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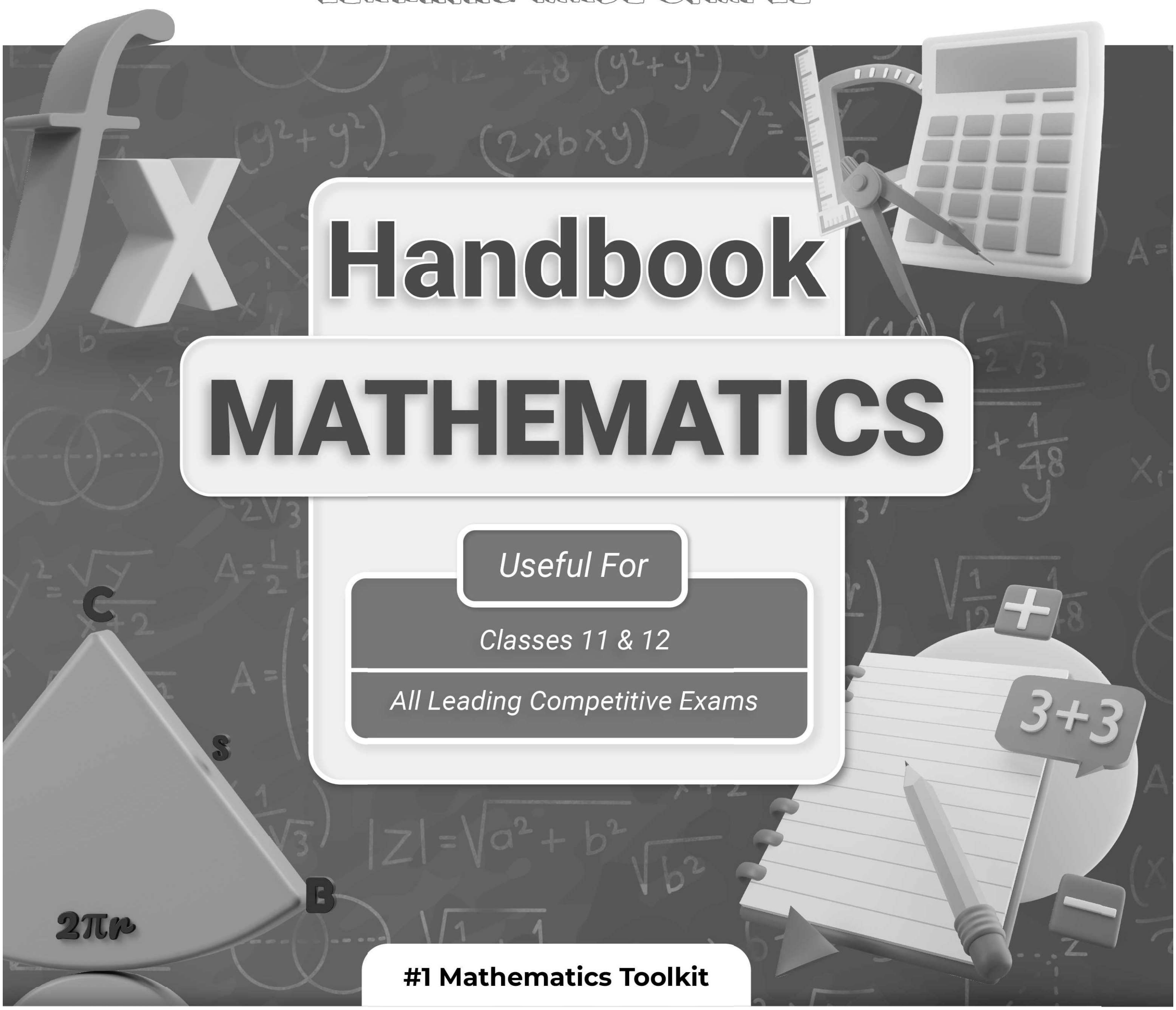

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PREFACE

There are numerous competitive exams conducted in India every year. In this age of cut throat competition, every student aspires to crack the ones he/she appears for in order to begin the journey towards his desired career. Students preparing for various competitive exams strictly follow the prescribed syllabus which has details about the official blueprint as well as typology of questions asked year after year. Along with this, what remains at the core of their preparation is the clarity of concepts.

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We have tried to make our books 100% error-free. Yet, we shall be grateful to the users who point out any errors or make suggestions for the betterment of this book.

We hope that OSWAAL HANDBOOKS will help you at every step as you move closer to your educational goal. We wish you all great success ahead!

All the Best!!

TEAM OSWAAL

CONTENTS

❑❑

SYLLABUS

1. **Sets and Representations :** Sets and their representations, Empty set, Finite and Infinite sets, Equal sets, Subsets of a set of real numbers especially intervals (with notations), Power set, Universal set, Venn diagrams, Union and Intersection of sets, Difference of sets, Complement of a set, Properties of Complement set, Algebraic properties of Union, Intersection and Complement of sets.
2. **Relations and Functions** : Ordered pairs, Cartesian product of sets, Number of elements in the Cartesian product of two finite sets, Cartesian product of the set of real with itself (upto R x R x R), Definition of relation, pictorial diagrams, domain, co-domain and range of a relation, types of relation, binary operation, Types of relations: reflexive, symmetric, transitive and equivalence relations. One to one and onto functions, composite functions,inverse of a function. Binary operations. Function as a special type of relation, Pictorial representation of a function, domain, co-domain and range of a function, real valued functions, domain and range of these functions, constant, identity, polynomial, rational, modulus, signum, exponential, logarithmic and greatest integer functions, with their graphs, composition of function, sum, difference, product and quotients of functions.
3. **Trigonometry**: Trigonometric Functions
   - Positive and negative angles
   - Measuring angles in radians and in degrees and conversion from one measure to another
   - Definition of trigonometric functions with the help of unit circle
   - Truth of the identity $\sin^2 x + \cos^2 x = 1$, for all x
   - Signs of trigonometric functions
   - Domain and range of trigonometric functions and their graphs
   - Expressing $\sin(x \pm y)$ and $\cos(x \pm y)$ in terms of $\sin x$, $\sin y$, $\cos x$ & $\cos y$ and their simple applications
   - Deducing identities like the following :
   - $\tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x.\tan y}$, $\cot(x \pm y) = \dfrac{\cot x.\cot y \mp 1}{\cot y \pm \cot x}$
   - $\sin\alpha \pm \sin\beta = 2\sin\frac{1}{2}(\alpha \pm \beta).\cos\frac{1}{2}(\alpha \mp \beta)$
   - $\cos\alpha + \cos\beta = 2\cos\frac{1}{2}(\alpha + \beta).\cos\frac{1}{2}(\alpha - \beta)$
   - $\cos\alpha - \cos\beta = -2\sin\frac{1}{2}(\alpha + \beta).\sin\frac{1}{2}(\alpha - \beta)$
   - Double angle, triple angle, half angle and one third angle formula as special cases.
   - Identities related to $\sin 2x$, $\cos 2x$, $\tan 2x$, $\sin 3x$, $\cos 3x$ and $\tan 3x$
   - Angle and Arc lengths
   - Graphs of simple trigonometric functions for all trigonometric ratios
   - Addition and subtraction formula : $\sin(A \pm B)$; $\cos(A \pm B)$; $\tan(A \pm B)$; $\tan(A+B+C)$ etc
   - Sum and differences as products
   - Product to sum or difference
   - Trigonometric equations: General solution of trigonometric equations of the type siny = sina, cosy = cosa and tany = tana
   - Properties of triangle in terms of
     - Sine formula: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$
     - Cosine formula: $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$, etc.
     - Area of triangle $= \frac{1}{2} bc \sin A$ etc.
   - Inverse Trigonometric Functions : Definition, range, domain, principal value branch. Graphs of inverse trigonometric functions. Elementary properties of inverse trigonometric functions.
4. **Complex Numbers**: Need for complex numbers, especially $\sqrt{-1}$, to be motivated by inability to solve some of the quadratic equations, algebraic properties of complex numbers, conjugate complex numbers, Argand plane and polar representation of complex numbers, modulus and argument, triangle inequality, loci on the Argand Diagram, product and quotient for complex numbers in modulus-argument form, De Moivre's theorem, square root of complex number, cube root of unity.
5. **Quadratic Equations** : Statement of Fundamental Theorem of Algebra, Solution of quadratic equations (with real coefficients), Equations reducible to quadratic form, Nature of roots – Product and sum of roots, Framing of quadratic, cubic, biquadratic equation with given roots, Conditions for common roots in quadratic equations, Properties of quadratic equation, Understanding the fact that a quadratic expression (when plotted on a graph) is a parabola, Minimum and Maximum value of quadratic equations, Sign when the roots are real and when they are complex, Interval of roots, Algebraic interpretation of Rolle's theorem, Descartes' rule of sign.
6. **Linear Inequalities** : Linear inequalities, Algebraic solutions of linear inequalities in one variable and their representation on the number line. Graphical solution of linear inequalities in two variables. Graphical method of finding a solution of system of linear inequalities in two variables.
7. **Principle of Mathematical Induction :** Process of the proof by induction, motivating the application of the method by looking at natural numbers as the least inductive subset of real numbers. The principle of mathematical induction and simple applications.
8. **Sequences and Series**: Sequence and series, Arithmetic Progression (AP), Arithmetic Mean (AM), Geometric Progression (GP),General Term of a GP, Sum of n terms of a GP, infinite GP and its sum, Geometric Mean (GM), Harmonic Progression (HP) , nth term of Harmonic Progression, Harmonic Mean, Relation in AM, GM and HM
   - Arithmetic - Geometric Progression (AGP), Sum of Arithmetic-Geometric Series
   - Formulae for the following special sums.
     $\sum_{k=1}^{k=n} k, \sum_{k=1}^{k=n} k^2 \, and \sum_{k=1}^{k=n} k^3$
9. **Permutations and Combinations :** Fundamental Principle of Counting, Factorial and its properties, Permutation and its properties, Circular Permutation, Combination and its properties, Exponent of Prime p in n!, Division into Groups, use of permutation and combination in geometry, use of permutation and combination in evaluating prime factor, use of permutation and combination in Integral solution, use of permutation combination in Sum of Digits, Derangements theorem.
10. **Binomial Theorem:** History, statement and proof of the binomial theorem for positive integral indices. Properties of binomial coefficients. Pascal's Triangle, General and middle term in binomial expansion, Binomial Theorem for Any Index, Simple Applications.
11. **Straight Lines:** Brief recall of two dimensional geometry from earlier classes, shifting of origin, slope of a line and

angle between two lines, Various forms of equations of a line, parallel to axis, point-slope form, slope-intercept form, two-point form and normal form, general equation of a line, equation of family of lines passing through the point of intersection of two lines, distance of a point from a line, angle between two lines, equation of lines bisecting the angle between two lines, condition for concurrence of three lines, coordinates of centroid, orthocenter and circumcenter of a triangle

12. **Conic Sections:** Sections of a cone: circles, ellipse, parabola, hyperbola, a point a straight line and a pair of intersecting lines as a degenerated case of a conic section. Standard equations and simple properties of parabola, ellipse and hyperbola, Standard equation of a circle.

