If wire is stretch n times then- $R' = n^2R$

If wire is compressed n times then- $R' = \frac{R}{n^2}$

Ohm's law for resistor- $R = \frac{V}{I}$

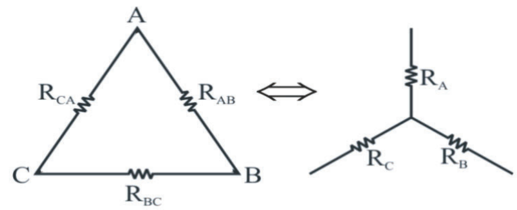
- **Series combination of resistor**

$$R_{eq} = R_1 + R_2 + R_3 + ... + R_n$$

- **Parallel combination of resistor-**

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ... + \frac{1}{R_n}$$

- **Conversion of Resistance–**



Delta to star	Star to Delta
$R_A = \frac{R_{AB} \times R_{CA}}{R_{AB} + R_{BC} + R_{CA}}$	$R_{AB} = R_A + R_B + \frac{R_AR_B}{R_C}$
$R_B = \frac{R_{AB} \times R_{BC}}{R_{AB} + R_{BC} + R_{CA}}$	$R_{BC} = R_B + R_C + \frac{R_BR_C}{R_A}$
$R_C = \frac{R_{BC} \times R_{CA}}{R_{AB} + R_{BC} + R_{CA}}$	$R_{CA} = R_C + R_A + \frac{R_CR_A}{R_B}$

- **Resistance Vs Temperature** $R_t = R_o\left(1 + \alpha\Delta T\right)$
- **AC Resistance** $R_{AC} = 1.6R_{DC}$
- **Resistivity of materials-**

$$\rho = \frac{1}{\sigma(\text{conductivity})}\ \Omega\text{-m}$$

- **Cable insulation Resistance-**

$$R = \frac{\rho}{2\pi\ell}\log_e\frac{r_2}{r_1} = \frac{2.303\rho}{2\pi\ell}\log_{10}\frac{r_2}{r_1}$$

- **Colour coding of Resistance**

Colour	Value	Multiplier	Tolerance
Black	0	1	–
Brown	1	10	± 1%
Red	2	10^2	± 2%
Orange	3	10^3	± 3%
Yellow	4	10^4	± 4%
Green	5	10^5	± 0.5%
Blue	6	10^6	± 0.25%
Violet	7	10^7	± 0.10%
Grey	8	10^8	± 0.05 %
White	9	10^9	–
Gold	–	10^{-1}	± 5%
Silver	–	10^{-2}	± 10%
(None)	–		± 20%

- **Formula for 4 Band resistor**

$R = AB \times 10^C \pm \text{Tolerance}$

Where A → I^{st} significant digit

B → 2^{nd} Significant digit

10^C → multiplier

- **Metal and melting point (in ^{0}C)**

Metal	Melting point (in ºC)	Metal	Melting point (in ºC)
Copper	1084	Chromium	1850
Magnesium	650	Molybdenum	2622
Zinc	419.5	Tungsten	3390
Aluminium	658.6	Iron	1538
Tin	231.8	Cobalt	1490
Lead	327.4	Nickel	1452
Silver	961	Carbon	3550

- **Behaviour of resistor**

Resistor is a linear, bilateral and passive element.

- **Material and dielectric constant**

Material	Dielectric Constant	Material	Dielectric Constant
Vacuum	1	Paper	3.6
Glass	5-12	Polystyrene	2.6
Mica	4-8	Air (100 atm)	1.0548
Germanium	16	Porcelain	5-6.2
Water	80.6	Rubber	2.5
Air (1 atm)	1.00059		

■ **Capacitor**

a. Capacitance of Capacitor $C = \frac{\varepsilon_o \varepsilon_r A}{d}$ Farad

b. Capacitance of different Dielectric having different thickness and relative permittivities –

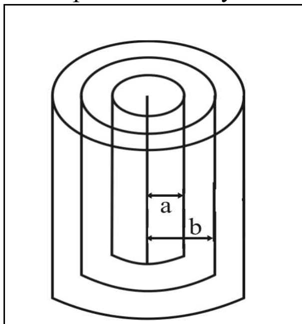
$$C = \frac{\varepsilon_o A}{\frac{t_1}{\varepsilon_{r_1}} + \frac{t_2}{\varepsilon_{r_2}} + \frac{t_3}{\varepsilon_{r_3}}} \text{ Farad}$$

c. When some part of parallel plate capacitor have air medium and some part have another medium then capacitance- $C = \frac{\varepsilon_o A}{\left[d - \left(t - \frac{t}{\varepsilon_r}\right)\right]}$ Farad

d. When some part of parallel plate capacitor have air medium in horizontal direction and some part have another medium then capacitance-

$$C = \frac{\varepsilon_o A}{d}\left[\frac{1+\varepsilon_r}{2}\right] = C_{air}\left[\frac{1+\varepsilon_r}{2}\right] \text{ Farad}$$

e. Capacitance of cylindrical capacitor-



$$C = \frac{2\pi\varepsilon_o\varepsilon_r \ell}{\log_e b/a} \text{ Farad}$$

$$C = \frac{2\pi\varepsilon_o\varepsilon_r \ell}{2.303 \log_{10} b/a} \text{ Farad}$$

f. **Capacitance of variable capacitor-**

$$C = \frac{(n-1)\varepsilon_o\varepsilon_r A}{d} \text{ Farad}$$

Where n = no. of plates

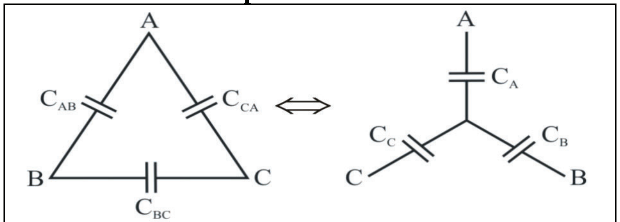
- **Series combination of capacitors**

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ... + \frac{1}{C_n}$$

- **Parallel combination of capacitors**

$$C_{eq} = C_1 + C_2 + C_3 + ... + C_n$$

- **Conversion of capacitors–**



Delta to star	Star to Delta
$C_A = C_{AB} + C_{CA} + \frac{C_{AB}\, C_{CA}}{C_{BC}}$	$C_{AB} = \frac{C_A\, C_B}{C_A + C_B + C_C}$
$C_B = C_{AB} + C_{BC} + \frac{C_{AB}\, C_{BC}}{C_{CA}}$	$C_{BC} = \frac{C_B\, C_C}{C_A + C_B + C_C}$
$C_C = C_{BC} + C_{CA} + \frac{C_{BC}\, C_{CA}}{C_{AB}}$	$C_{CA} = \frac{C_C\, C_A}{C_A + C_B + C_C}$

- **Charging and Discharging of capacitor**

Charging	Discharging	Time constant for capacitor
Current equation $I_C(t) = I_0 e^{-t/\tau}$	Current equation $I_C(t) = -I_0 e^{-t/\tau}$	$\tau = R_{th} C_{eq}$
Voltage equation $V_C(t) = V_0\left(1 - e^{-t/\tau}\right)$	Voltage equation $V_C(t) = V_0\, e^{-t/\tau}$	
Charge equation $q_c(t) = Q_0\left(1 - e^{-t/\tau}\right)$	Charge equation $q_{c(t)} = Q_0\, e^{-t/\tau}$	

- **Transient equation for capacitor–**

$$V_c(t) = V_c(\infty) + \left[V_c(0^+) - V_c(\infty)\right] e^{-t/\tau} t > 0$$

- **Concept of short circuit and open circuit of capacitor with respect to time-**

$$i_c = \frac{CdV}{dt} \; ; \qquad V_c(t) = \frac{1}{C}\int_0^t i_c(t)dt + V(0^-)$$

- **Behaviour of capacitor with initial condition-**

at $t = 0$ → act as a voltage source
at $t = \infty$ → act as open circuit

- **Behaviour of capacitor without initial condition–**

at $t = 0$ → act as short circuit
at $t = \infty$ → act as open circuit

- **Some important point Regarding to capacitor–**

a. Capacitor opposes rate of change of voltage.
$V_C(0^-) = V_C(0^+)$

b. Capacitor Stores the energy in electric field.

c. While charging, $I_c(t) = \dfrac{dV_c(t)}{dt}$ increased and $I_c(t)$ must be positive.

d. While discharge, $I_c(t) = \dfrac{dV_c(t)}{dt}$ decreased but $I_c(t)$ must be negative.

e. While a capacitor charge and discharge polarity of dc voltage of capacitor never change.

- **Energy stored in capacitor –**

$$E = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C} \text{ Joules}$$

■ Inductor

- **Faraday's law for an inductor**

$$V_L = L\frac{di}{dt} \qquad i(t) = \frac{1}{L}\int_0^t V_L dt + i(0^-)$$

- **Coefficient of self inductance**

$$L = \frac{N\phi}{i} = \frac{V_L}{di/dt} = \frac{N^2}{S} = \frac{\mu_o\mu_r aN^2}{\ell}$$

Where N = no. of turns in the coil, 'a' is cross sectional area and ℓ is the length of the coil.
Also,
μ_r = Relative permeability
$\mu_0 = 4\pi \times 10^{-7}$ Henry/meter
S = Self inductance

- **Coefficient of mutual Inductance**

$$M = \frac{N_2\phi_1}{i_1} = \frac{V_{L_2}}{di_1/dt} = \frac{\mu_o\mu_r N_1N_2\pi r^2}{\ell_1}$$

$$M = \frac{\mu_o\mu_r N_1N_2A}{\ell_1}$$

- **Coupling factor of Inductance** $K = \dfrac{M}{\sqrt{L_1L_2}}$

- **Inductor Voltage & Current**

a. Inductor with initial condition

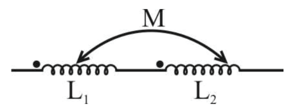
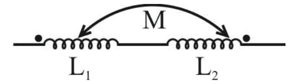
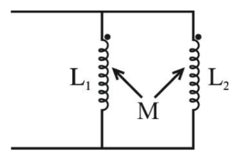
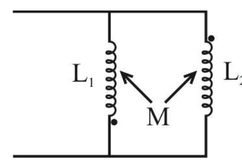
at $t = 0 \rightarrow$ act as a current source
at $t = \infty \rightarrow$ act as short circuit

b. Inductor without initial condition

at $t = 0 \rightarrow$ act as an open circuit
at $t = \infty \rightarrow$ act as short circuit

- **Energy stored in inductor-** $E = \dfrac{1}{2}Li^2$

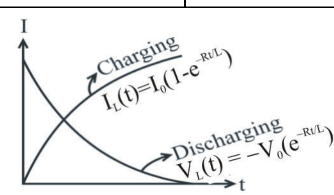
- **Magnetic coupling of Inductor**

Series connection of Inductor	
i. Adding nature $L = L_1 + L_2 + 2M$ 	**ii. Subtracting nature** $L = L_1 + L_2 - 2M$ 
Parallel connection of Inductor	
i. Adding nature $L_{eq} = \dfrac{L_1L_2 - M^2}{L_1 + L_2 - 2M}$ 	**ii. Subtracting nature** $L_{eq} = \dfrac{L_1L_2 - M^2}{L_1 + L_2 + 2M}$ 

- **Transient equation for inductor**

$$i_L(t) = i_L(\infty) + \left[i_L(0^+) - i_L(\infty)\right]e^{-t/\tau} \quad t > 0$$

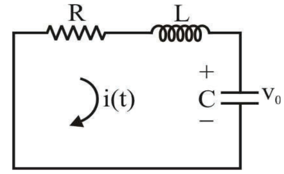
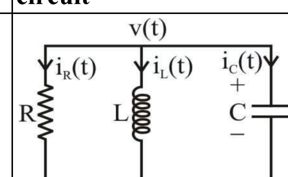
- **Charging and discharging of inductor**

Charging	**Discharging**
Current $I_L(t) = I_o\left(1 - e^{-t/\tau}\right)$	Current $I_L(t) = I_o e^{-t/\tau}$
Voltage $V_L(t) = V_o e^{-t/\tau}$	Voltage $V_L(t) = -V_o e^{-t/\tau}$
	
• **Time constant for inductor** $\tau = \dfrac{L}{R}$ second	

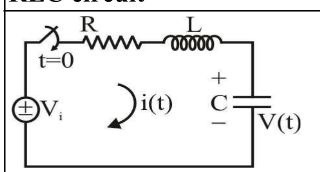
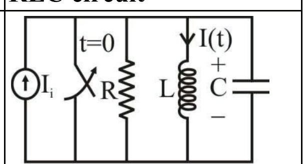
- ## Behaviour of Inductor

a. Inductor is linear, bilateral and passive element.
b. Inductor opposes the sudden change of current.
$I_L(0^-) = I_L(0^+)$
c. Inductor allows the sudden change of voltage.
d. Inductor is an energy storing element.
e. Inductor does not dissipate any power, it only stores energy.

- **Transient equation for RLC circuit -**

Source free series RLC circuit	**Source free parallel RLC circuit**
	
roots $s_1, s_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2}$	roots $s_1, s_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2}$

Overdamped	Overdamped
• $\alpha > \omega_o \Rightarrow C > 4L/R^2$ • $i(t) = A_1e^{s_1t} + A_2e^{s_2t}$ • $\xi > 1$	• $\alpha > \omega_o \Rightarrow L > 4R^2C$ • $v(t) = A_1e^{s_1t} + A_2e^{s_2t}$ • $\xi > 1$
Critically damped	**Critically damped**
• $\alpha = \omega_o \Rightarrow C = 4L/R^2$ • $i(t) = (A_1 + A_2t)e^{-\alpha t}$ • $\xi = 1$	• $\alpha = \omega_o \Rightarrow L = 4R^2C$ • $v(t) = (A_1 + A_2t)e^{-\alpha t}$ • $\xi = 1$
Under damped	**Under damped**
• $\alpha < \omega_o \Rightarrow C < 4L/R^2$ • $s_1, s_2 = -\alpha \pm j\omega_d$ Where $\omega_d = \sqrt{\omega_o^2 - \alpha^2}$ • $i(t) = e^{-\alpha t}(A_1\cos\omega_dt + A_2 \sin\omega_dt)$ • Time constant = $\frac{1}{\alpha}$ • Period (T) = $\frac{2\pi}{\omega_o}$ • $\xi < 1$	• $\alpha < \omega_o \Rightarrow L < 4R^2C$ • $s_1, s_2 = -\alpha \pm j\omega_d$ Where $\omega_d = \sqrt{\omega_o^2 - \alpha^2}$ • $v(t) = e^{-\alpha t}(A_1\cos\omega_dt + A_2 \sin\omega_dt)$ • Time constant = $\frac{1}{\alpha}$ • Period (T) = $\frac{2\pi}{\omega_o}$ • $\xi < 1$
Note : The constant A_1 & A_2 can be determined from initial condition $i(0^+)$ & $di(0^+)/dt$ and $v(0^+)$ & $dv(0^+)/dt$	

Step response of series RLC circuit	Step response of parallel RLC circuit
	
Complete solution	**Complete solution**
• Over damped - $V(t) = V_i + A_1e^{s_1t} + A_2e^{s_2t}$ • Critically damped $V(t) = V_i + (A_1 + A_2)e^{-\alpha t}$ • Under damped - $V(t) = V_i + e^{-\alpha t}(A_1\cos\omega_dt + A_2\sin\omega_dt)$	• Over damped - $I(t) = I_i + A_1e^{s_1t} + A_2e^{s_2t}$ • Critically damped $I(t) = I_i + (A_1 + A_2)e^{-\alpha t}$ • Under damped - $I(t) = I_i + e^{-\alpha t}(A_1\cos\omega_dt + A_2\sin\omega_dt)$

❑ Electric Circuit

- **Classification of circuit element**

➢ **Active and Passive elements**

Active	Passive
Capable of delivering the energy for infinite time	Not capable of delivering energy for infinite time.
Ex. Current source op-amp, BJT etc.	**Ex.** R,L,C, Bulb, Transformer etc.

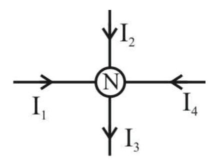
➢ If the ratio of voltage to current at any point on characteristic curve is negative then the element is active otherwise it will be passive.

➢ **Bidirectional and unidirectional element**

Bidirectional	Unidirectional
If the characteristic or property of element is independent of direction of flow of current, element is bidirectional. Ex. R, L & C	If the property or characteristic of element is dependent on direction of flow of current, element is unidirectional. Ex. Diode, BJT op-amp

➢ When characteristic curve is similar in opposite quadrant part the element is bidirectional otherwise it is unidirectional.

❑ Kirchhoff's Current Law



$\sum I = 0$

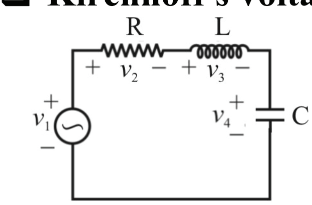
- Outgoing current = incoming current.

$$I_3 = I_1 + I_2 + I_4$$

$$q_3 - (q_1 + q_2 + q_4) = 0$$

KCL is also known as law of conservation of charge.

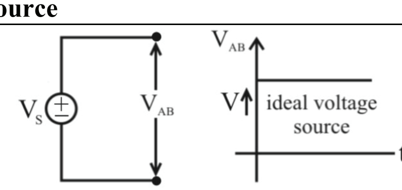
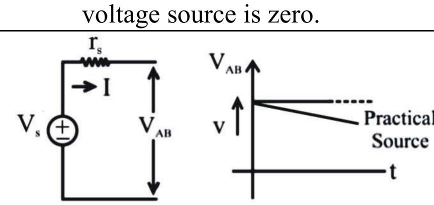
❑ Kirchhoff's voltage Law (KVL)



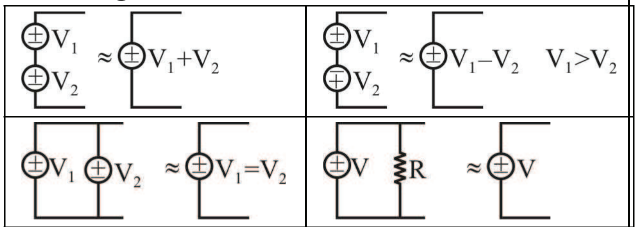
$\sum V = 0$, $-v_1 + v_2 + v_3 + v_4 = 0$

KVL is known as law of conservation of energy.

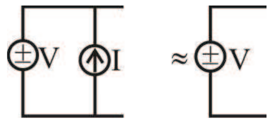
- **Voltage source**

Ideal voltage source	 r = 0, where r is internal resistance $V = V_S$ **Note:** Internal resistance of ideal voltage source is zero.
Practical voltage source	 $r_s \neq 0$ $V = V_s - Ir_s$ **Note:** The internal resistance of practical voltage source is not zero

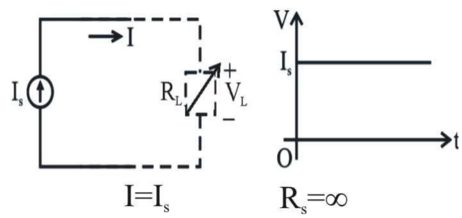
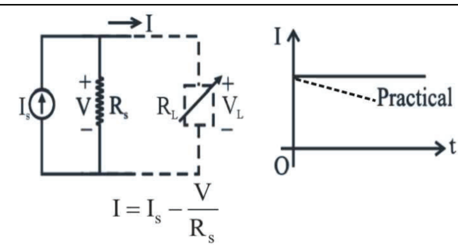
- **Voltage source connection**



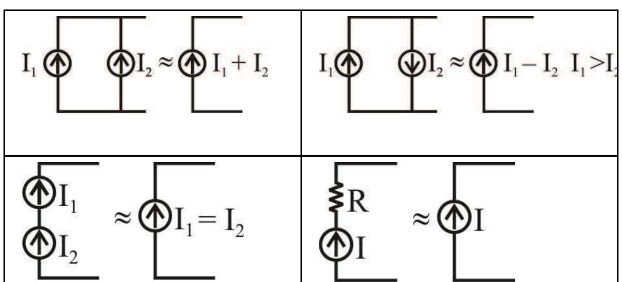
In parallel, voltage source is dominant.



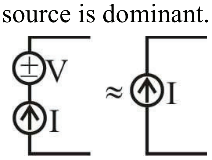
■ Current Source

Ideal Current Source	 **Note:** The internal resistance of an ideal current source is infinite
Practical Current Source	 **Note:** The internal resistance of practical current source is not infinite.

- **Current source connection**

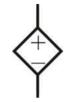
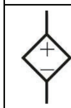
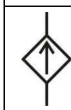
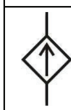


In series, current source is dominant.

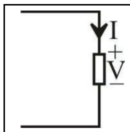
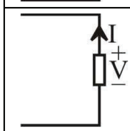


Source	Ideal	Practical
Voltage Source	$R_s = 0$	R_s = Low
Current Source	$R_s = \infty$	R_s = High
Voltmeter	$R_m = \infty$	R_m = High
Ammeter	$R_m = 0$	R_m = Low

■ Dependent source

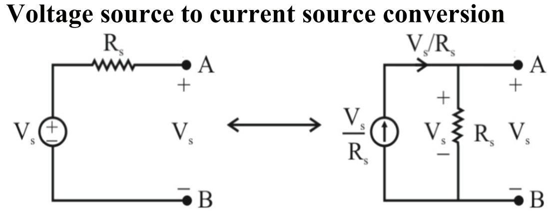
Source	Parameter to control
 V(t)= ki	Current controlled voltage source
 V(t)= kv	Voltage controlled voltage source
 I(t)= kv	Voltage controlled current source
 I(t)= ki	Current controlled current source

■ Power absorbed and delivered by element

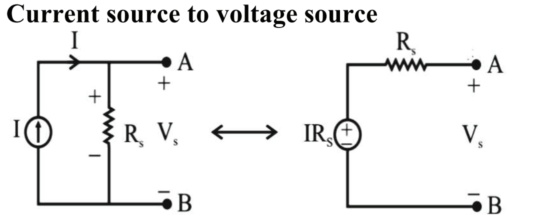
	When current is entering into positive terminal, element absorbs power.
	When current is leaving from positive terminal, element delivers power.