13. **Three Dimensional Geometry:** Coordinate axes and coordinate planes in three dimensions. Coordinates of a point. Distance between two points and section formula, direction cosines and direction ratios of a line joining two points, cartesian equation and vector equation of a line, coplanar and skew lines, shortest distance between two lines. Cartesian and vector equation of a plane. Intersection of a line and a plane in different form, angle between (i) two lines, (ii) two planes, (iii) a line and a plane, distance of a point from a plane.

14. **Vector Algebra**: Derivative introduced as rate of change both as that of distance function and geometrically, Intuitive idea of limit, Fundamental theorems on limits, Limits of polynomials and rational functions trigonometric, exponential and logarithmic functions, Definition of derivative relate it to scope of tangent of the curve, Derivative of sum, difference, product and quotient of functions, Derivatives of polynomial and trigonometric functions, Derivative of composite functions, Parametric functions, Derivative of function with respect to other function, Successive differentiation, Differentiation using first principles.

15. **Limits and Derivatives:** Derivative introduced as rate of change both as that of distance function and geometrically, Intuitive idea of limit, Fundamental theorems on limits, Limits of polynomials and rational functions trigonometric, exponential and logarithmic functions, Definition of derivative relate it to scope of tangent of the curve, Derivative of sum, difference, product and quotient of functions, Derivatives of polynomial and trigonometric functions, Derivative of composite functions, Parametric functions, Derivative of function with respect to other function, Successive differentiation, Differentiation using first principles.

16. **Continuity, Differentiability and Application of Derivatives:** Continuity and differentiability, derivative of composite functions, chain rule, derivatives of inverse trigonometric functions, derivative of implicit functions. Concept of exponential and logarithmic functions.
    - Derivatives of logarithmic and exponential functions. Logarithmic differentiation,derivative of functions expressed in parametric forms. Second order derivatives. Rolle's and Lagrange's Mean Value Theorems (without proof) and their geometric interpretation.
    - Applications of derivatives: rate of change of bodies, increasing/decreasing functions, tangents and normals, use of derivatives in approximation, maxima and minima (first derivative test motivated geometrically and second derivative test given as a provable tool). Simple problems (that illustrate basic principles and understanding of the subject as well as real-life situations).

17. **Determinants:** Determinant of a square matrix (up to 3 × 3 matrices), properties of determinants, minors, cofactors and applications of determinants in finding the area of a triangle. Adjoint and inverse of a square matrix. Consistency, inconsistency and number of solutions of system of linear equations by examples, solving system of linear equations in two or three variables (having unique solution) using inverse of a matrix.

18. **Matrices:** Concept, notation, order, equality, types of matrices, zero and identity matrix, transpose of a matrix, symmetric and skew symmetric matrices, Conjugate of matrix, Orthogonal matrix, idempotent matrix and Involutory matrix, Operation on matrices : Addition and multiplication and multiplication with a scalar. Simple properties of addition, multiplication and scalar multiplication. Non-commutativity of multiplication of matrices and existence of non-zero matrices whose product is the zero matrix (restrict to square matrices of order 2 and 3). Concept of elementary row and column operations. Invertible matrices and proof of the uniqueness of inverse, if it exists; (Here all matrices will have real entries).

19. **Integral and its Applications:** Integration as inverse process of differentiation. Integration of a variety of functions by substitution, by partial fractions and by parts, Evaluation of simple integrals of the different types and problems based on them. Definite integrals as a limit of a sum, fundamental Theorem of Calculus (without proof). Basic properties of definite integrals and evaluation of definite integrals. Applications in finding the area under simple curves, especially lines, circles/ parabolas/ellipses (in standard form only), Area between any of the two above said curves (the region should be clearly identifiable).

20. **Differential Equations:** Definition, order and degree, general and particular solutions of a differential equation. Formation of differential equation whose general solution is given. Solution of differential equations by method of separation of variables, solutions of homogeneous differential equations of first order and first degree. Solutions of linear differential equation of the type. $\frac{dy}{dx} + py = q$,where p and q are functions of x or constants. $\frac{dx}{dy} + px = q$,where p and q are functions of y or constants.

21. **Mathematical Reasoning:** Mathematically acceptable statements, Connecting words/phrases - consolidating the understanding of "if and only if (necessary and sufficient) condition", "implies", "and/or", "implied by", "and", "or", "there exists" and their use through variety of examples related to real life and Mathematics, Tautology, Contradiction and Duality, Algebra of statements, Validating the statements involving the connecting words, difference among contradiction, converse and contrapositive

22. **Statistics:** Measures of Dispersion : Range & Mean deviation, Variance and standard deviation of ungrouped / grouped data, The Median, Quartiles, Decile, Percentile, Mode of grouped and ungrouped data, analysis of frequency distributions with equal means but different variances. Correlation, types of correlation, Covariance, Karl Pearson Coefficient of correlation, Regression and its analysis.

23. **Commercial Mathematics**: Annuity, Partnership, Bill of Exchange, Foreign Exchange.

24. **Probability**: Random experiments, Outcomes, Sample spaces (set representation), Events, Occurrence of events: 'Not', 'And' and 'Or' events, Exhaustive events and mutually exclusive events, Axiomatic (set theoretic) probability, Connections with other theories of earlier classes, Probability of an event, Probability of 'net', 'and' 'or' events. Conditional probability, Multiplication theorem on probability, Independent events, Total probability, Bayes' theorem, Random variable and its probability distribution, Mean and variance of random variable, Repeated independent (Bernoulli) trials and Binomial distribution, Laws of probability addition theorem, De Morgan's Law, Poisson's Distribution, Normal Distribution.

❑❑❑

MATHEMATICS MNEMONICS

Sets, Relations and Functions

Sets And Representations (a)

Today's **S**cenario, **E**qually **T**alented **S**ingers **F**ind **I**nfinite **N**ew **S**ongs **T**o **S**ing.

Today's → Types; **S**cenario, → Sets; **E**qually talented → Equivalent, Equal

Singers → Singleton; **F**ind → Finite; **I**nfinite → Infinite

New → Null; **S**ongs to sing. → Sets

Interpretation :

Types of **S**ets :

1. **E**mpty or **N**ull **S**et - A set which has no element.
2. **F**inite **S**et - A **s**et having **f**inite **n**umber of **e**lements.
3. **I**nfinite **S**et - A set having infinite number of elements.
4. **E**quivalent **S**et - Two finite sets A and B are said to be equivalent if n(A)=n(B).
5. **E**qual **S**et - Two sets A and B are equal if every element of A is in B.
6. **S**ingleton **S**et - A sets having one element is called singleton set.

Sets And Representations (b)

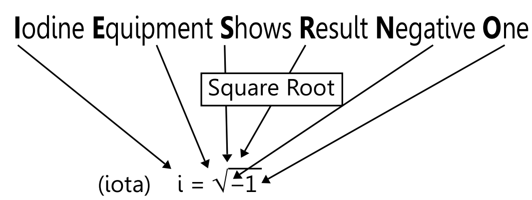
Laws of **A**lgebra of **S**tatements :

Iacd and **I**cai are friends

Interpretation :

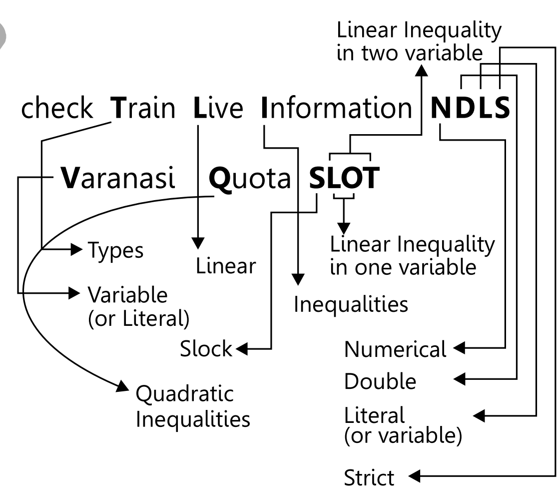
1. **I**dempotent **L**aw -
   (i) $(A \land A) \Leftrightarrow A$
   (ii) $(A \lor A) \Leftrightarrow A$
2. **A**ssociative **L**aw -
   (i) $(A \land B) \land C \Leftrightarrow A \land (B \land C)$
   (ii) $(A \lor B) \lor C \Leftrightarrow A \lor (B \lor C)$
3. **C**ommutative **L**aw -
   (i) $A \lor B \Leftrightarrow B \lor A$
   (ii) $A \land B \Leftrightarrow B \land A$
4. **D**istributive **L**aw -
   (i) $A \lor (B \land C) \Leftrightarrow (A \lor B) \land (A \lor C)$
   (ii) $A \land (B \lor C) \Leftrightarrow (A \land B) \lor (A \land C)$
5. **I**dentity **L**aws -
   (i) $A \lor T \Leftrightarrow A$
   (ii) $A \land F \Leftrightarrow F$
   (iii) $A \lor T \Leftrightarrow T$
   (iv) $A \lor F \Leftrightarrow A$
6. **C**omplement **L**aws -
   (i) $A \lor (\sim A) \Leftrightarrow T$
   (ii) $A \land (\sim A) \Leftrightarrow F$
   (iii) $\sim T \Leftrightarrow F$
   (iv) $\sim F \Leftrightarrow T$
7. **A**bsorption **L**aw -
   (i) $A \lor (A \land B) \Leftrightarrow A$
   (ii) $A \land (A \lor B) \Leftrightarrow A$
   (iii) $\sim (A \land B) \Leftrightarrow (-A) \lor (-B)$
8. **I**nvolution **L**aw -
   (i) $\sim (\sim A) \Leftrightarrow A$

Complex Numbers and Quadratic Equations



Interpretation: Complex numbers are expressed in the form of a+ib where 'i' is an imaginary number called 'iota' and the value of iota is $\sqrt{-1}$

Types of Linear Inequalities



Interpretation :

1. Numerical Inequality - 3<5, 8>4
2. Literal or Variable Inequalities - $x<5$, $y>8$
3. Double Inequality- $5<x<9$, $3<y<10$

4. Strict Inequality- $x<9$, $5<10$
5. Slack Inequality- $x\geq7$, $y\leq9$
6. linear Inequality in One Variable- $x<9$, $y>12$
7. linear Inequality in Two Variable- $5x+7y<12$
8. Quadratic Inequality- $x^2+5x\leq10$

Matrices and Determinants

Identity Matrix-

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad a_{ij} = 0 \text{ when } i \neq j, \quad a_{ij} = 1 \text{ when } i = j$$

Zero Matrix-

$$A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Singular Matrix

A square matrix is said to be singular matrix if determinant of matrix denoted by $|A|$ is zero otherwise it is non zero matrix

Inverse Of a Matrix

Determinant

"a **D**etermined **A**rtist **C**an become a **S**inger,
if he is **O**ptimistic. → (Zero)

"a **D**etermined **A**rtist Can **N**ever be **S**inger
if he is **N**ot **O**ptimistic → Non Singular

if $|A| = 0$, then A is Singular Otherwise,
A is non-Singular

≠0 (Zero)

"If **D**etermined **A**rtist is **N**ot **O**ptimistic
then **ADJ**ust **B**elow **I**nternational
↓ Adjoint ↓ By ↓ Inverse
Musicians"
↓ Matrix