■ Source Transformation Technique

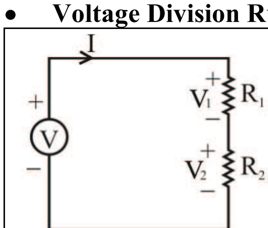
- **Voltage source to current source conversion**



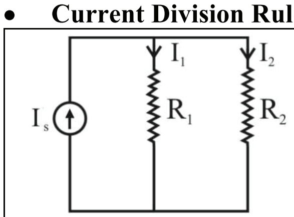
- **Current source to voltage source**



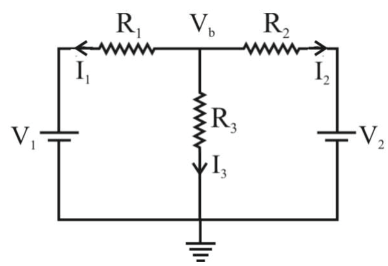
- **Voltage Division Rule**

	$V_1 = \frac{R_1}{R_1 + R_2} \times V$ $V_2 = \frac{R_2}{R_1 + R_2} \times V$

- **Current Division Rule :**

	$I_1 = \frac{R_2}{R_1 + R_2} \times I_s$ $I_2 = \frac{R_1}{R_1 + R_2} \times I_s$

- **Nodal voltage Analysis circuit :**



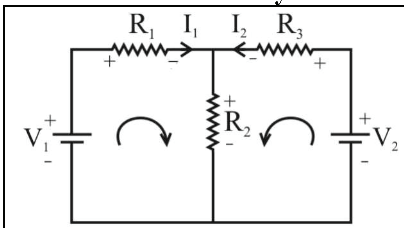
$$\frac{V_b - V_1}{R_1} + \frac{V_b - V_2}{R_2} + \frac{V_b}{R_3} = 0\,, \quad I_1 + I_2 + I_3 = 0$$

Nodal analysis is an application of Kirchhoff's current law.

Number of equation in nodal analysis = N-1

N = Number of nodes

- **Mesh Current Analysis Circuit**



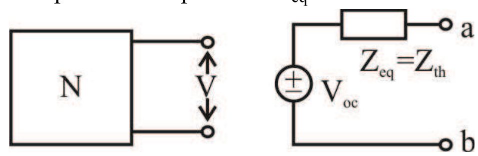
$$V_1 - I_1R_1 - (I_1 + I_2)R_2 = 0\,,\; V_2 - I_2R_3 - (I_1 + I_2)R_2 = 0$$

Mesh analysis is used only for planer network.

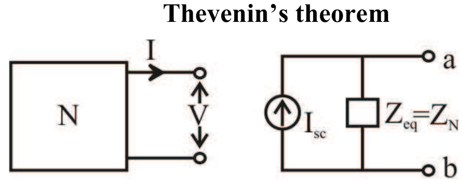
Number of equations in mesh analysis = B–(N–1)

❑ Network theorems

- **Superposition theorem-** It is applicable only for linear and bilateral network. When more than one independent voltage or current source is present, then response across any element in the circuit, is the sum of the responses obtained from each source.
- **Thevenin's & Norton's theorem -** It states that any linear, bilateral and active RLC network which contains one or more independent or dependent voltage or current source can be replaced by a single voltage source V_{oc} in series with equivalent impedance Z_{eq} for thevenin's theorem and can be replaced by a single current source I_{sc} in shunt with equivalent impedance Z_{eq} for norton's theorem



Thevenin's theorem



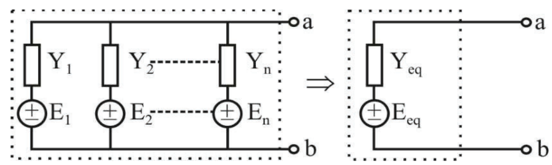
Norton's Theorem

- **Tellegen's theorem-** States that the summation of power delivered is zero for each branch of any electrical network at any instant of time.

$$\sum_{k=1}^{b} v_k i_k = 0 \qquad \text{b= number of branches}$$

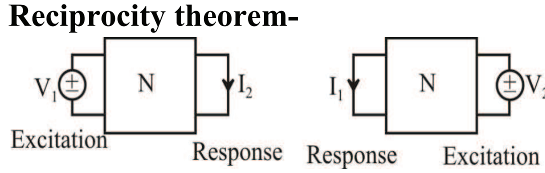
➢ This theorem is valid for any network where kVL and KCL equation are valid.

- **Millman's theorem-**



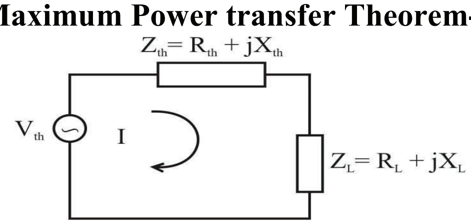
$$E_{eq} = \frac{\sum_{i=1}^{n} E_i Y_i}{\sum_{i=1}^{n} Y_i} \text{ and } Y_{eq} = \sum_{i=1}^{n} Y_i$$

- **Reciprocity theorem-**



$$\frac{V_1}{I_2} = \frac{V_2}{I_1};\; Z_{12} = Z_{21}$$

- **Maximum Power transfer Theorem-**



$$I = \frac{V_{th}}{\sqrt{(R_{th} + R_L)^2 + (X_{th} + X_L)^2}}$$

- **Condition for maximum power transfer-**

$$Z_L = Z_{th}^*$$

Z_L^* = Complex conjugate of load impedance

Where Z_{th} = Thevenin's equivalent impedance

- **Only R_L is variable & X_L fixed**.

$$R_L = \sqrt{R_{th}^2 + (X_L + X_{th})^2}$$

- **Only X_L is variable & R_L fixed.**

$$X_L = -X_{th}$$

- **If both are variable.**

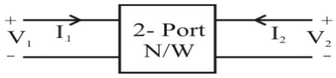
$|Z_L| = |Z_{th}|^*$

- **If load is purely Resistive**

$$R_L = \sqrt{R_{th}^2 + X_{th}^2}$$

➢ Efficiency of maximum power transfer theorem is 50%.

❑ Two Port Network



Maximum number of possible parameters for analysis of two port network is given by $N_o = {}^4C_2$

- **Parameters of two port network**

• Z-Parameter (Impedance Parameter)

$$V_1 = I_1Z_{11} + I_2Z_{12};\quad V_2 = I_1Z_{21} + I_2Z_{22};\quad \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}_{2\times2} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}_{2\times1}$$

$\because I_1, I_2 \rightarrow$ Independent Variable

$V_1, V_2 \rightarrow$ Dependent Variable

• Y – Parameters (Admittance Parameter)

$$I_1 = Y_{11}V_1 + Y_{12}V_2;\quad I_2 = Y_{21}V_1 + Y_{22}V_2;\quad \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}_{2\times2} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}_{2\times1}$$

$\because V_1, V_2 \rightarrow$ Independent Variable

$I_1, I_2 \rightarrow$ Dependent Variable

• h–Parameter (hybrid Parameter)

$$V_1 = I_1h_{11} + V_2h_{12};\quad I_2 = I_1h_{21} + V_2h_{22};\quad \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}_{2\times2} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}_{2\times1}$$

$\because V_1, I_2 \rightarrow$ Dependent Variable

$I_1, V_2 \rightarrow$ Independent Variable

• g- Parameter (Inverse hybrid Parameter)

$$I_1 = g_{11}V_1 + g_{12}I_2;\quad V_2 = g_{21}V_1 + g_{22}I_2;\quad \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}_{2\times2} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}_{2\times1}$$

$\because I_1, V_2 \rightarrow$ Dependent Variable

$V_1, I_2 \rightarrow$ Independent Variable

• ABCD- Parameter (Transmission Parameter)

$$V_1 = AV_2 - BI_2;\quad I_1 = CV_2 - DI_2;\quad \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$\because V_2, I_2 =$ Independent variable

$V_1, I_1 =$ Dependent variable

• abcd (t)- Parameter (Inverse Transmission Line Parameter)

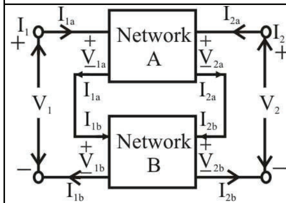
$$V_2 = aV_1 - bI_1;\quad I_2 = cV_1 - dI_1;\quad \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} V_1 \\ -I_1 \end{bmatrix}$$

- **Condition for network to be symmetrical & reciprocal.**

Reciprocal	Symmetrical
$Z_{12} = Z_{21}$	$Z_{11} = Z_{22}$
$Y_{12} = Y_{21}$	$Y_{11} = Y_{22}$
$\begin{vmatrix} A & B \\ C & D \end{vmatrix} = 1$	A=D
$h_{12} = -h_{21}$	$\begin{vmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{vmatrix} = 1$
$g_{12} = -g_{21}$	$\begin{vmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{vmatrix} = 1$

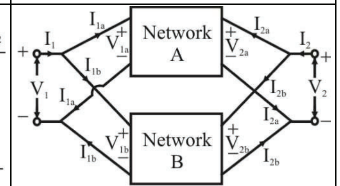
- **Connections of Two-port networks**

Series series connection



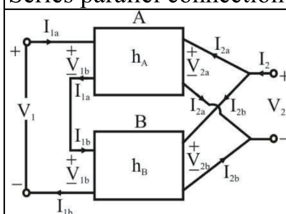
$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Z_{11a} + Z_{11b} & Z_{12a} + Z_{12b} \\ Z_{21a} + Z_{21b} & Z_{22a} + Z_{22b} \end{bmatrix}$$

Parallel Parallel connection



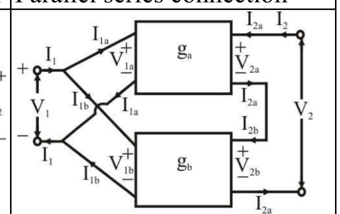
$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} Y_{11a} + Y_{11b} & Y_{12a} + Y_{12b} \\ Y_{21a} + Y_{21b} & Y_{22a} + Y_{22b} \end{bmatrix}$$

Series parallel connection



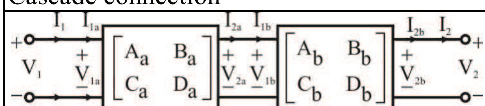
$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} h_{11a} + h_{11b} & h_{12a} + h_{12b} \\ h_{21a} + h_{21b} & h_{22a} + h_{22b} \end{bmatrix}$$

Parallel series connection



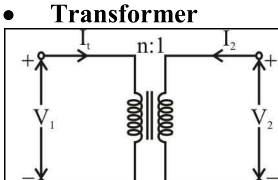
$$\begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} = \begin{bmatrix} g_{11a} + g_{11b} & g_{12a} + g_{12b} \\ g_{21a} + g_{21b} & g_{22a} + g_{22b} \end{bmatrix}$$

Cascade connection



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix}$$

- **Transformer**

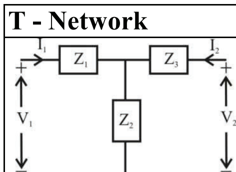


$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} n & 0 \\ 0 & 1/n \end{bmatrix} = \text{T- parameter}$$

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} 0 & n \\ -n & 0 \end{bmatrix} = \text{h- parameter}$$

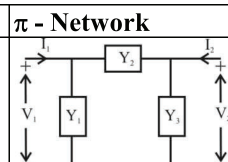
- **T & π Network**

T - Network



$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Z_1 + Z_2 & Z_2 \\ Z_2 & Z_2 + Z_3 \end{bmatrix}$$

π - Network



$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} Y_1 + Y_2 & -Y_2 \\ -Y_2 & Y_2 + Y_3 \end{bmatrix}$$

❑ AC Circuit Analysis

- **Average and RMS value of periodic signals**
- **Average value = DC value**

$$V_{avg} = V_{DC} = \frac{1}{T}\int_0^T v(t)\,dt \quad , \quad V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$$

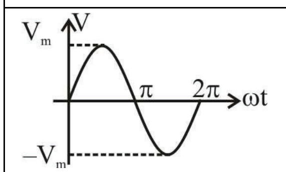
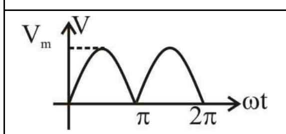
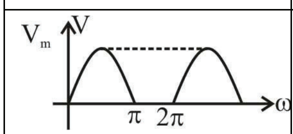
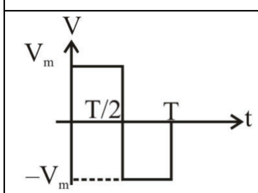
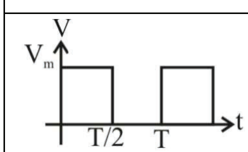
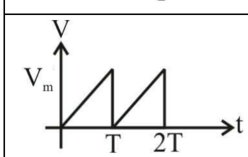
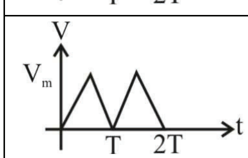
$$\text{Peak Factor} = \frac{V_{max}}{V_{rms}} \quad ; \quad \text{Form Factor} = \frac{V_{rms}}{V_{avg}}$$

Note:

If $f(t) = a_0 + a_1\cos\omega t + a_2\cos 2\omega t + ... + b_1\sin\omega t + b_2\sin_2 2\omega t + ...$

$$f(t)_{RMS} = \sqrt{a_0^2 + \frac{1}{2}\left(a_1^2 + a_2^2 + ... + b_1^2 + b_2^2 + ...\right)}$$

- **RMS and Average of Signal**

Time domain Signal	**RMS value**	**Average Value**
	$V_{rms} = \frac{V_m}{\sqrt{2}}$	$V_{avg} = 0$
	$V_{rms} = \frac{V_m}{\sqrt{2}}$	$V_{avg} = \frac{2V_m}{\pi}$
	$V_{rms} = \frac{V_m}{2}$	$V_{avg} = \frac{V_m}{\pi}$
	$V_{rms} = V_m$	$V_{avg} = 0$
	$V_{rms} = \frac{V_m}{\sqrt{2}}$	$V_{avg} = \frac{V_m}{2}$
	$V_{rms} = \frac{V_m}{\sqrt{3}}$	$V_{avg} = \frac{V_m}{2}$
	$V_{rms} = \frac{V_m}{\sqrt{3}}$	$V_{avg} = \frac{V_m}{2}$

❑ Phasor Diagram

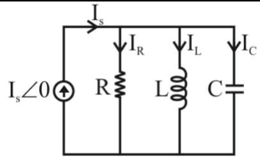
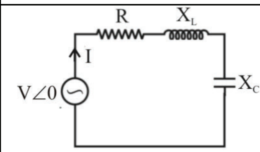
- **Series circuit**

RL	**RC**	**RLC**
$V_{RL} = \sqrt{V_R^2 + V_L^2}$ $V_R = IR$ $V_L = IX_L\angle 90^0$	$V_{RC} = \sqrt{V_R^2 + V_C^2}$ $V_C = IX_C\angle -90^0$	$V = \sqrt{V_R^2 + (V_L \sim V_C)^2}$
$\phi = \tan^{-1}\frac{V_L}{V_R}$	$\phi = \tan^{-1}\frac{V_C}{V_R}$	$\phi = \tan^{-1}\frac{V_L \sim V_C}{V_R}$
$\cos\phi = \frac{V_R}{V}(lag)$	$\cos\phi = \frac{V_R}{V}$ (lead)	$\cos\phi = \frac{V_R}{V}$ If $V_L > V_C (lag)$ $V_C > V_L (lead)$ If $V_L = V_C$ (Resonance) $\cos\phi = 1$

- **Parallel Circuit-**

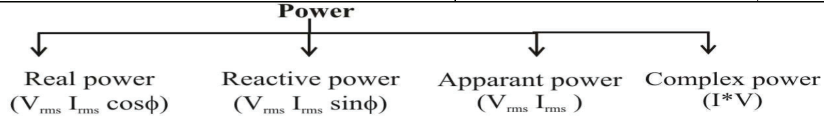
RL	**RC**	**RLC**
$I = \sqrt{I_R^2 + I_L^2}$ $I_R = \frac{V}{R}, I_L = \frac{V}{X_L}\angle -90^0$	$I = \sqrt{I_R^2 + I_C^2}$ $I_C = \frac{V}{X_C}\angle 90^0$	$I = \sqrt{I_R^2 + (I_L \sim I_C)^2}$
$\cos\phi = \frac{I_R}{I}$ (lag)	$\cos\phi = \frac{I_R}{I}$ (lead)	$\cos\phi = \frac{I_R}{I}$ (Lead); $I_C > I_L$ (Lag); $I_L > I_C$ $\cos\phi = 1$; $I_L = I_C$

- **Resonance Circuit**

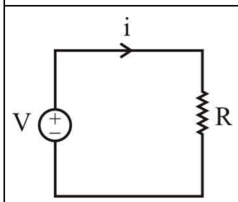
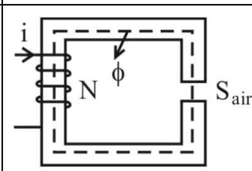
Parallel Resonance circuit	**Series Resonance Circuit**
	
• **Resonance condition** $\lvert I_L\rvert = \lvert I_C\rvert$	• **Resonance Condition** $X_C = X_L$, $\lvert V_L\rvert = \lvert V_C\rvert$
• $\downarrow I_s = \frac{V}{R} = \frac{V}{Z\uparrow} = I_R$ = minimum Z = R = maximum , Y = minimum	• $\uparrow I = \frac{V}{R} = \frac{V}{Z\downarrow}$ = maximum Z = R = minimum , Y = maximum
• **Resonance in Practical RLC Circuit** $\omega = \frac{1}{\sqrt{LC}}\sqrt{\frac{R_L^2 - \frac{L}{C}}{R_C^2 - \frac{L}{C}}}$ rad/sec	$\omega_o = \frac{1}{\sqrt{LC}}$ rad / sec $\quad f_0 = \frac{1}{2\pi\sqrt{LC}}$ Hz

ω	**Power factor**	**Y (Admittance)**	ω	**Power factor**	**Z (Impedance)**
$\omega < \omega_0$	lag	Inductive	$\omega < \omega_0$	lead	Capacitive
$\omega > \omega_0$	lead	Capacitive	$\omega > \omega_0$	lag	Inductive
$\omega = \omega_0$	Unity power factor	Resistive	$\omega = \omega_0$	Unity power factor	Resistive

$\omega_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$, $\omega_2 = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$ $\omega_0 = \sqrt{\omega_1\ \omega_2}$ rad/sec $\quad f_0 = \sqrt{f_1 f_2}$ Hz Where, ω_2 = Upper frequency, ω_1 = Lower frequency	$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$, $\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$ $\omega_0 = \sqrt{\omega_1\ \omega_2}$ rad/sec $\quad f_0 = \sqrt{f_1 f_2}$ Hz
• $B.W = \omega_2 - \omega_1 = \frac{1}{RC} = \frac{\omega_0}{\omega_0 RC} = \frac{\omega_0}{Q}$ $\omega_1 = \omega_r - \frac{\Delta\omega}{2}$, $\quad \omega_2 = \omega_r + \frac{\Delta\omega}{2}$	$\Delta\omega = B.W = \frac{R}{L} = \omega_2 - \omega_1 = \frac{\omega_0}{Q}$ $\omega_2 = \omega_o + \frac{\Delta\omega}{2}$, $\quad \omega_1 = \omega_o - \frac{\Delta\omega}{2}$
Quality factor(Q) $= 2\pi\left(\frac{\text{Maximum energy stored in the circuit}}{\text{Total energy dissipated by the circuit}}\right)$ $Q = \frac{R}{X_L} = \frac{R}{X_C} = \omega CR = R\sqrt{\frac{C}{L}} = \frac{\lvert I_L\rvert}{I} = \frac{\lvert I_C\rvert}{I} = \frac{f_r}{B.W}$	$\text{Quality factor}(Q) = \frac{1}{\text{Dissipation factor}}$ $Q = \frac{\lvert V_L\rvert}{V} = \frac{\lvert V_C\rvert}{V} = \frac{X_L}{R} = \frac{X_C}{R} = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{\omega_o L}{R} = \frac{1}{\omega_0 CR}$
• It is current magnifier circuit. • At resonance Z=R and $Z = \frac{L}{CR}$ (tank circuit) circuit becomes purely resistive circuit • A parallel resonance circuit behave as - band stop filter For Band stop filter- $P \ge 2P_{min}, I \ge \sqrt{2}\ I_{min}$ • $\text{Selectivity} = \frac{\text{resonance frequency}}{\text{Bandwidth}}$	• Net reactive power of a series resonant circuit is zero. $\lvert P_{LC}\rvert = 0$ • Net reactive voltage of series resonant circuit is zero. $\lvert V_{LC}\rvert = 0$ • Phase angle is zero $P \ge \frac{P_{max}}{2}, I \ge \frac{I_{max}}{\sqrt{2}}$ • Series resonance circuit is voltage magnifier circuit. • $\omega_C = \omega_0\sqrt{1 - \frac{R^2C}{2L}}$, $\quad \omega_L = \frac{\omega_o}{\sqrt{1 - \frac{R^2C}{2L}}}$