Determinant

"A is non-singular *i.e.* $|A| \neq 0$ then

$$A^{-1} = \frac{1}{|A|}.\,(\text{adj } A)$$

Interpretation :

Singular & Non Singular Matrix -

if $|A| = 0$, then A is singular. Otherwise A is non-singular

Inverse of a Matrix -

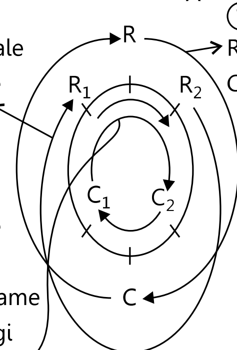
Inverse of a Matrix exists if A is non- singular *i.e* $|A| \# 0$, and is given by

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

Properties Of |A|

Possible Clockwise **D**irections of
↓ Properties ↓ Determinant (of Distance)

Action
↓
A



② Route 1 wale Route 2 Se Jaye and Route 2 Wale Route 1 Se Jaye to distance Same **N**ahi rahegi
↓ (–ve)

① R - Society wale
C - Society me Shift hojaye and C Society Wale R me Shift ho jaye, Distance Same hi Rahegi

③ Circle 1 Wale Circle 2 Se Jaye and Circle 2 Wale Circle 1 Se to Distance same **N**ahi rahegi
↓ (–ve)

④ Distance = 0 if $R_1=R_2$ (Route 1=Route 2)
↓ $|A| = 0$ if $R_1=R_2$

⑤ Distance = 0 if $C_1=C_2$ (Circle 1=Circle 2)

Interpretation :

Properties of |A| -

(i) $|A|$ remains unchanged, if the rows and columns of A are interchanged i.e. $|A| = |A'|$

(ii) If any two rows (or columns) of A are interchanged, then the sign of $|A|$ changes.

(iii) If any two rows (or Columns) of A are identical then $|A|$ = a

Principle of Mathematical Induction

San **F**rancis **P**rincipal **OM** **I**nvited **P**arents

SFPOMIP

Principle of Mathematical Induction (B)

Provided **T**est **P**aper of **1**st **T**erm

PTP(1)T

Principle of Mathematical Induction (C)

Also **T**est **P**aper of **K**th **T**erm

ATP(K)T

Principle of Mathematical Induction (D)

Then **T**est **P**aper of (**K+1)th** **T**erm

TPTP(K+1)T

Principle of Mathematical Induction (E)

Hence **P**aper of **nth** is **T**rustworthy **F**or **A**ll **N**ecessary **N**umbers

HP(n)TFANN

Principle of Mathematical Induction (F)

SFPOMIP-Steps for Principle of Mathematical Induction Proof

Interpretation :
Step1: Let P(n) be a result or statement formulated in terms of n in a given equation.

Principle of Mathematical Induction (G)

PTP(1)T-Prove that P(1) is true.

Interpretation :

Step2: Prove that P(1) is true.

Principle of Mathematical Induction (H)

ATP(K)T-Assume that P(K) is true.

Interpretation :

Step3: Assume that P(k) is true.

Principle of Mathematical Induction (I)

TPTP(K+1)T-prove that P(k+1) is true.

Interpretation :
Step4: Using step 3, prove that P(k+1) is true.

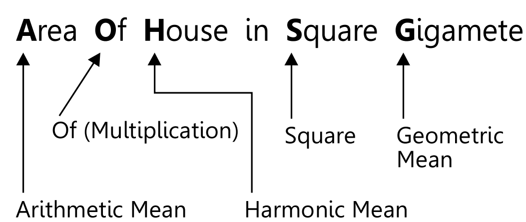
Principle of Mathematical Induction (J)

HP(n)TFANN - Hence, by the principle of mathematical induction, P(n) is true for all natural numbers n

Interpretation :
Step5: Thus, P(1) is true and P(k+1) is true whenever P(k) is true. Hence, by the principle of mathematical induction, P(n) is true for all natural numbers n.

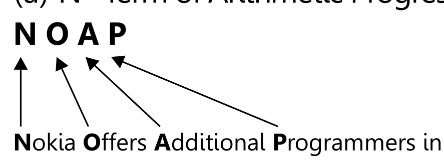
Sequence and Series

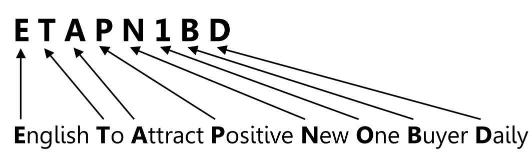
Relationship between **A**M, **G**M and **H**M



Arithmetic Progression (AP)

(a) Nth Term of Arithmetic Progression -

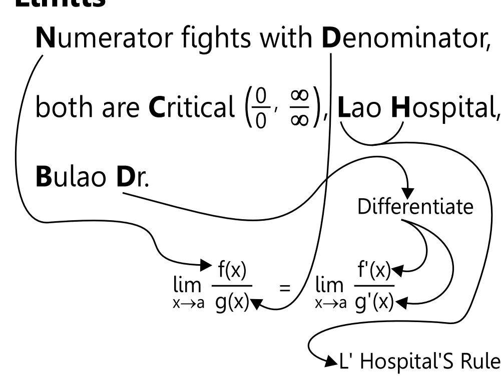




Nth Term of AP = $a + (n - 1)d$

Limits Continuity and Differentiability

L' Hospital's Rule for Evaluating Limits



Interpretation :

if $\lim_{x\to a} \frac{f(x)}{g(x)}$ takes $\frac{0}{0}$ or $\frac{\infty}{\infty}$ form

then $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$

where $f'(x) = \frac{df(x)}{dx}$ and $g'(x) = \frac{dg(x)}{dx}$

Sandwich Theorem for Evaluating Limits

Likhil always uses **S**amesize **(L) M**iddle bread to make **T**hree layer Sandwich

(Three functions f(x), g(x) & h(x))

If $f(x) \le g(x) \le h(x)$ and

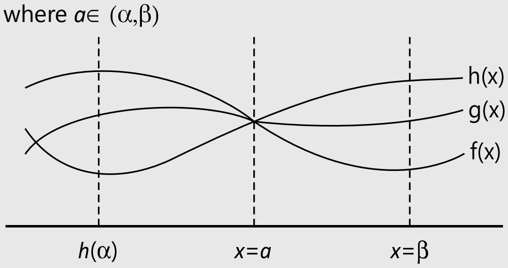
$\lim_{x\to a} f(x) = L = \lim_{x\to a} h(x) = L$ Then $\lim_{x\to a} g(x) = L$

Interpretation :

If $f(x) \le g(x) \le h(x)\ \forall\ x \in (\alpha, \beta) - \{a\}$

and $\lim_{x\to a} f(x) = L = \lim_{x\to a} h(x)$ then $\lim_{x\to a} g(x) = L$

where $a \in (\alpha, \beta)$



Mean Value Theorem & Rolle's Theorem

Film **a b c d MO**vie **V**s **T**heater **RO**le,

- **F** → function → f (a, b)→R
- **a b** → f (a, b)→R
- **c** → Continuous an closed interval [a,b]
- **d** → differentiable on open interval
- **MO** → Mean; **O** → on open interval
- **V** → Value
- **T** → Theorem
- **R** → Rolle's Theorem
- **O** → O shows f'(c)=0 in Rolle's Theorem

∃ **S**ome **C**haracter **O**f **A**mitabh **B**achchan

∃ Some C (a, b)

Interpretation :

Mean Value Theorem -

if f: [a,b] → R Continuous on [a,b] and differential on (a,b) , then ∃ some c in (a,b) such that-

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Rolle's Theorem -

If f: [a,b]→ R continuous on [a,b] and differentiable on [a,b] and f(a) = f(b) then ∃ some c in (a,b) s.t. f'(c) =0

Se**C**ond **F**und**A**mental Theorem of **D**efinite Integration

f c c   F a d   F b F a

(f c c: Continuous, Closed; F a d: anti derivative)

You can also remember

fcc is small **f**ashionable **C**lothes **C**ounter **F**or **a**dorable **d**resses at **Fb**b **Fa**shion

(Closed, anti, Continuous, derivative, F(b) – F(a))

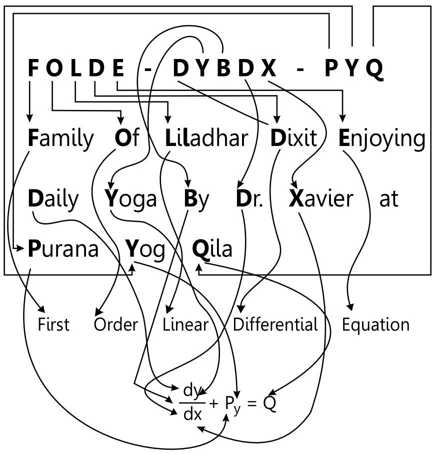
Interpretation :

Let f be a continuous function defined on a closed interval [a,b] and F be an anti derivative of f. Then $\int_a^b f(x)\,dx = [F(x)]_a^b = F(b) - F(a)$,

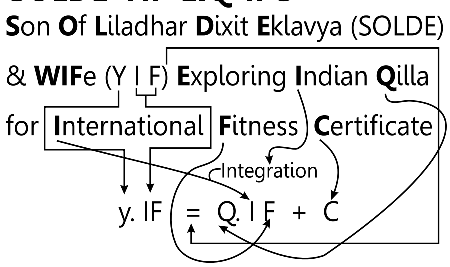
where a and b are called limit of Integration.

Differential Equations

Linear Differential Equations



SOLDE-YIF-EIQ-IFC



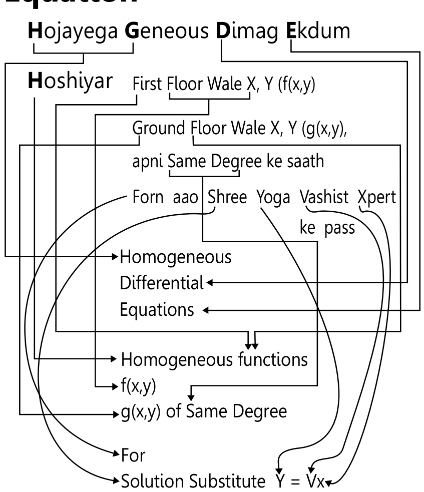
S — Solution
O — Of
L — Linear
D — Differential
E — Equation

Interpretation :

Differential equation is of the form $\frac{dy}{dx}+py=Q$, where P and Q are constants or the function of 'x' is called a first order linear differential equations. Its solution is given as

$$Y.IF=\int Q.IF+C$$

Homogeneous Differential Equation



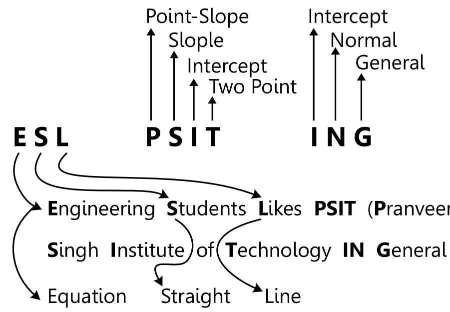
Interpretation :

Differential equation can be expressed in the form $\frac{dy}{dx}=f(x,y)$ or $\frac{dx}{dy}=g(x,y)$

where f(x,y) and g(x,y) are homogeneous functions of sum is called a homogeneous Differential equation. These equations can be solved by substituting y=vx so that dependent variable y is changed to another variable v, where v is some unknown function.

Coordinate Geometry

Equation of Straight Line in Various forms :



Interpretation :

(1) Point Slope form :- $y-y_1=m(x-x_1)$

(2) Slope intercept form :- $y= mx+c$

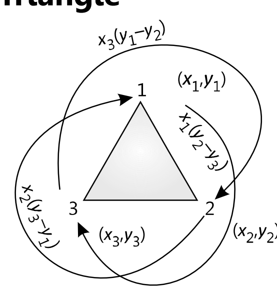
(3) Two point form :- $y - y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$

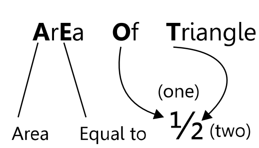
(4) Intercept form:- $\frac{x}{a}+\frac{y}{b}=1$

(5) Normal / Perpendicular form :-
$x\cos\alpha + y\sin\alpha = P$

(6) General Form :- $ax + bx + c = 0$

Area of Triangle





Area $=\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_3)]$

Eccentricity of conic Sections

English alphabet **C**ounting **S**equence

C --------- **e** --------- **P**

Circle ($e=0$) — ellipse ($0<e<1$) — Parabola ($e=1$)

Hyp**E**rbola → Highest ECcentricity i.e. $e>1$

Interpretation :

Eccentricity of Conic Sections-

(a) If e=1, the conic is called parabola.
(b) If 0<e<1, the conic is called ellipse.
(c) If e>1, the conic is called hyperbola.
(d) If e=0, the conic is called circle.

Three Dimensional Geometry

Direction Cosines

Dance **C**horeographer **P**refer **D**ieting

Direction — Cosines — Positive — Direction

1 glass **L** e **M** o **N** juice

$l^2 + m^2 + n^2 = 1$

Direction Ratios

Director Remo a **P**rofessional **D**ancer

Direction — Ratios — Proportional — Direction

Choreographer created

Cosines

3 **L**ifetime **M**ovies with **N**ew faces **a b c**

$l = \frac{a}{\sqrt{a^2+b^2+c^2}}$, $m = \frac{b}{\sqrt{a^2+b^2+c^2}}$, $n = \frac{c}{\sqrt{a^2+b^2+c^2}}$

Interpretation :

Direction cosines of a line are the cosines of the angles made by the line with the positive directions of the co. ordinate axes. If l, m, n are the D. cs of a line, then $l^2+m^2+n^2=1$

Vector Algebra

Types Of Vectors (A)

Teachers of **V**arious **N**ew **C**olleges **U**sing

Types — Vectors — Negative — Collinear — Unit

Zoom **C**onferencing during **E**pidemic

Zero — Coinitial — Equal

Interpretation :

Types of Vectors-

1. Zero Vector - Initial and terminal points coincide

2. Unit Vector - Magnitude is unity

3. Coinitial Vectors - Same initial points

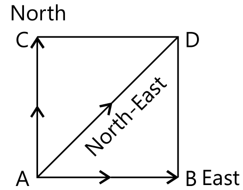
4. Collinear vectors - Parallel to the same Line

5. Equal Vectors - Same magnitude and direction

6. Negative of a vector- Same magnitude, opp. direction

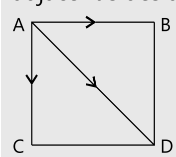
Properties Of Vectors(B)

"Neither choose East nor choose north, always choose North-East and save your time".



Interpretation :

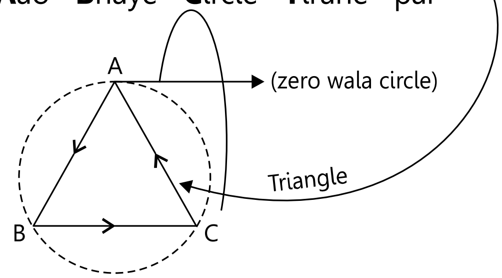
The vector sum of two coinitial vectors is given by the diagonal of the parallelogram whose adjacent sides are given vectors.



$\vec{AB} + \vec{AC} = \vec{AD}$

Properties Of Vectors(C)

Aao **B**naye **C**ircle **T**irahe par



Interpretation:

The vector sum of the three sides of a triangle taken in order is $\vec{O}$ i.e

$\vec{AB} + \vec{BC} + \vec{CA} = \vec{O}$

Statistics & Probability

Mutually Exclusive Events

MEE-Mutual Enemies Everywhere

Morning **E**vening **E**veryday **C**annot **O**ccur
↓ Mutually ↓ Exclusive ↓ Events ↓ cannot ↓ occur

Sametime
↓ simultaneously

Interpretation :

Events A & B are called mutually exclusive events if occurrence of any one of them excludes occurrence of other event, i.e. they cannot occur simultaneously.

eg: A die is thrown. Event A=All even outcomes & events B=All odd outcomes. then, A & B are mutually exclusive events, they cannot occur simultaneously

Poisson Distribution

DPD – Directions for **P**ure **D**ishes

Distribution–Poisson Distribution

Perfect **R**estaurant **E**stablished
near **L**ajpat **N**agar **I**n **NCR** **P**rovides
Rich **Q**uality **N**aan **R**ice

Equal

$$P(r) = \lim_{n\to\infty} {}^nC_r p^r q^{(n-r)}$$

Here **L**emo**N** **Q**uinoa **I**s **C**ostliest **P**ure **D**ish
↓ Here ↓ $\lambda = nq$ is called Poisson Distribution

Normal Distribution

DND — Do **N**ot **D**isturb

Distribution Normal Distribution

Parking **x** **E**ntry **T**icket is **O**ne rs.
Per **S**ingle **R**oot for **2** **P**ersons and
Entry **P**oint is **M**id **H**alf **Z**one **2.**

Power Minus ½

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$$

Here **Z**one **E**ntry for **EXMS**
(Electronic Exam Management System)
in **R**ajasthan for **N**ational **P**rofessional
Qualification

Here Minus Meu Sigma Root

$$Z = \frac{x-\mu}{\sigma}, \sigma = \sqrt{npq}$$

Variance and standard deviation for ungrouped data-

(a) Standard deviation for un-grouped data-

Subject me **D**istinction **U**n **G**roups ki
(Standard) (Deviation) (Ungrouped data)

Rhti, jo "**A**dd kre **X**tra **M**ehnont
(Root) ('Σ' sign) (minus)

X **B**ar **D**ouble **P**ower ke saath"

Distinction wale **N**umber ke Saath".

$$\text{S. D. }(\sigma) = \sqrt{\frac{1}{n}\Sigma(x_i - \bar{x})^2}$$

Divided by

"Mere Naam me mera Sign-S se sigma

Variance for ungrouped data

"**V**edic **F**undamentals **U**nder **G**raduates
(Variance) (for) (Ungrouped data)

lagaao **S**quare me **D**istinction
(Square) (Standard) (Deviation)

number Paao"

Variance = (Standard deviation)^2

Interpretation :

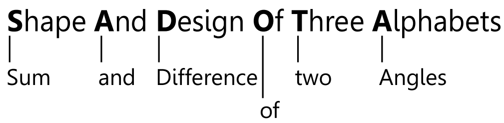
Standard deviation of ungrouped data :

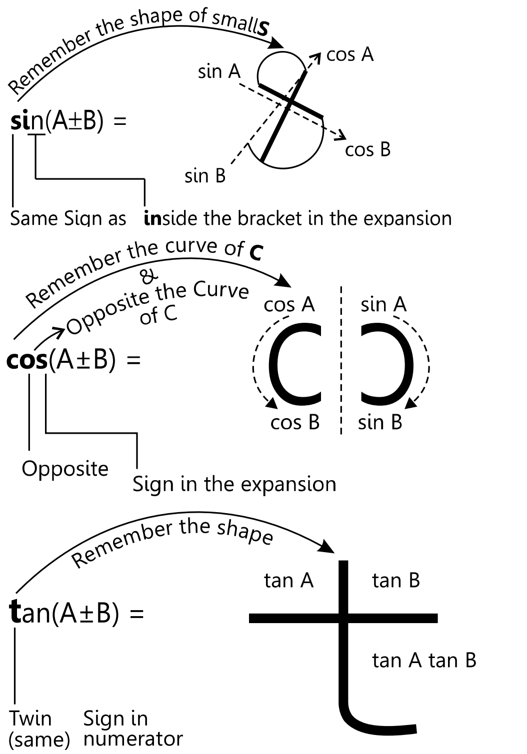
S.D. of ungrouped data is the square root of squared deviation from the mean of data. It is denoted by the symbol " 6 "

Variance for ungrouped data :

Variance for ungrouped data is defined as the square of S.D. It is denoted by " 6^2"

Sum and Difference of two Angles

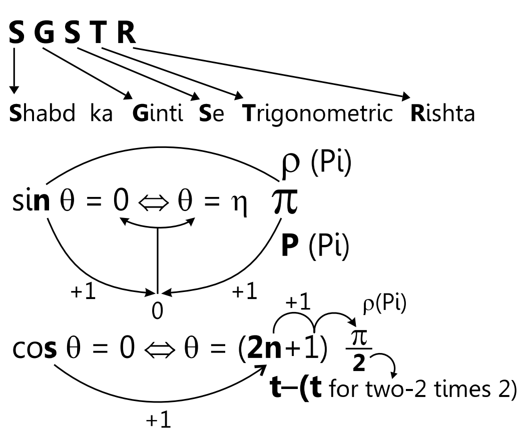


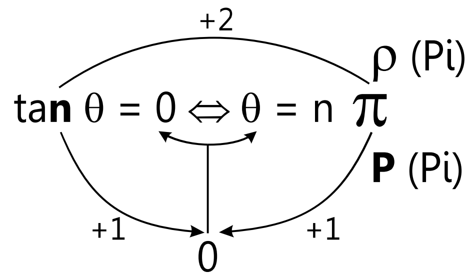


Interpretation :

* $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$
* $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$
* $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

Standard General Solution of Trigonometric Ratios





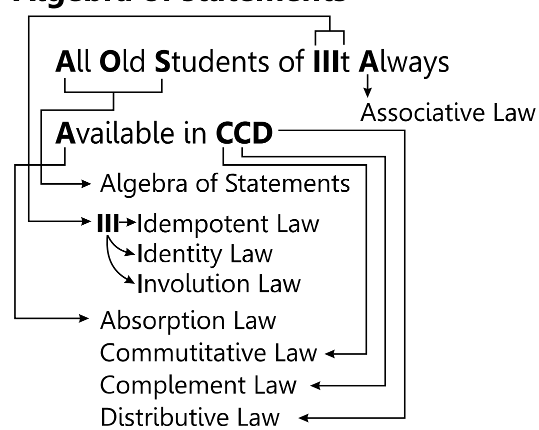
Interpretation :

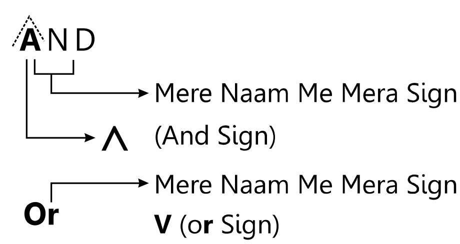
The solution consisting of all possible solutions of a trigonometric equation is called its general solution.