■ **Analogy between electric and magnetic circuit**

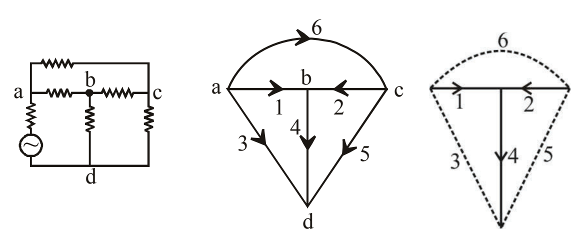
Electric Circuit	Magnetic Circuit
Voltage/EMF is the 'cause' [Units-volts]	mmf→Magneto Motive force is the cause = N.I
Current is the effect [Unit-Ampere]	Flux is the effect → [Unit → weber (Wb)]
Ohm law → $R = \frac{V}{I}\Omega$	Ohms Law- $S = \frac{mmf}{\phi} = \frac{NI}{\phi}$
$R = \frac{\rho\ell}{a}$, Electrical material , ρ = Resistivity (Ω-m)	$S = \frac{\ell}{\mu a}$, magnetic material μ = Permeability (H/m)
$E = \frac{V}{d}, [\text{volt}/\text{m}]$, E = Electric field intensity	$H = \frac{mmf}{\ell} = \frac{N.I}{\ell}$, H = Magnetic field intensity
$J = \frac{I}{a} \quad \left[\frac{A}{m^2}\right]$, J = Current density	$B = \frac{\phi}{a} \quad \left[\frac{Wb}{m^2}\right]$ or [Tesla], B = Magnetic field density
$J = \sigma E$ J = Electric current density σ = Conductivity (mho/m)	$B = \mu H$ (B → Mag. Flux density) $\mu = \mu_0 \mu_r$ μ_r = Relative permeability
 Electric Conductor (Cu, Al, etc)	 ϕ = flux , N = No. of turns in the coil Magnetic Conductor (Fe, Ni cobalt etc)

■ **Three Phase System**

In star connection	In Delta Connection
$i_R + i_Y + i_B = 0$; $I_L = I_{Ph}$ $V_L = \sqrt{3}V_{Ph}$; $V_{Ph} = \frac{V_L}{\sqrt{3}}$	$V_R + V_Y + V_B = 0$ $V_L = V_{Ph}$ $I_L = \sqrt{3}\, I_{Ph}$
$P = 3V_{Ph}I_{Ph}\cos\phi$ or $P = \sqrt{3}\, V_L I_L \cos\phi$	$P = \sqrt{3} V_L I_L \cos\phi$

❑ Graph Theory

- Graph is defined as collection of node and branch
- Rank of graph = N–1 Where N= Number of node
- Degree of node = number of incoming branch at any node.



Terminology	Definition	Notation	Formula
Node	Intersection point of branch	[a,b,c,d]	N
Branch	The portion of circuit between two nodes	[1,2,3,4,5,6]	N(N-1)/2
Tree	The part of graph containing all nodes. Possible trees = $\det[[A_r][A_r]^T]$	[a,b,c,d]	N^{N-2}
Twig	The branches of a tree are twig	[1,2,4]	N–1
Co-tree	The part of graph is not covered by tree	[3,5,6]	B–N+1
Link or chord	The branch of a co-tree	[3,5,6]	B–N+1
Where N= Number of node, B = Number of branch			

- **Incidence Matrix**

$$\begin{array}{c} \text{Nodes} \\ a \\ b \\ c \\ d \end{array} \overset{\text{Branch}}{\begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 1 \\ -1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & -1 & -1 & 0 \end{bmatrix} \end{array}}$$

➢ Rank of incidence matrix = N-1

➢ Order of incidence matrix $[A]_{N\times B}$

➢ Sum of all column mmf of incidence matrix is zero

- **Reduced Incidence Matrix**

On deleting any row we get reduced incidence matrix.

$$\begin{array}{c} \text{Nodes} \\ a \\ b \\ c \end{array} \overset{\text{Branch}}{\begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 1 \\ -1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & -1 \end{bmatrix} \end{array}}$$

➢ Order of reduce incidence matrix = $[R]_{(N-1)\times B}$

- **Tie-set matrix-** Tie-set matrix is a fundamental loop matrix

➢ Rank of tie-set matrix = B–N+1

➢ Number of KVL equation = Number of fundamental loop current

- **Cut set matrix**- Cut set matrix is a twig dependent matrix.

➢ Number of cut set (T) = N–1

➢ Rank of cut set matrix = N–1

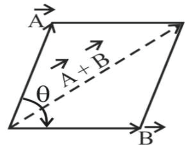
➢ Order of cut set matrix = $[\text{Cut set matrix}]_{T\times B}$

- **Principle of Duality**

Resistance (R)	Conductance (G)
Inductance (L)	Capacitance (C)
Impedance (Z)	Admittance (Y)
Voltage (V)	Current (I)
Voltage Source	Current Source
Node	Mesh
Series path	Parallel path
Open circuit	Short circuit
KCL	KVL
Thevenin	Norton
Star Network	Delta Network
Ri (t)	GV (t)
$\frac{Ldi(t)}{dt}$	$C\frac{dv(t)}{dt}$
$\frac{1}{C}\int i(t)dt$	$\frac{1}{L}\int v(t).dt$
V D R	C D R
V_{oc}	I_{sc}

ELECTROMAGNETIC FIELD THEORY

■ Vector analysis-

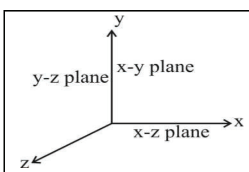


Addition	$\vec{A}+\vec{B}=\vec{B}+\vec{A}$
Subtraction	$\vec{A}-\vec{B}=\vec{A}+(-\vec{B})$
Cumulative law	$\vec{A}+\vec{B}=\vec{B}+\vec{A}$
Associative law	$(\vec{A}+\vec{B})+\vec{C}=\vec{A}+(\vec{B}+\vec{C})$

■ Multiplication

Scalar or Dot product	$\vec{A}.\vec{B}=\lvert A\rvert\lvert B\rvert\cos\theta$
Vector or cross product	$\vec{A}\times\vec{B}=\lvert A\rvert\lvert B\rvert\sin\theta\,\hat{n}$ θ = angle between $\vec{A}$ and $\vec{B}$ $\hat{n}$ = Unit vector perpendicular to $\vec{A}$ and $\vec{B}$
Scalar triple product	$\vec{A}\cdot(\vec{B}\times\vec{C})=\vec{B}\cdot(\vec{C}\times\vec{A})=\vec{C}\cdot(\vec{A}\times\vec{B})$
Vector triple product	$\vec{A}\times(\vec{B}\times\vec{C})=\vec{B}(\vec{A}\cdot\vec{C})-\vec{C}(\vec{A}\cdot\vec{B})$

■ Cartesian Co-ordinate System (x, y, z)

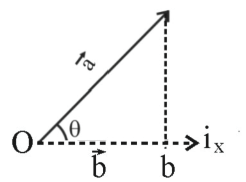


$-\infty < x < \infty$
$-\infty < y < \infty$
$-\infty < z < \infty$

Unit vector cross product	Unit vector dot product
$\hat{a}_x\times\hat{a}_y=\hat{a}_z$	$\hat{a}_x.\hat{a}_y=\hat{a}_y.\hat{a}_z=\hat{a}_z.\hat{a}_x=0$
$\hat{a}_y\times\hat{a}_z=\hat{a}_x$	$\hat{a}_x.\hat{a}_x=\hat{a}_y.\hat{a}_y=\hat{a}_z.\hat{a}_z=1$
$\hat{a}_z\times\hat{a}_x=\hat{a}_y$	
$\hat{a}_x\times\hat{a}_x=\hat{a}_y\times\hat{a}_y=\hat{a}_z\times\hat{a}_z=0$	

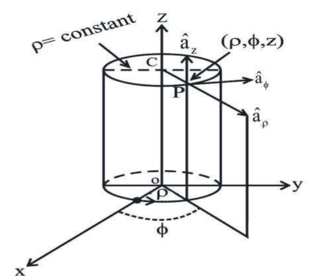
Position vector	$\vec{r}=x\hat{a}_x+y\hat{a}_y+z\hat{a}_z$
Displacement vector	$\vec{d\ell}=dx\,\hat{a}_x+dy\hat{a}_y+dz\hat{a}_z$ $\lvert\vec{d\ell}\rvert=\sqrt{(dx)^2+(dy)^2+(dz)^2}$
Differential normal surface area	$d\vec{S}_x=dydz\hat{a}_x$, $d\vec{S}_y=dzdx\hat{a}_y$ $d\vec{S}_z=dxdy\hat{a}_z$
Differential volume	dV = dxdydz

- **Unit vector** $\hat{r}=\dfrac{\vec{r}}{\lvert\vec{r}\rvert}=\dfrac{x\hat{a}_x+y\hat{a}_y+z\hat{a}_z}{\sqrt{x^2+y^2+z^2}}$
- **Projection of a vector**



Projection of $\vec{a}$ on $\hat{i}_x$ direction of $\vec{b}$ (ob)
$=\vec{a}.\hat{i}_x$

■ Cylindrical Co-ordinate system (ρ, ϕ, z)



$\begin{cases}0\le\rho<\infty\\0\le\phi<2\pi\\-\infty<z<\infty\end{cases}$

$x=\rho\cos\phi$
$y=\rho\sin\phi$
$z=z$

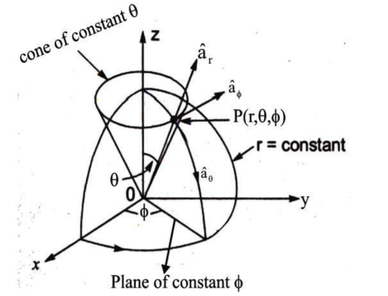
$\rho=\sqrt{x^2+y^2}$

Position vector of a point p (ρ,ϕ,z)	$\vec{r}=\rho\hat{a}_\rho+z\hat{a}_z$
Displacement vector	$\vec{d\ell}=d\rho\hat{a}_\rho+\rho d\phi\hat{a}_\phi+dz\hat{a}_z$ $\lvert\vec{d\ell}\rvert=\sqrt{(d\rho)^2+(\rho d\phi)^2+(dz)^2}$
Differential length element	$d\rho\hat{a}_\rho,\ \rho d\phi\hat{a}_\phi,\ dz\hat{a}_z$
Differential normal surface areas	$d\vec{S}_\rho=\rho d\phi dz\hat{a}_\rho$, $d\vec{S}_\phi=d\rho dz\hat{a}_\phi$ $d\vec{S}_z=\rho d\phi d\rho\hat{a}_z$
Differential volume	$dV=\rho d\rho\cdot d\phi\cdot dz$

- **Comparison between Cartesian and cylindrical co-ordinate system**

Cylindrical to Cartesian	$x=\rho\cos\phi$, $y=\rho\sin\phi$ $z=z$
Cartesian to cylindrical	$\rho=\sqrt{x^2+y^2}$ $\phi=\tan^{-1}\left(\dfrac{y}{z}\right)$, $z=z$

■ Spherical Co-ordinate System (r,θ,ϕ)

	$\begin{cases} 0 \le r < \infty \\ 0 \le \theta \le \pi \\ 0 \le \phi < 2\pi \end{cases}$ $x = r\sin\theta\cos\phi$ $y = r\sin\theta\sin\phi$ $z = r\cos\theta$