* $\sin\theta = 0 \Leftrightarrow \theta = n\pi$
* $\cos\theta = 0 \Leftrightarrow \theta = (2n+1)\dfrac{\pi}{2}$
* $\tan\theta = 0 \Leftrightarrow \theta = n\pi$

Mathematical Reasoning

Algebra of statements -





CHAPTER : 1 SETS AND REPRESENTATIONS - PART -I

Sets and Representations

Introduction

A set is a collection of well-defined distinguished objects. The sets are usually denoted by capital letters A, B, C etc., and the members or elements of the set are denoted by lower case letters a, b, c etc.., If x is a member of the set A, we write $x \in$ A (read as 'x' belongs to A) and if x is not a member of set A, we write $x \notin$A(read as 'x' doesn't belongs to A). If x and y both belong to A, we write x, y $\in$ A.

Some examples of sets are: A: odd numbers less than 10
N: the set of all natural numbers
V : the vowels in the English alphabates
R: the set of all real numbers.

Cardinal Number

The number of elements in a finite set is represented by n (A), known as cardinal number.
Eg.: A = {a, b, c, d, e} Then, n (A) = 5

Subset & Power Set

Let A and B be two sets. If every element of A is an element of B,then A is called a subset of B and written as A⊂B or B⊃A(read as 'A' is contained in 'B' or 'B contains A'). B is called superset of A.

Note:
1. Every set is a subset and superset of itself.
2. If A is not a subset of B, we write A⊄B.
3. The empty set is the subset of every set.
4. Power Set: If A is a set with n (A) = m, then no. of subset in power set n [P(A)]=2^m

e.g. Let A = {3, 4}, then subsets of A are ϕ, {3}, {4}, {3, 4}. Here, n(A) = 2 and number of subsets of A = 2^2=4.

Representation of Sets

Set builder form or Rule Method

In this form, we write a variable (say x) representing any member of the set followed by property satisfied by each member of the set.
e.g.: The set A of all prime number less than 10 in set builder form is written as
A ={x | x is a prime number less than10}
The symbol "|" stands for the word "such that". Sometimes, we use symbol ":" in place of symbol "|"

Roaster or Tabular form

In this form, we first list all the members of the set within braces (curly brackets) and separate these by commas.
e.g: The set of all odd natural number less than 10 in this form is written as: A = {1, 3, 5, 7, 9}
In roaster form, every element of the set is listed only once.
The order in which the elements are listed is immaterial.
e.g. Each of the following sets denotes the same set
{1, 2, 3}, {3, 2, 1}, {1, 3, 2}

Types of Sets

Empty set or Null set

A set which has no element is called null set. It is denoted by symbol ϕ or {}.
e.g: Set of all real numbers whose square is −1.
In set-builder form: {x : x is a real number whose square is −1}
In roaster form: { } or ϕ

Finite and Infinite set

A set which has finite number of elements is called a finite set. Otherwise, it is called an infinite set.
e .g.: The set of all days in a week is a finite set whereas the set of all integers, denoted by
{……−2, −1, 0, 1, 2,……} or {x | x is integers} is an infinite set.
An empty set ϕ which has no element is a finite set is called empty or void or null set.

Equal set

Two sets A and B are set to be equal, written as A=B, if every element of A is in B and every element of B is in A.
e.g.: (i) A = {1, 2, 3,4} and B = {3, 1, 4, 2}, then A =B
(ii) A = {x : x−5 =0} and B = {x : x is an integral positive root of the equation $x^2 - 2x - 15 = 0$}
Then A= B

Singleton set

A set having one element is called singleton set.
e.g.: (i) {0} is a singleton set, whose only member is 0.
(ii) A = {x : 1<x <3, x is a natural number} is a singleton set which has only one member which is 2.

Equivalent set

Two finite sets A and B are said to be equivalent, if n(A) = n(B). Note: equal set are equivalent but equivalent sets need not to be equal.
e.g.: The sets A = {4, 5, 3, 2} and B = {1, 6, 8, 9} are equivalent, but are not equal.

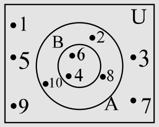
CHAPTER : 1 SETS AND REPRESENTATIONS - PART -II

Sets and Representations

Venn Diagram

A Venn diagram is an illustration of the relationships between and among sets, groups of objects that share something common. These diagrams consist of rectangle and closed curves usually circles

E.g.:



In the given venn diagram U={1,2,3,.......10} universe set of which A={2,4,6,8,10} and B={4,6} are subsets and also B⊂A

Universal Set

The set containing all objects of element and of which all other sets are subsets is known as **universal sets** and denoted by U.
e.g : For the set of all intergers, the universal set can be the set of rational numbers or the set R of real numbers

Algebra of sets

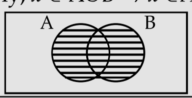
1. For any set A, we have
   (a) A∪A=A, (b) A∩A=A, (c) A∪ϕ=A, (d) A∩ϕ=ϕ, (e) A∪ U=U
   (f) A∩U=A, (g) A– ϕ=A, (h)A–A=ϕ
2. For any two sets A and B we have
   (a) A∪B = B∪A, (b) A∩B= B∩A, (c) A–B⊆ A, (d) B–A⊆B
3. For any three sets A,B and C, we have
   (a) A∪(B∪C)= (A∪B)∪C, (b) A∩(B∩C)= (A∩B)∩C
   (c) A∪(B∩C)=(A∪B)∩(A∪C), (d) A∩(B∪C)=(A∩B) ∪(A∩C)
   (e) A–(B∪C)=(A–B))∩(A–C), (f) A–(B∩C)=(A–B)∪(A–C)

Operations on Sets

Union

The union of two sets A and B, written as A∪B (read as A union B) is the set of all elements which are either in A or in B or in both. Thus, A∪B = {x : x ∈A or x ∈B}
clearly, x ∈ A∪B ⇒ x ∈A or x ∈ B and x ∉A∪B⇒x ∉A and x ∉B

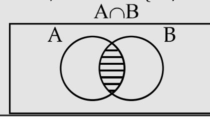
A∪B



eg: If A = {a, b, c, d} and B ={c, d, e,f } then A∪B = {a, b, c, d, e, f}

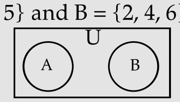
Intersection

The intersection of two sets A and B, written as A∩B (read as 'A' intersection 'B') is the set consisting of all the common elements of A and B.
Thus, A∩B = {x : x∈Aand x∈B}
Clearly, x∈A∩B⇒ {x∈A and x∈B} and x∉A∩B ⇒ {x ∉A or x∉B}.
e.g: If A={a,b,c,d} and B={c,d,e,f}
Then A∩B = {c,d}



Disjoint sets

Two sets A and B are said to be disjoint, if A∩B =ϕ i.e, A and B have no common element. e.g: if A = {1, 3, 5} and B = {2, 4, 6}
Then, A∩B =ϕ , so A and B are disjoint.

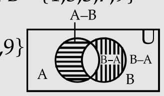


Symmetric Difference

The symmetric difference of two sets A and B, denoted by A Δ B, in defined as (A Δ B)=(A–B) ∪ (B – A)
e.g. If A = {1, 2,3,4,5}
and B = {1,3,5,7,9} then
(A Δ B)= (A–B) ∪ (B – A)
= {2, 4,} ∪ {7, 9}
= {2, 4, 7, 9}

Difference of two sets

If A and B are two sets, then their difference A-B is defined as:
A–B = {x : x ∈ A and x ∉B}
Similarly, B – A = {x : x ∈B and x ∉A}
e.g. If A={1,2,3,4,5,} and B={1,3,5,7,9}
then
A–B={2,4} and B–A={7,9}



Complement set

If U is a universal set and A is a subset of U, then complement of A is the set which contains those elements of U, which are not present in A and is denoted by A' or A^C. Thus,
A^c = {x : x∈U and x∉A}
e.g.: If U = {1, 2, 3, 4, …}
and A = {2, 4, 6, 8, ….}
then A^c = {1, 3, 5, 7, …}

Properties of complement

- Complement law:
  (i) A∪A' = U (ii) A∩A'= ϕ
- De morgan's Law:
  (i) (A∪B)' = A'∩B' (ii) (A∩B)' = A'∪B'
- Double Complement law:
  (A')' =A
- Law of empty set and universal set
  ϕ'=U and U' = ϕ

Subsets of a set of real numbers 'R'

- The set of natural numbers
  N={1, 2, 3, 4, 5, – – –}
- The set of integers
  Z={. . . –3, –2, –1, 0, 1, 2, 3,– – –}
- The set of irrational numbers,
  T = {x: x∈R and x∉Q}
- The set of rational number
  Q={x : $x = \frac{p}{q}$, p, q ∈Z and $q \neq 0$}

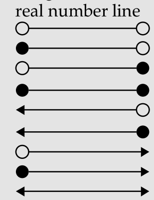
Relation among these subsets are
N ⊂ Z ⊂ Q, Q ⊂ R,T ⊂ R, N ⊄T

Interval Notation

Let a and b be real numbers with $a < b$

Set of Real Numbers	Interval Notation
• {x|a< x < b}	(a, b)
• {x|a ≤ x < b}	[a, b)
• {x|a< x ≤ b}	(a, b]
• {x|a ≤ x ≤ b}	[a, b]
• {x|x < b}	(–∞, b)
• {x|x ≤ b}	(–∞, b]
• {x|x >a}	(a, ∞)
• {x|x ≥a}	[a, ∞)
• R	(–∞, ∞)

Region on the real number line



CHAPTER

1 SETS AND REPRESENTATIONS

Chapter Objectives

Sets and their representations, Empty set, Finite and Infinite sets, Equal sets, Subsets of a set of real numbers especially intervals (with notations), Power set, Universal set, Venn diagrams, Union and Intersection of sets, Difference of sets, Complement of a set, Properties of Complement set, Algebraic properties of Union, Intersection and Complement of sets

STUDY MATERIAL

I. Concept Clarified

For more details, scan the code

1. Sets and their notation :

A set is a collection of well-defined and well distinguished objects of our perception or thought which are distinct from each other.

➢ **Notations**

The sets are usually denoted by capital letters *A*, *B*, *C*, etc. and the members or elements of the set are denoted by lowercase letters *a*, *b*, *c*, etc.

If x is a member of the set *A*, we write $x \in A$ (read as 'x belongs to *A*') and if x is not a member of the set *A*, we write $x \notin A$ (read as 'x does not belong to A'). If x and y both belong to *A*, we write $x, y \in A$.

➢ **Standard Notation**

$\mathbb{N}$: A set of natural number.

$\mathbb{W}$: A set of whole number.

$\mathbb{Z}$: A set of integers.

$\mathbb{Z}^+/\mathbb{Z}^-$: A set of all positive/negative Integers.

$\mathbb{Q}$: A set of rational number.

$\mathbb{Q}^+/\mathbb{Q}^-$: A set of all positive/negative rational number.

$\mathbb{R}$: A set of real number.

$\mathbb{R}^+/\mathbb{R}^-$: A set of all positive/negative real number.

$\mathbb{C}$: A set of complex number.

For more details, scan the code

➢ **Representation of a set**

Sets are represented in the following two ways:

(i) Roster form or Tabular form

(ii) Set Builder form or Rule Method

Roster Form or Tabular Form

In this method a set is described by listing elements, separated by commas and enclose then by curly brackets. For example, the set A of all odd natural numbers less than 10 in the Roster form is written as : A = {1, 3, 5, 7, 9}

Note

(i) In roster form, every element of the set is listed only once.

(ii) The order in which the elements are listed is immaterial. For example, each of the following sets denotes the same set {1, 2, 3}, {3, 2, 1}, {1, 3, 2}.

Set-Builder Form or Rule Method

In this case we write down a property or rule which gives us all the elements of the set by that rule.

For example, the set A of all prime numbers less than 10 in the set-builder form is written as A = {x | x is a prime number less that 10} The symbol '|' stands for the words 'such that'. Sometimes, we use the symbol':' in place of the symbol'|'.

➢ **Types of Sets**

1. **Null set or Empty set :** A set having no element is called an empty set or a null set or void set. It is denoted by ϕ or {} e.g. a set a = { x : x is an integer whose square root is negative natural number}
2. **Finite Set**: A set which has only finite number of elements is called a finite set.
   Example : $A = \{x : x^2 + 5x + 6 = 0\}$
3. **Infinite set:** A set which has an infinite number of elements is called an Infinite set.
   Example : $A = \{y = 2x : x \text{ and } y \in \mathbb{R}\}$ since x can be any real number so the set contains any real number. Since the number of elements in the set is not defined so the given set is an infinite set.
4. **Singleton set:** A set consisting of a single element is called a singleton set.
   Example : set A= {3}. Collection of SUN in solar system.
5. **Order of a finite set**: The number of elements in a finite set is called the order of the set A and is denoted by $O(A)$ or $n(A)$. It is also called cardinal number of the set.
   Example : The order of set $B = \{-2, -3\}$ is $O(B)$ or $n(B) = 2$.
6. **Equal sets :** Two sets A and B are said to be equal if every element of A is a member of B, and every element of B is a member of A. We write A = B, if sets A and B are equal and A ≠ B when A and B are not equal.
   Example : Consider set $A = \{x : x^2 + 5x + 6 = 0\}$ and set $B = \{-2, -3\}$. Since, A and B both have exactly same elements, hence set A is equal to set B.
7. **Equivalent set** : Two finite sets A and B are equivalent if their number of elements are same *i.e.* $n(A) = n(B)$.
   Example : Let set A contain the vowel in the English alphabet and set B is defined as $B = \{x^2 : 1 \le x \le 5, x \in \mathbb{N}\}$.
   Since, $n(A) = n(B) = 5$, so they are equivalent set.
   Note : Equal sets will always be equivalent but equivalent sets may not be equal sets.
8. **Subsets** : Let A and B be two sets. If every element of A is the element of B, then A is called a subset of B. If A is a subset of B, we write $A \subseteq B$.
9. **Proper subset :** If ***A*** is a subset of B and $A \ne B$ then A is a proper subset of ***B*** and we *write* $A \subset B$.
   In other words, if A is a proper subset of B, then all elements of A are in B but B contains at least one element that is not in A.
   Example : Let set $A = \{x^2 : 1 < x < 5, x \in \mathbb{N}\}$ and $B = \{x^2 : 2 \le x \le 4, x \in \mathbb{N}\}$, so $A \subseteq B$.
   Example : Let set $A = \{x^2 : 1 < x < 5, x \in \mathbb{N}\}$ and $B = \{x^2 : 1 < x < 10, x \in \mathbb{N}\}$, so $A \subset B$
   Note : Every set is a subset of itself i.e. $A \subseteq A$ for all A. Empty set is a subset of every set
   $N \subset W \subset Z \subset Q \subset R \subset C$
   The total number of subsets of a finite set containing n elements is 2^n.
   A set is a subset of itself but not a proper subset of itself.
   If A has n elements, then its power set $P(A)$ contains exactly 2^n elements.
10. **Power set** : Let A be any set. The set of all the subsets of A is called power set of A and is denoted by $P(A)$.
    Example : Let set A = {a, b, c}, then Power Set of A is given as $P(A)$ = { {}, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c}}
11. **Universal set** : A set consisting of all possible elements which occur in the fixed domain, is called a Universal set and is denoted by U**.** All sets are contained in the Universal set.
    Example : Set of Real numbers is a universal set for natural numbers, whole numbers and rational numbers**.**
12. **Comparable sets** : Two sets A and B are comparable, if $A \subseteq B$ or $B \subseteq A$.
    Example : Let two sets $A = \{x^2 : 1 < x < 5, x \in \mathbb{N}\}$ and $B = \{4, 9, 16\}$
    Since, $A \subseteq B$ or $B \subseteq A$ so we can compare two sets.

For more details, scan the code

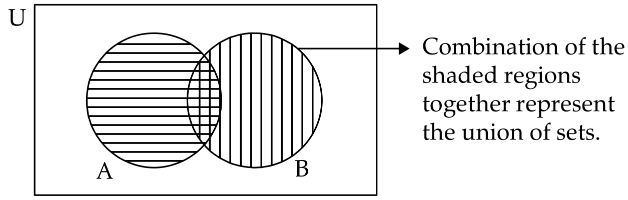
13. **Venn diagram** : The diagram drawn to represent sets are called venn – diagrams, where universal set U is represented by rectangle and its subsets represented by closed curves within the rectangle.

For more details, scan the code

➢ **Operations on Sets with Venn Diagram**

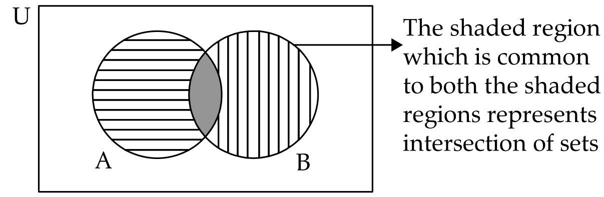
1. **Union of Two Sets :** The union of two sets A and B, written as $A \cup B$ (read as 'A union B'), is the set consisting of all the elements which are either in A or in B or in both.

Example : Let two sets $A = \{x^2 : 1 < x < 5, x \in \mathbb{N}\}$ and $B = \{1, 2, 3, 5\}$ then $A \cup B = \{1, 2, 3, 4, 5, 9, 16\}$



2. **Intersection of Two sets** : The intersection of two sets *A* and *B*, written as $A \cap B$ (read as 'A' intersection 'B') is the set consisting of all the common elements of *A* and *B*.

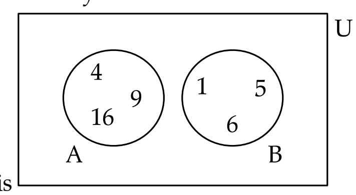
Example : Let two sets $A = \{x^2 : 1 < x < 5, x \in \mathbb{N}\}$ and $B = \{1, 2, 3, 4, 5\}$ then $A \cap B = \{4\}$.



3. **Disjoint Sets** : Two sets *A* and *B* are called disjoint, if $A \cap B = \phi$. They do not have any common element.

Example : Let two sets $A = \{x^2 : 1 < x < 5, x \in \mathbb{N}\}$ and $B = \{1, 5, 6\}$

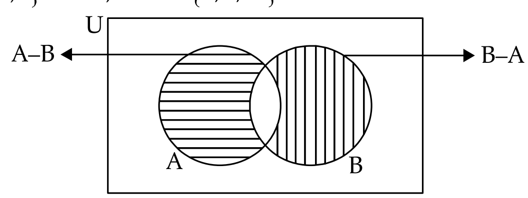
Since, there are no elements in common, therefore, $A \cap B = \phi$



4. **Difference of Two Sets** : If *A* and *B* are two sets, then their difference *A* – *B* is defined as $A - B = \{x : x \in A \text{ and } x \notin B\}$. Similarly, $B - A = \{x : x \in B \text{ and } x \notin A\}$.

Example : Let two sets $A = \{x^2 : 1 < x < 5, x \in \mathbb{N}\}$ and $B = \{1, 2, 3\}$ Then, $A - B = \{4, 9, 16\}$

5. **Complement of a set :** Let A is a subset of universal set U, then the complement of A with respect to U is the set of all those element of U which are not in A. It is denoted by A' or A^c or U – A. The union of a set *A* and its complement *A′* gives the universal set *U* of which *A* and *A′* are a subset, i.e., $A \cup A' = U$



Also, the intersection of a set *A* and its complement *A′* gives the empty set φ. *i.e.*, $A \cap A' = \varphi$

Law of Double Complementation : According to this law if we take the complement of the complemented set *A′* then, we get the set *A* itself. *i.e.*, $(A')' = A$

Law of empty set and universal set : According to this law the complement of the universal set gives the empty set and vice-versa *i.e.* $U' = \phi$ and $\phi' = U$.

➢ LAWS OF ALGEBRA OF SETS

For more details, scan the code

For three sets A, B and C

1. **Idempotent Law**
   - **(a)** $A \cup A = A$
   - **(b)** $A \cap A = A$
2. **Identity Law**
   - **(a)** $A \cup \phi = A$
   - **(b)** $A \cap U = A$
3. **Commutative Law**
   - **(a)** $A \cup B = B \cup A$
   - **(b)** $A \cap B = B \cap A$
4. **Associative Law**
   - **(a)** $(A \cup B) \cup C = A \cup (B \cup C)$
   - **(b)** $A \cap (B \cap C) = (A \cap B) \cap C$
5. **Distributive Law**
   - **(a)** $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
   - **(b)** $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
6. **De-Morgan's Law**
   - **(a)** $(A \cup B)' = A' \cap B'$
   - **(b)** $(A \cap B)' = A' \cup B'$
7. **Symmetric difference property**

   $A \Delta B = (A - B) \cup (B - A)$
8. **Results on operation on sets**
   - **(a)** $A - (B \cap C) = (A - B) \cup (A - C)$
   - **(b)** $A - (B \cup C) = (A - B) \cap (A - C)$
   - **(c)** $A - B = A \cap B'$
   - **(d)** $B - A = B \cap A'$
   - **(e)** $A - B = A \Leftrightarrow A \cap B = \phi$
   - **(f)** $(A - B) \cup B = A \cup B$
   - **(g)** $(A - B) \cap B = \phi$
   - **(h)** $A \cap B \subseteq A$ and $A \cap B \subseteq B$
   - **(i)** $A \cup (A \cap B) = A$
   - **(j)** $A \cap (A \cup B) = A$
9. **More on symmetric difference property**
   - **(a)** $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$

For more details, scan the code

(b) $A \cap (B - C) = (A \cap B) - (A \cap C)$

(c) $A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$

(d) $(A \cap B) \cup (A - B) = A$

(e) $A \cup (B - A) = A \cup B$

10. Universal Set property

(a) $U' = \phi$

(b) $\phi' = U$

(c) $(A')' = A$

(d) $A \cap A' = \phi$

(e) $A \cup A' = U$

(f) $A \subseteq B \Leftrightarrow B' \subseteq A'$

II. Important Formulae

For more details, scan the code

(1) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(2) $n(A \cup B) = n(A) + n(B) \Leftrightarrow$ A, B are disjoint non – void sets.