Position vector of point p	$\vec{r} = r\hat{a}_r$
Displacement vector	$d\vec{\ell} = dr\hat{a}_r + rd\theta\hat{a}_\theta + r\sin\theta d\phi\hat{a}_\phi$ $\lvert d\vec{\ell}\rvert = \sqrt{(dr)^2 + (rd\theta)^2 + (r\sin\theta d\phi)^2}$
Differential length element	$dr\hat{i}_r,\ rd\theta\hat{i}_\theta,\ r\sin\theta d\phi\hat{i}_\phi$
Differential normal surface area	$d\vec{S}_r = r^2\sin\theta d\theta d\phi\hat{a}_r$ $d\vec{S}_\theta = r\sin\theta dr d\phi\hat{a}_\theta$, $d\vec{S}_\phi = rdrd\theta\hat{a}_\phi$
Differential volume	$dV = r^2\sin\theta d r d\theta d\phi$

- **Relation between spherical and Cartesian Co-ordinate system**

Spherical to Cartesian	$x = r\sin\theta\cos\phi$ $y = r\sin\theta\sin\phi$ $z = r\cos\theta$
Cartesian to Spherical	$r = \sqrt{x^2 + y^2 + z^2}$ $\theta = \cos^{-1}\dfrac{z}{\sqrt{x^2+y^2+z^2}}$

■ Special Derivatives

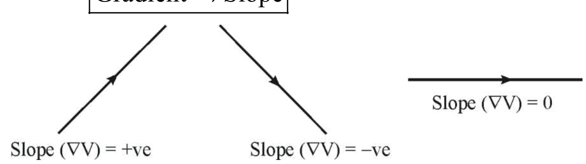
- **Del (∇) Operator-**

In Cartesian co-ordinate system	$\nabla = \dfrac{\partial}{\partial x}\hat{a}_x + \dfrac{\partial}{\partial y}\hat{a}_y + \dfrac{\partial}{\partial z}\hat{a}_z$
In Cylindrical co-ordinate system	$\nabla = \dfrac{\partial}{\partial \rho}\hat{a}_\rho + \dfrac{1}{\rho}\dfrac{\partial}{\partial \phi}\hat{a}_\phi + \dfrac{\partial}{\partial z}\hat{a}_z$
In spherical co-ordinate system	$\nabla = \dfrac{\partial}{\partial r}\hat{a}_r + \dfrac{1}{r}\dfrac{\partial}{\partial \theta}\hat{a}_\theta + \dfrac{1}{r\sin\theta}\dfrac{\partial}{\partial \phi}\hat{a}_\phi$

- **Gradient of a Scalar field**

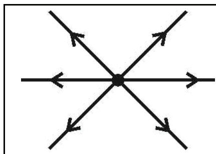
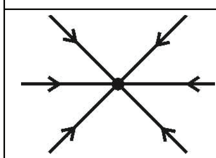
Gradient of scalar field represents the both magnitude and the direction of the maximum space rate of increase of a scalar.

Gradient → Slope



In Cartesian co-ordinate system	$\nabla V = \dfrac{\partial V}{\partial x}\hat{a}_x + \dfrac{\partial V}{\partial y}\hat{a}_y + \dfrac{\partial V}{\partial z}\hat{a}_z$
In cylindrical co-ordinate system	$\nabla V = \dfrac{\partial V}{\partial \rho}\hat{a}_\rho + \dfrac{1}{\rho}\dfrac{\partial V}{\partial \phi}\hat{a}_\phi + \dfrac{\partial V}{\partial z}\hat{a}_z$
In spherical co-ordinate system	$\nabla V = \dfrac{\partial V}{\partial r}\hat{a}_r + \dfrac{1}{r}\dfrac{\partial V}{\partial \theta}\hat{a}_\theta + \dfrac{1}{r\sin\theta}\dfrac{\partial V}{\partial \phi}\hat{a}_\phi$

➢ **Divergence of a vector-** Divergence of a vector field simply measure how much the flow is expanding at a given point.

 Source	$\nabla.\vec{A} = +ve$ going away from a point in different direction.
 Sink	$\nabla.\vec{A} = -ve$ Coming towards the point from the different direction
$\nabla.\vec{A} = 0$	divergence less or solenoidal

$$\text{Div}\vec{A} = \nabla.\vec{A} = \lim_{\nabla V \to 0}\frac{\oint_s \vec{A}.d\vec{S}}{\Delta V}$$

In cartesian co-ordinate system	$\nabla.\vec{D} = \dfrac{\partial D_x}{\partial x} + \dfrac{\partial D_y}{\partial y} + \dfrac{\partial D_z}{\partial z}$
In cylindrical co-ordinate system	$\nabla.\vec{D} = \dfrac{1}{\rho}\dfrac{\partial}{\partial \rho}(\rho D_\rho) + \dfrac{1}{\rho}\dfrac{\partial D_\phi}{\partial \phi} + \dfrac{\partial D_z}{\partial z}$

➢ In spherical co-ordinate system

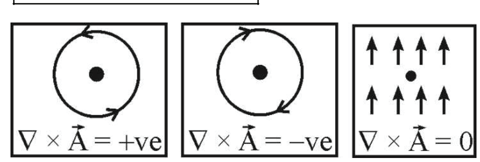
$$\nabla.\vec{D} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 D_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta D_\theta) + \frac{1}{r\sin\theta}\frac{\partial D_\phi}{\partial \phi}$$

- **Curl of a Vector field**

Curl of any vector point function gives the measure of angular velocity at any point of the vector field.

$$\nabla \times \vec{A} = \lim_{\Delta S \to 0}\frac{\oint_L \vec{A}.d\vec{\ell}}{\Delta S}\hat{i}_n$$

Curl → rotation



- **Curl of vector**

Curl in rectangular co-ordinate	$\text{Curl}\vec{A} = \nabla\times\vec{A} = \begin{bmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \\ A_x & A_y & A_z \end{bmatrix}$
Curl in cylindrical co-ordinate	$\nabla\times\vec{A} = \begin{bmatrix} \frac{\hat{a}_\rho}{\rho} & \hat{a}_\phi & \frac{\hat{a}_z}{\rho} \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{bmatrix}$
Curl in spherical co-ordinate	$\nabla\times\vec{A} = \begin{bmatrix} \frac{\hat{a}_r}{r\sin\theta} & \frac{\hat{a}_\theta}{r\sin\theta} & \frac{\hat{a}_\phi}{r} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ A_r & rA_\theta & r\sin\theta A_\phi \end{bmatrix}$

➢ **Important identities**

$\nabla.(\nabla\times\vec{A}) = 0$	Divergence of curl is always zero.
$\nabla\times(\nabla\vec{A}) = 0$	Curl of gradient is always zero.
$\nabla.\vec{A} = 0$	$\vec{A}$ is divergence less or solenoidal
$\nabla\times\vec{A} = 0$	$\vec{A}$ is irrotational

- **Laplace operator**

∇^2 (Scalar)

Scalar (f): $\nabla^2 f$ — Scalar Laplacian

Vector(f): $\nabla^2.\vec{f}$ — Vector Laplacian

$$\nabla^2 f = \nabla.(\nabla f)$$

$$\nabla^2.\vec{f} = \nabla(\nabla.\vec{f}) - \nabla\times\nabla\times\vec{f}$$

☞ Remember point

If Laplacian of any scalar 'F' is zero in the given region, then that scalar 'f' is said to be harmonic.

In Cartesian co-ordinate system	$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$
In cylindrical co-ordinate system	$\nabla^2 f = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial f}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial\phi^2} + \frac{\partial^2 f}{\partial z^2}$
In spherical co-ordinate system	
$\nabla^2 f = \frac{1}{\rho^2}\frac{\partial}{\partial\rho}\left(\rho^2\frac{\partial f}{\partial\rho}\right) + \frac{1}{\rho^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{\rho^2\sin^2\theta}\left(\frac{\partial^2 f}{\partial\phi^2}\right)$	

Divergence Theorem	Stokes Theorem
The divergence of $\vec{A}$ at a point P is the outward flux per unit volume shrinks about P. $\oint_S \vec{A}.d\vec{S} = \int_V (\nabla.\vec{A})dV$	The curl is a vector operation that can be used to state, if there is a rotation associated with a vector. $\oint_L \vec{A}.d\vec{\ell} = \iint_S (\nabla\times\vec{A}).d\vec{S}$

■ **Electro-statics**

- **Source of Charges**

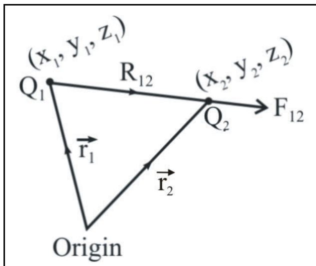
Line charge density (ρ_L)	$\frac{dQ}{dL} = \rho_L\ C/m$, $Q = \int_L \rho_L dL\ C$
Surface charge density (ρ_S)	$\frac{dQ}{dS} = \rho_S\ C/m^2$, $Q = \iint_S \rho_S\, dS\ C$
Volume charge density (ρ_V)	$\frac{dQ}{dV} = \rho_V\ C/m^3$, $Q = \iiint_V \rho_V dV\ C$

- **Relation between Network theory and electromagnetic field**

Network Theory	EMFT
Voltage (V)-Volt	Electric field intensity $(\vec{E})\frac{\text{Volt}}{\text{m}}$
Current (I)-Amp	Magnetic field intensity $(\vec{H})\ A/m$ Current density $(\vec{J})\ A/m^2$
Capacitance (C)-Farad	Permittivity (ε) Farad/m
Inductance (L)-Henry	permeability (μ) Henry/m
Charge (Q)-Coulomb	Electric flux density (D)C/m^2
Magnetic flux (ϕ) Weber/m^2	Magnetic flux density (B)Wb/m^2
Resistance (R)-ohm	Conductance $(G) - \mho/m$

$Q = CV$ ⇓ $\vec{D} = \varepsilon\vec{E}$	$\phi = LI$ ⇓ $\vec{B} = \mu\vec{H}$	$W_C = \frac{1}{2}CV^2$ ⇓ $W_E = \frac{1}{2}\varepsilon\vec{E}^2$

■ **Coulomb's Law**



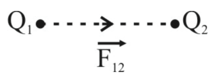
$$\vec{r}_1 = x_1\hat{a}_x + y_1\hat{a}_y + z_1\hat{a}_z$$

$$\vec{r}_2 = x_2\hat{a}_x + y_2\hat{a}_y + z_2\hat{a}_z$$

- **Force on Q_2 due to Q_1**

➢ $$F_{12} = \frac{1}{4\pi\varepsilon_o}\frac{Q_1Q_2}{|R_{12}|^2}\hat{a}_{R_{12}} = \frac{1}{4\pi\varepsilon_o}\frac{Q_1Q_2}{|R_{12}|^3}\vec{R}_{12}$$