(3) $n(A - B) = n(A) - n(A \cap B)$

(4) $n(B - A) = n(B) - n(A \cap B)$

(5) $n(A \Delta B) = n(A) + n(B) - 2n(A \cap B)$

(6) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

(7) number of elements in exactly two of the sets A, B, C $= n(A \cap B) + n(B \cap C) + n(C \cap A) - 3n(A \cap B \cap C)$

(8) number of elements in exactly one of the sets A,B,C

$= n(A) + n(B) + n(C) - 2n(A \cap B) - 2n(B \cap C) - 2n(A \cap C) + 3n(A \cap B \cap C)$

(9) $n(A' \cup B') = n((A \cap B)') = n(U) - n(A \cap B)$

(10) $n(A' \cap B') = n((A \cup B)') = n(U) - n(A \cup B)$

CHAPTER - 2 : RELATIONS AND FUNCTIONS - PART- I

Relations and Functions

Composition of functions and Invertible functions

Composition of functions

The composition of functions $f : A \to B$ and $g : B \to C$ is denoted by gof, and is defined as $gof : A \to C$ given by $gof(x) = g(f(x))$ $\forall x \in A$. *e.g.* let $A = N$ and $f, g : N \to N$ such that $f(x) = x^2$ and $g(x) = x^3$ $\forall$ $x \in N$. Then $gof(2) = g(f(2)) = g(2^2) = 4^3 = 64$.

Invertible functions

A function $f : X \to Y$ is invertible, if $\exists$ a function $g : Y \to X$ such that $gof = I_X$ and $fog = I_Y$. Then, g is the inverse of f. If f is invertible, then it is both one-one and onto and vice-versa. *eg*. If $f(x) = x$ and $f : N \to N$, then f is invertible.

Theorem 1 : If $f : X \to Y$, $g : Y \to Z$ and $H : Z \to S$ are functions, then $ho(gof) = (hog)of$.

Theorem 2 : Let $f : X \to Y$ and $g : Y \to Z$ be two invertible functions, then gof is invertible with $(gof)^{-1} = f^{-1} o\, g^{-1}$.

Types of Relations

Reflexive relation

A relation $R : A \to A$ is reflexive if aRa $\forall$ $a \in A$

Symmetric relation

A relation $R : A \to A$ is symmetric if $aRb \Leftrightarrow bRa$ $\forall$ $a, b \in A$

Transitive relation

A relation $R : A \to A$ is transitive if aRb and $bRc \Rightarrow aRc$ $\forall$ $a,b,c \in A$.

Reflexive + Symmetric + Transitive →

Equivalence relation

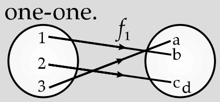
(If a relation has reflexive, symmetric and transitive relations) *e.g.*, Let T = the set of all triangles in a plane and $R : T \to T$ defined by $R = \{(T_1, T_2)\} : T_1$ is congruent to $T_2\}$. Then, R is equivalence.

Types of Functions

One-one (injective)

$f : X \to Y$ is one-one if $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$ $\forall$ $x_1, x_2 \in X$. Other wise, f is many- one,

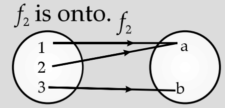
f_1 is one-one.



Onto (surjective)

$f : X \to Y$ is onto if every $y \in Y$, $\exists$ $x \in X$ such that $f(x) = y$, Then f is surjective

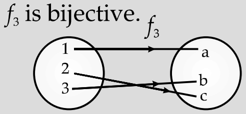
f_2 is onto.



Bijective

$f : X \to Y$ is both one-one and onto, then f is bijective.

f_3 is bijective.

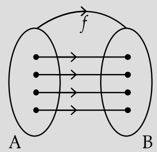


CHAPTER : 2 RELATIONS AND FUNCTIONS - PART- II

Relations and Functions

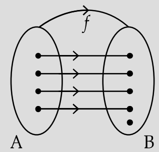
Kinds of Functions

One-One Onto function



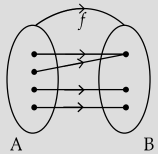
Range=Codomain

One-One Into function



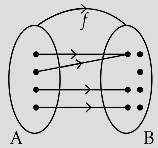
Range ⊂ Codomain

Many-One Onto function



Range=Codomain

Many One Into function

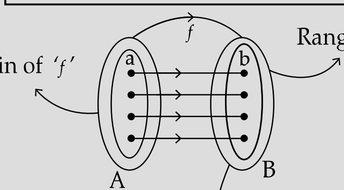


Range⊂Codomain

Functions

Definition: A relation 'f' from a set A to set B is said to be a function if every element of set A has one and only one image in set B.

Notations: Domain(Input) $\xrightarrow{x}$ $\boxed{f}$ ⟶ $y=f(x)$ Range(Output)



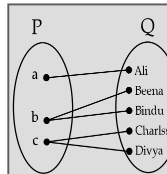
Cartesian product of sets

Given two non empty sets A&B. The cartesian product A×B is the set of all ordered pairs of elements from A&B i.e., A×B={(a,b) : a ∈ A ; b ∈ B}. If n(A)=p and n(B)=q , then n(A×B)=pq

Relation

Let A&B be two non- empty sets. Then any subset ' R ' of A x B is a relation from A to B.If (a,b) ∈R, then we write a R b, which is read as ' a is related to b ' by a relation R, 'b' is also called image of'a' under R. The total number of relations that can be defined from a set A to a set B is the number of possible subsets of A × B. If n(A)=p and n(B)=q, then n (A × B)=pq and total number of relations is 2^{pq}.

Pictorial representation of a relation



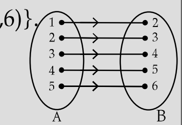
Given, R={(x,y): x is the first letter of the name y, x ∈P, y ∈Q} Then, R={(a, Ali), (b, Beena), (c, charu)}This is a visual or pictorial representation of relation R (called an arrow diagram)is shown in figure.

Domain & range of a Relation

If R is a relation from A to B, then the set of first elements in R is called domain & the set of second elements in R is called range of R. Symbolically, Domain of R={x:(x,y) ∈ R}; Range of R={y:(x,y) ∈ R} The set B is called co-domain of relation R.

Note: the range ⊆ Codomain.

e.g. Given, R={(1,2),(2,3),(3,4),(4,5),(5,6)}, then Domain of R={1,2,3,4,5} Range of R={2,3,4,5,6} and codomain of R={1,2,3,4,5,6}



Inverse relation

Let A&B be two sets and R be a relation from set A to set B.Then inverse of R ,denoted by R^{-1}, is a relation from B to A and is defined by R^{-1}={(b,a) : (a,b) ∈ R}. Note: (a,b) ∈R ⇔ (b,a) ∈ R^{-1} Also, Dom(R)=Range (R^{-1}) and Range(R)=Dom(R^{-1})

Even and odd function

Even function
$f(-x)=f(x)$, ∀x∈ Domain

Odd function
$f(-x)=-f(x)$, ∀x∈ Domain

Algebra of functions

Let $f : x \to$ R and g : $x \to$ R be any two real functions where $x \subset$ R.
Addition: $(f+g)\ x = f(x)+g(x)$; ∀x∈ R
Subtraction: $(f-g)\ x = f(x)-g(x)$; ∀ x∈ R
Product: $(fg)\ x = f(x).g(x)$; ∀ x ∈ R
Quotient: $(f/g)\ (x) = f(x)/g(x)$; provided $g(x) \neq 0$, ∀ x ∈ R

Some standard real functions

Log function: $f(x)=\log_a x$, a>0 , a ≠ 1 Domain=x ∈ (0, ∞) Range=y ∈ R

Identity function: The function f:R → R defined by y=$f(x)$=x ∀ x ∈R is called identity function. Domain=R and Range=R

Constant function: The function f:R → R defined by y=$f(x)$=c, ∀ x ∈ R, where c is a constant, is called constant function. Domain=R \ and Range ={c}

Modulus function: The function$f$:R → R defined by $f(x)=\begin{cases} x; & x \geq 0 \\ -x; & x < 0 \end{cases}$ is called modulus function. It is denoted by $y=f(x)=|x|$. Domain=R and Range =(0, ∞)

Signum function: The function f:R → R defined by $f(x)=\begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$ is called signum function. It is usually denoted by $y=f(x)$=sgn(x) Domain=R and Range ={ –1, 0, 1}

Greatest integer function: The function f:R → R defined by as the greatest integer less than or equal to x .It is usually denoted by y=$f(x)$=[x]. Domain=R and Range=Z(All integers)

Exponential function: $f(x)=a^x$, a>0 , a ≠ 1, Domain: x∈R: Range:f(x)∈(0,∞)

CHAPTER

2 RELATIONS AND FUNCTIONS

Chapter Objectives

Ordered pairs, Cartesian product of sets, Number of elements in the Cartesian product of two finite sets, Cartesian product of the set of real with itself (up to R x R x R), Definition of relation, pictorial diagrams, domain, co-domain and range of a relation, types of relation, binary operation, Types of relations : reflexive, symmetric, transitive and equivalence relations. One to one and onto functions, composite functions, inverse of a function. Binary operations.

Function as a special type of relation, Pictorial representation of a function, domain, co-domain and range of a function, real valued functions, domain and range of these functions, constant, identity, polynomial, rational, modulus, signum, exponential, logarithmic and greatest integer functions, with their graphs, composition of function, sum, difference, product and quotients of functions.

STUDY MATERIAL

I. Concept Clarified

1. Ordered Pair :

An ordered pair consists of two objects or elements in a given fixed order. If $a \in A$ and $b \in B$, then all pairs in the form (a, b) is called the ordered pair.

Example : Let $2^a = 7^b$. the solution of this set that is (0, 0) will be ordered pair where $a = 0$ and $b = 0$.

Equality of ordered pairs : Two ordered pairs (a_1, b_1) and (a_2, b_2) are equal, if $a_1 = a_2$ and $b_1 = b_2$.

2. Cartesian product of sets

For two sets A and B (non-empty sets), the set of all ordered pairs (a, b) such that $a \in A$ and $b \in B$ is called cartesian product of the sets A and B, denoted by $A \times B$.

For more details, scan the code

$\mathrm{A} \times \mathrm{B} = \{(a, b) : a \in \mathrm{A} \text{ and } b \in \mathrm{B}\}$

Example : Let Set $A = \left\{ f(x) f(4) - \frac{1}{2}\left(f\left(\frac{x}{4}\right) + f(4x) \right) \text{ where } f(x) = \cos\log x \right\}$ and

Set $B = \{b, c \; where \; f(x) = bx^2 + cx \; + d \; and \; f(x+1) - f(x) = 8x + 3\}$

The element Sets are,

$$\text{Set } A = \left\{ \begin{array}{l} f(x) f(4) - \frac{1}{2}\left(f\left(\frac{x}{4}\right) + f(4x) \right) \textit{ where } f(x) = \cos\log x \\ \Rightarrow \cos(\log x)\cos(\log 4) - \frac{1}{2}\left(\cos\log\frac{x}{4} + \cos\log 4x \right) \\ \Rightarrow \cos(\log x)\cos(\log 4) - \frac{1}{2}[2\cos(\log x)\cos(\log 4) \\ \Rightarrow 0 \end{array} \right\}$$

Set $A = \{0\}$

$$\text{Set } B = \left\{ \begin{array}{l} b, c \text{ where } f(x) = bx^2 + cx + d \text{ and } f(x+1) - f(x) = 8x + 3 \\ b = 4, c = -1 \end{array} \right\}$$

Set $B = \{4, -1\}$

Cartesian product of $A \times B$ is $\{(0,4)(0,-1)\}$ where $0 \in \text{Set } A$ and $4, -1 \in \text{Set } B$.