➢ $$\varepsilon_o = 8.854\times10^{-12}\,F/m = \frac{10^{-9}}{36\pi}$$



➢ $$\frac{1}{4\pi\varepsilon_o} = K = 9\times10^9\,m/F$$

➢ If two similar charges with same sign are placed at a distance then the force acted on it is repulsive in nature. $\vec{F}_1 = -\vec{F}_2$

- **Electric Field Intensity-** It is measure of strength of electric field at any point. $E = \lim_{Q\to0}\frac{\vec{F}}{Q}\,N/C$

- **Electric field due to point charge**

$$\vec{E} = \frac{\vec{F}}{Q_t} = \frac{Q}{4\pi\varepsilon_oR^2}\hat{a}_r, V/m$$

- **Electric field due to various charge distribution:**

For Line charge	$E = \int_L \frac{\rho_L dL}{4\pi\varepsilon_oR^2}\hat{a}_r$ ρ_L = Line Charge Density
For Surface charge	$E = \int_S \frac{\rho_s dS}{4\pi\varepsilon_oR^2}\hat{a}_r$ ρ_S = Surface Charge Density
For Volume charge	$E = \int_V \frac{\rho_V dV}{4\pi\varepsilon_oR^2}\hat{a}_r$ ρ_v = Volume Charge Density
Electric field due to infinite long line charge	$\vec{E} = \frac{\rho_L}{2\pi\varepsilon_o\rho}\hat{a}_\rho$, $E \propto \frac{1}{\rho}$
Electric field due to circular loop	$\vec{E} = \frac{\rho_L R_h}{2\varepsilon_o\left(R^2+h^2\right)^{3/2}}\hat{a}_x$
Electric field due to circular disc	$\vec{E} = \frac{\rho_s}{2\varepsilon_o}\left(1-\frac{h}{\sqrt{R^2+h^2}}\right)\hat{a}_x$
Electric field due to infinite sheet	$\vec{E} = \frac{\rho_s}{2\varepsilon_o}\hat{a}_n$
Electric field due to infinite sheet for external	$\vec{E} = \frac{\rho_s}{2\varepsilon_o}(-\hat{a}_n)$

- Electric/Displacement flux Density $\vec{D} = \varepsilon\vec{E}\ C/m^2$

- **Gauss's law** $\psi = \oint_S \vec{D}.d\vec{S} = Q = \int_V \rho_V dv$

➢ Gauss's law in differential or point form $\nabla.\vec{D} = \rho_v$

➢ For charge free region, $\rho_v = 0$, $\nabla.\vec{D} = 0$

- **Energy Density in Electrostatic Field**

➢ Electrostatic Energy Density–

$$W_E = \frac{dW_E}{dV} = \frac{1}{2}\vec{D}.\vec{E} = \frac{1}{2}\varepsilon_oE^2 = \frac{D^2}{2\varepsilon_o}\ J/m^3$$

➢ Total Electrostatic Energy

$$W_E = \frac{1}{2}\int_V \vec{D}.\vec{E}\,dV = \frac{1}{2}\int_V \varepsilon_oE^2dV$$

- **Electric Potential difference**

$$V_{AB} = -\int_A^B \vec{E}.d\vec{\ell} = \frac{Q}{4\pi\varepsilon_o}\left[\frac{1}{r_B}-\frac{1}{r_A}\right]$$

- **Potential at a point P due to point charge**

$$(V) = \frac{Q}{4\pi\varepsilon R},\ V = \int \vec{E}.\vec{dl}$$

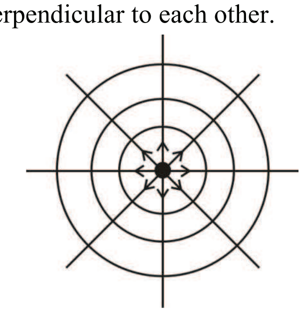
- **Relationship between $\vec{E}$ & V ,** $\vec{E} = -\nabla V$

- **Capacitance**

Parallel plate	$C = \frac{\varepsilon_0\varepsilon_rA}{d}$
Coaxial cable	$C = \frac{2\pi\varepsilon L}{\ell n(b/a)}$
Capacitance of sphere	$C = \frac{4\pi\varepsilon}{\left(\frac{1}{a}-\frac{1}{b}\right)}$
Capacitance of isolated sphere	$C = 4\pi\varepsilon a$
Composite parallel plate capacitor	$C = \frac{A}{d_1/\varepsilon_1 + d_2/\varepsilon_2 .....d_n/\varepsilon_n}$

■ **Equipotential Surface**

1. Concentric sphere whose centre is at point charge (source).
2. Equipotential surface and electric field line are always perpendicular to each other.



Poisson's equation	Laplace Equation
$\nabla^2V = \frac{-\rho_V}{\varepsilon}$	$\nabla^2V = 0$

- **Electric dipole moment** $\boxed{\vec{p} = Q.\vec{d}}$
- **Inside a Conductor-** $\boxed{E = 0, \rho_V = 0, V_{ab} = 0}$
- **Power (Joule's law)**

  $\boxed{P = \int_V \vec{E}.\vec{J}\,dV}$ J →current density (A/m^2)
- **Continuity Equation** $\boxed{\nabla.J = -\frac{\partial \rho_V}{\partial t}}$
- **Dielectric material** - In any dielectric material electric dipole are present. Dielectric material stores electric energy. Using these electric dipole by rotating on their axis.

  In free space no electric dipole, Magnetic dipole and free electron.

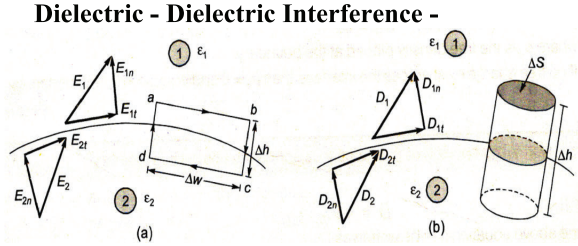
  $\bar{D} = \varepsilon_0\vec{E} + \vec{P}$ → [True all material]

  For linear dielectric material. $\vec{P} = \chi_e\varepsilon_0\vec{E}$

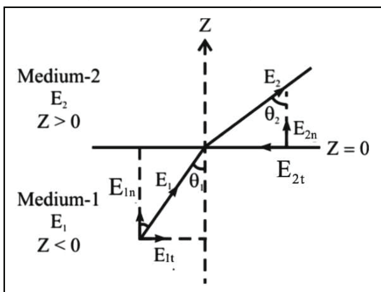
  $\boxed{\bar{D} = (1+\chi_e)\varepsilon_0\vec{E}}$ and $\boxed{\varepsilon_r = 1+\chi_e}$

Boundary Condition- Boundary condition govern the behaviour of electric field at the boundary between two different media. If field exist in a region consisting of 2 different media then there is some condition which must be satisfy at the interface separating media is known as boundary condition.

- **Dielectric - Dielectric Interference -**



- Tangential component of electric field in medium (1) and medium (2) are equal $\boxed{E_{1t} = E_{2t}}$
- If the both medium are equal, then surface charge on the boundary $\boxed{D_{1n} = D_{2n},\ \rho_s = 0}$
- The field is directed from medium (1) to medium **(2)** $\boxed{D_{2n} - D_{1n} = \rho_S}$



$\because \tan\theta_1 = \frac{E_{1t}}{E_{1n}}$

$\tan\theta_2 = \frac{E_{2t}}{E_{2n}}$

$\frac{\tan\theta_1}{\tan\theta_2} = \frac{E_{1t}}{E_{1n}} \times \frac{E_{2n}}{E_{2t}}$

$\because E_{1t} = E_{2t}$

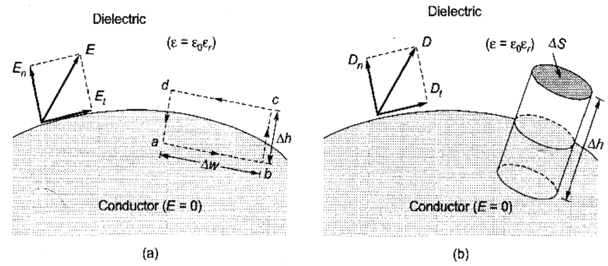
$\frac{\tan\theta_1}{\tan\theta_2} = \frac{E_{2n}}{E_{1n}}$

$\because D_{1n} = D_{2n}, \rho_s = 0$

$\varepsilon_1.E_{1n} = \varepsilon_2 E_{2n}$

$\frac{E_{2n}}{E_{1n}} = \frac{\varepsilon_1}{\varepsilon_2}$

- **Conductor - dielectric boundary**



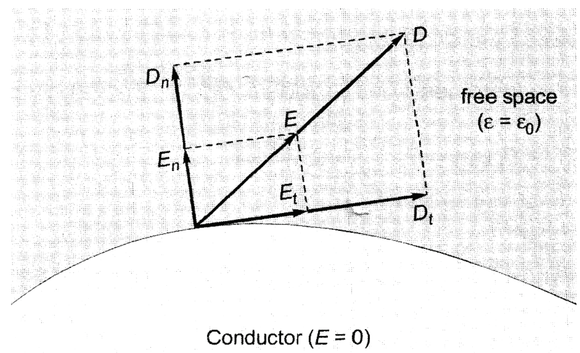
$\boxed{E_t = 0,\ D_t = \varepsilon_0 E_t = 0}$ $\boxed{D_n = \rho_s,\ D_{2n} = \varepsilon_2 E_{2n}}$

- **Law of Refraction**

  From boundary condition

$\vec{E}_{1t} = \vec{E}_{2t}$ or $E_1\sin\theta_1 = E_2\sin\theta_2$
$D_{1n} = D_{2n}$ or $\varepsilon_1 E_1\cos\theta_1 = \varepsilon_2 E_2\cos\theta_2$
$\frac{\tan\theta_1}{\varepsilon_1} = \frac{\tan\theta_2}{\varepsilon_2}$ or $\frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_1}{\varepsilon_2}$

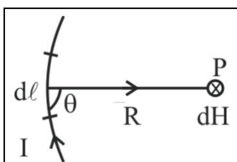
- Image theory applicable problem solving of electrostatic field only.
- **Conductor and free space boundary condition**



$E_t = 0,\ D_t = \varepsilon_0 E_t = 0$	$D_n = \rho_s,\ \ E_n = \frac{\rho_s}{\varepsilon_0}$

■ **Magneto static Field**

- **Biot-Savart's law**



$\boxed{\vec{H} = \oint \frac{Id\vec{\ell} \times \hat{a}_r}{4\pi R^2} A/m}$

$\boxed{dH \propto \frac{I d\ell \sin\theta}{R^2}}$

- **Magnetic Field Intensity for Distributed Current**

Line current	Surface current	Volume current
$\vec{H} = \int_L \frac{Id\vec{\ell} \times \hat{a}_r}{4\pi R^2}$	$\vec{H} = \int_S \frac{\rho_v d\vec{S} \times \hat{a}_r}{4\pi R^2}$	$\vec{H} = \int_V \frac{\vec{J}.dV \times \hat{a}_r}{4\pi R^2}$

- **Magnetic field intensity due to straight current carrying filamentary conductor**

For finite length	$\bar{H}=\frac{I}{4\pi\rho}(\cos\alpha_2-\cos\alpha_1)\hat{a}_\phi$
For semi finite length	$\bar{H}=\frac{I}{4\pi\rho}\hat{a}_\phi$
For infinite length	$\bar{H}=\frac{I}{2\pi\rho}\hat{a}_\phi$

- **Relation between μ_o & ε_0**

$$c=\frac{1}{\sqrt{\mu_o\varepsilon_o}}, c=\text{Speed of light}\left(3\times10^8\,m/s\right)$$

- **Magnetic Energy Density in $\vec{H}$ field (W_m)**

$$W_m=\frac{1}{2}\mu\vec{H}^2=\frac{1}{2}\vec{B}.\vec{H}$$

- **Magnetic Energy**

$$W_m=\iiint_V W_m dV=\frac{1}{2}LI^2\,(\text{Joule})$$

- **Force on a charge particle placed in a magnetic field**

Magnetic force	$\vec{F}_m=Q(\vec{v}\times\vec{B})$ newton
Electric force	$\vec{F}_e=Q\vec{E}$
Lorentz force Equation	$\vec{F}=\vec{F}_e+\vec{F}_m=Q(\vec{E}+\vec{v}\times\vec{B})$
Force on a current carrying conductor	$\vec{F}=\oint_L Id\vec{\ell}\times\vec{B}$

➢ Force between 2-infinite line $F=\frac{\mu_0 I_1 I_2 \ell}{2\pi d}$

Same direction current → attraction

- **Magnetic Torque**

$T=BINA\sin\alpha$
$\vec{T}=\vec{M}\times\vec{B}$
A → area of the loop

- **Magnetic Dipole Moment** $\vec{M}=iA.\hat{n}$

■ **Maxwell's Equations**

- **Maxwell's equation for Static Electric and Magnetic Fields**

	Differential or point form	Integral form	Remarks
First Law	$\vec{\nabla}.\vec{D}=\rho_v$	$\oint_S \vec{D}.d\vec{S}=\int_V \rho_V dv$	Gauss's law
Second Law	$\vec{\nabla}.\vec{B}=0$	$\oint_S \vec{B}.d\vec{S}=0$	Non existence of magnetic monopole
Third Law	$\vec{\nabla}\times\vec{E}=0$	$\oint_L \vec{E}.d\vec{\ell}=0$	Conservative property of electrostatic field
Fourth Law	$\vec{\nabla}\times\vec{H}=\vec{J}$	$\oint_L \vec{H}.d\vec{\ell}=\int_S \vec{J}.d\vec{S}$	Ampere's circuital law

- **Maxwell's equation for time-varying field**

Differential form or point form	Integral form	Remarks
$\vec{\nabla}.\vec{D}=\rho_v$	$\oint_S \vec{D}.d\vec{S}=\int_V \rho_V dv$	Gauss's law
$\vec{\nabla}.\vec{B}=0$	$\oint_S \vec{B}.d\vec{S}=0$	Non existence of magnetic charge
$\vec{\nabla}\times\vec{E}=\frac{-\partial\vec{B}}{dt}$	$\oint_L \vec{E}.d\vec{\ell}=\frac{-\partial}{\partial t}\int_S \vec{B}.d\vec{S}$	Faraday's law
$\vec{\nabla}\times\vec{H}=\vec{J}+\frac{\partial\vec{D}}{\partial t}$	$\oint_L \vec{H}.d\vec{\ell}=\int_S\left(\vec{J}+\frac{\partial\vec{D}}{\partial t}\right).d\vec{S}$	Ampere's circuital law

➢ Vector magnetic potential (A) and magnetic field density relation $\vec{B}=\nabla\times\vec{A}$

- **Time-varying potentials** $\vec{E}=-\nabla.V-\frac{\partial\vec{A}}{\partial t}$

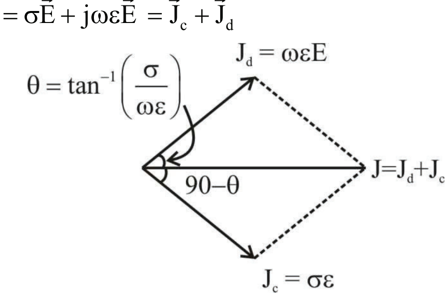
- **Lorentz condition for potential** $\nabla.\vec{A}=-\mu\varepsilon\frac{\partial V}{\partial t}$

■ **Electromagnetic Wave Propagation**

Free space	$\sigma=0,\ \varepsilon=\varepsilon_o,\mu=\mu_o$
Perfect dielectric	$\sigma=0$, ε and μ can have any value
Good dielectrics	$\sigma\simeq0,\ \varepsilon=\varepsilon_r\varepsilon_o,\mu=\mu_r\mu_o$ or $\sigma<<\omega\varepsilon$
Perfect conductor	$\sigma\simeq\infty$, ε and μ can have any vlaue
Good conductor	$\sigma\simeq\infty,\ \varepsilon=\varepsilon_o,\mu=\mu_r\mu_o$ or $\sigma>>\omega\varepsilon$
Lossy Dielectric	$\sigma\neq0,\ \varepsilon=\varepsilon_0\varepsilon_r,\ \mu=\mu_0\mu_r$
For isotopic medium	independent of μ,ε,σ

- **Relation between conduction current density and displacement current density**

$\nabla\times\vec{H}=\sigma\vec{E}+j\omega\varepsilon\vec{E}=\vec{J}_c+\vec{J}_d$



$\vec{J}_C=\sigma\vec{E}$ → Conduction current density

$\vec{J}_d=\frac{\partial\vec{D}}{dt}$ → Displacement current density

$\left|\frac{\vec{J}_c}{\vec{J}_d}\right|=\frac{\sigma}{\omega\varepsilon}$ → $\vec{J}_c$ & $\vec{J}_d$ are out of phase (not 180^0)

In time and they are in the same direction in space

- **Loss tangent/Dissipation factor**

$$\tan\delta = \left|\frac{\vec{J}_c}{\vec{J}_d}\right| = \frac{\sigma}{\omega\varepsilon_0} \quad (\delta \to \text{loss angle})$$

- **Vector wave equation or vector Helmholtz's equation**

➢ **Case I- For frequency domain**

For Electric field	$\nabla^2\vec{E}_s - \gamma^2\vec{E}_s = 0$
For magnetic-field	$\nabla^2\vec{H}_s - \gamma^2\vec{H}_s = 0$

➢ **Case II-For time domain**

Electric wave equation	$\nabla^2\overline{E} = \mu\sigma\frac{d\overline{E}}{dt} + \mu\varepsilon\frac{d^2\overline{E}}{dt^2}$
Magnetic wave equation	$\nabla^2\overline{H} = \mu\sigma\frac{d\overline{H}}{dt} + \mu\varepsilon\frac{d^2\overline{H}}{dt^2}$

- **Field Equation of EM wave in S-domain**.

$$E_S(z) = E_o e^{-\alpha z}.e^{j(\omega t - \beta z)_{a_x}}$$
$$H_S(z) = H_o e^{-\alpha z}.e^{j(\omega t - \beta z)_{a_y}}$$

- **Field equation of EM wave in the time Domain**

$$E(z,t) = E_o e^{-\alpha z}\cos(\omega t - \beta z)a_x$$
$$H(z,t) = H_o e^{-\alpha z}\cos(\omega t - \beta z)a_y$$

Intrinsic Impedance (η)	$\eta = \frac{E}{H}\Omega$, $\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$ For free space $\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi = 377\Omega$
Phase velocity	$v = \frac{\omega}{\beta} = \frac{C}{\sqrt{\mu_r\varepsilon_r}}\ m/s$
Skin Depth	$\delta = \frac{1}{\alpha} = \sqrt{\frac{1}{\pi f\mu\sigma}}$ meter $\quad \delta \propto \frac{1}{\sqrt{f}}$
Surface or skin Resistance	$R_S = \frac{1}{\sigma\delta} = \sqrt{\frac{\pi f\mu}{\sigma}}\ \Omega/m^2$

- Plane waves in good conductor

$\sigma >> \omega\varepsilon$	$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\pi f\mu\sigma}$
$u = \frac{\omega}{\beta} = \sqrt{\frac{2\omega}{\mu\sigma}}$, $\lambda = \frac{2\pi}{\beta}$	$\eta = \sqrt{\frac{\omega\mu}{\sigma}}\angle 45^o$

- **Poynting Theorem-** The net power flowing through volume V is equal to the sum of the ohmic loss and decrease in rate of energy with V.

$$-\oint_S(\vec{E}\times\vec{H}).d\vec{S} = \frac{\partial}{\partial t}\int_V\left[\frac{1}{2}\varepsilon(\vec{E})^2 + \frac{1}{2}\mu(\vec{H})^2\right]dV + \int_V(\sigma\vec{E}^2 J)dV$$

$-\oint_s(\vec{E}\times\vec{H})d\vec{S} =$ Total power leaving the volume
$\int_v(\sigma\vec{E}^2 J)dV$ =Ohmic power dissipation
$\frac{\partial}{\partial t}\int_V\left[\frac{1}{2}\varepsilon(\vec{E})^2 + \frac{1}{2}\mu(\vec{H})^2\right]dV =$ rate of decrease in energy stored in electric and magnetic field.

- **Poynting vector -** $\vec{P} = \vec{E}\times\vec{H}$ W/m^2

Instantaneous power density	$\vec{P} = \frac{E_0^2}{\eta}\cos^2(\omega t - \beta z)\hat{a}_z$
Average power density in lossless medium	$P_{avg} = \frac{1}{2}\frac{E_x^2}{\eta_0}\hat{a}_z = \frac{1}{2}E_x H_y\hat{a}_z = \frac{E_{rms}^2}{\eta}\hat{a}_z = E_{rms}H_{rms}\hat{a}_z$

- **Incidence of EM wave**

Reflection coefficient	$\Gamma = \frac{E_r}{E_i} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\sqrt{\varepsilon_1} - \sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}}$
Transmission Coefficient	$T = \frac{2\eta_2}{\eta_2 + \eta_1} = \frac{2\sqrt{\varepsilon_1}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}}\{T = 1 + \Gamma\}$
Standing Wave Ration (SWR)	$S = \frac{1 + \lvert\Gamma\rvert}{1 - \lvert\Gamma\rvert}$ or $S = \frac{1 + \rho}{1 - \rho}$ or $S = \frac{V_{max}}{V_{min}}$
Snell's law for EM waves	$\frac{\sin\theta_t}{\sin\theta_i} = \frac{\eta_1}{\eta_2} = \sqrt{\frac{\varepsilon_1}{\varepsilon_2}}$
Brewster's Angle (θ_B)	$\tan\theta_B = \frac{\eta_2}{\eta_1} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$
Total internal reflection	$\theta_c = \sin^{-1}\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$ $\theta_c =$ Critical angle

- **Polarization (P)-** Polarization is electric field orientation of electromagnetic wave at a fixed position in space with respect to time. $[E \perp H \perp P]$.