3. Number of elements in the Cartesian product of two finite sets :

$$n\ (\mathrm{A}\times\mathrm{B}) = n(\mathrm{A}).n(\mathrm{B})$$

4. Cartesian product of the set of real with itself (R x R x R)

The set $R \times R \times R$ represent the coordinates of all the points in three dimensional plane.

$$\text{Set } B = \begin{cases} b, c \text{ where } f(x) = bx^2 + cx + d \text{ and } f(x+1) - f(x) = 8x + 3 \\ b = 4, c = -1 \end{cases}$$

Set $B = \{4, -1\}$

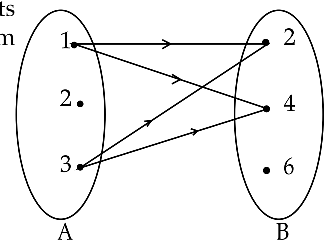
Then Cartesian product of $B \times B \times B$ will contain{(4, 4, 4), (4, 4, –1), (4, –1, 4), (–1, 4, 4), (4, –1, –1), (–1, 4, –1), (–1, –1, 4), (–1, –1, –1)}

These are also known as ordered triplet.

5. Relation

If A and B are two non-empty sets then any subset of A× B is called relation from A to B. Such relations between two non-empty sets is called binary relation and if R is a relation from a to b and $(a, b) \in$ R, then it is written *as a R b* and read as a is related to b.

Example : Consider a set A = $\{x : x^3 - 6x^2 + 11x - 6 = 0\}$ and set B contains elements as Set B = $\{x : x^3 - 12x^2 + 44x - 48 = 0\}$. Then the relation between Set A and B from A to B will be set of any combinations from set A to set B.



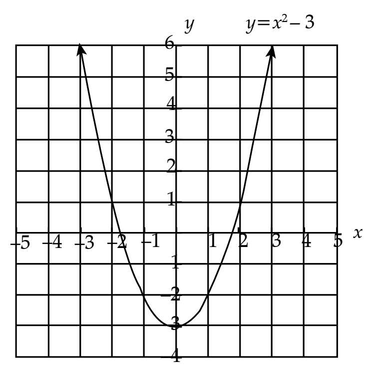
6. Domain And Range of A Relation :

Let R be a relation from a set A to set B. Then, set of all first components or x-coordinates of the ordered pairs belonging to R is called the domain of R, while the set of all second components or y-coordinates of the ordered pairs belonging to R is called the range of R.

Thus, domain of R = $\{a : (a, b) \in \mathrm{R}\}$ and range of R = $\{b : (a, b) \in \mathrm{R}\}$

Example $R = \{y = x^2 - 3, \text{where } x \in \mathrm{N} < 4\}$, then the domain of the function is $\{1, 2, 3\}$ correspondingly the range of the function is {–2, 1, 6}. The graph of the following relation is as follow.

For more details, scan the code



7. Types of Relation

(i) Void Relation :

As $\phi \subset \mathrm{A}\times\mathrm{A}$, for any set A, so ϕ is a relation on A, called the empty or void relation.

Example : The relation R on the set A = {1, 2, 3, 4} defined by R = {(a, b) : $a + b = 10$} since $a + b \neq 10$ for any two elements of set A. Therefore, $(a, b) \notin$ R for any $a, b \in$ A $\Rightarrow$ R does not contain any element A × A

$\Rightarrow$ R is an empty set.

$\Rightarrow$ R is the void relation on A.

(ii) Universal Relation :

Since, $\mathrm{A}\times\mathrm{A} \subseteq \mathrm{A}\times\mathrm{A}$, so $\mathrm{A}\times\mathrm{A}$ is a relation on A, called the universal relation.

Example : Relation on the set A = {1, 2, 3, 4, 5, 6} by R = $\{(a, b) \in \mathrm{R} : |a - b| \geq 0\}$ since $|a - b| \geq 0$ for all $a, b \in$ A

$\Rightarrow (a, b) \in$ R for all $(a, b) \in A \times A$.

$\Rightarrow$ each element of set A is related to every element of set A.

$\Rightarrow R = A \times A$.

$\Rightarrow$ R is a universal relation on set A.

(iii) Identity Relation :

The relation $I_A = \{(a, a) : a \in A\}$ is called the identity relation on A..

Example : suppose A={1, 2, 3}, then the set of ordered pairs {(1, 1), (2, 2), (3, 3)} is the identity relation on set 'A'.

(iv) Reflexive Relation :

A relation R is said to be reflexive relation, if every element of A is related to itself. Thus,

$(a, a) \in R, \forall a \in A \Rightarrow R$ is reflexive.

For more details, scan the code

Example : A relation R is defined on the set Z (set of all integers) by "a R b if and only if $2a + 3b$ is divisible by 5", for all $a, b \in Z$. So let say, $a \in Z$. Now $2a + 3a = 5a$, which is divisible by 5. Therefore a R a holds for all a in Z *i.e.* R is reflexive.

(v) Symmetric Relation :

A relation R is said to be symmetric relation, iff

$(a, b) \in R \Rightarrow (b, a) \in R, \forall a, b \in A$

For more details, scan the code

i.e. $a R b \Rightarrow b R a, \forall a, b \in A$

$\Rightarrow$ R is symmetric. The set A of natural numbers.

Example : A relation R is defined on the set Z by "a R b if $a - b$ is divisible by 5" for $a, b \in Z$. So let say $a, b \in Z$ and aRb hold. Then $a - b$ is divisible by 5 and therefore $b - a$ is divisible by 5.

Thus, $aRb \Rightarrow bRa$ and therefore R is symmetric.

(vi) Transitive Relation :

A relation R is said to be transitive relation, iff $(a, b) \in R$ and $(b, c) \in R$

$\Rightarrow (a, c) \in R, \forall a, b, c \in A$

For more details, scan the code

Example : in the set A of natural numbers if the relation R be defined by 'x less than y' then $a < b$ and $b < c$ imply $a < c$, that is, aRb and $bRc \Rightarrow aRc$. Hence this relation is transitive.

(vii) Equivalence Relation :

A relation R is said to be an equivalence relation, iff it is simultaneously reflexive , symmetric and transitive on A.

For more details, scan the code

Example : Let assume that R be a relation on the set R of real numbers defined by xRy if and only $x - y$ is an integer. Prove that R is an equivalence relation on R.

Reflexive : Consider x belongs to R, then $x - x = 0$ which is an integer. Therefore xRx.

Symmetric : Consider x and y belongs to R and xRy. Then $x - y$ is an integer. Thus, $y - x = -(x - y)$, $y - x$ is also an integer. Therefore yRx.

Transitive : Consider x and y belongs to R, xRy and yRz. Therefore $x - y$ and $y - z$ are integers. According to the transitive property, $(x - y) + (y - z) = x - z$ is also an integer. So that xRz.

Thus, R is an equivalence relation on R.

(viii) Partial Order Relation :

A relation R is said to be a partial order relation, iff it is simultaneously reflexive, anti symmetric and transitive on A.

For more details, scan the code

Example : $(4, 2) \in R$ and $(2, 1) \in R$, implies $(4, 1) \in R$. As the relation is reflexive, anti-symmetric and transitive. Hence, it is a partial order relation.

(ix) Equivalence classes of an equivalent relation.

Let R be equivalence relation in A $(\neq \phi)$. Let $a \in A$. Then, the equivalence class of a denoted by $[a]$ or

$\{\bar{a}\}$ is defined as the set of all those points of A which are related to a under the relation R.

For more details, scan the code

Example : $f(x)$ is the set of all integers, we can define the equivalence relation ~ by saying '$a \sim b$ if and only if $(a - b)$ is divisible by 9'. Then the equivalence class of 4 would include –32, –23, –14, –5, 4, 13, 22, and 31.

(x) Composition of relation :

For more details, scan the code

Let R and S be two relations from sets A to B and B to C respectively, then we can define relation composition of S over R or SoR from A to C such that $(a, c) \in \text{SoR} \Leftrightarrow \exists\, b \in \text{B}$ such that $(a, b) \in \text{R}$ and $(b, c) \in \text{S}$.

This relation S o R is called the composition of R and S.

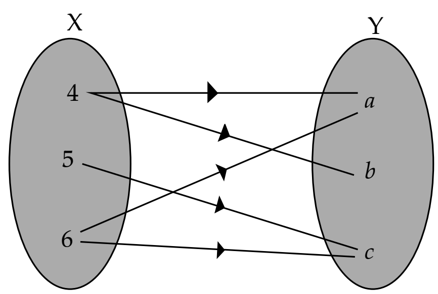
(a) $RoS \neq SoR$

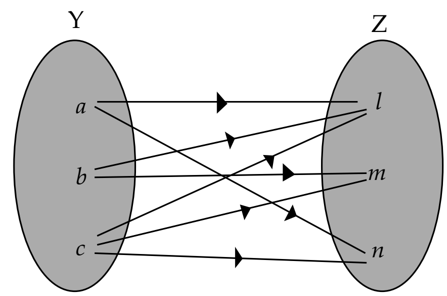
(b) $(SoR)^{-1} = R^{-1} o S^{-1}$ known as reversal rule.

Example : Let X = {4, 5, 6}, Y = {*a*, *b*, *c*} and Z = {*l*, *m*, *n*}. Consider the relation R_1 from X to Y and R_2 from Y to Z.

R_1 = {(4, *a*), (4, *b*), (5, *c*), (6, *a*), (6, *c*)}

R_2 = {(*a*, *l*), (*a*, *n*), (*b*, *l*), (*b*, *m*), (*c*, *l*), (*c*, *m*), (*c*, *n*)}





The composition of relation (i) R_2 o R_1

The composition relation R_2 o R_1 as shown in fig :

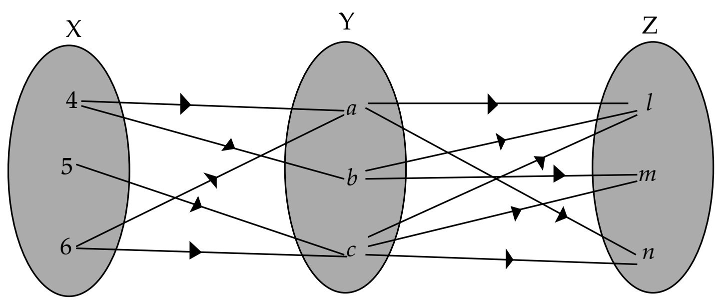


Fig : R_1 o R_2

R_2 o R_1 = {(4, *l*), (4, *n*), (4, *m*), (5, *l*), (5, *m*), (5, *n*), (6, *l*), (6, *m*), (6, *n*)}

(xi) Congruence Modulo *m* :

For more details, scan the code

Let *m* be an arbitrary but fixed integer. Two integers *a* and *b* are said to be congruence modulo *m*, if $a - b$ is divisible by *m* and we write $a \equiv b \pmod{m}$.

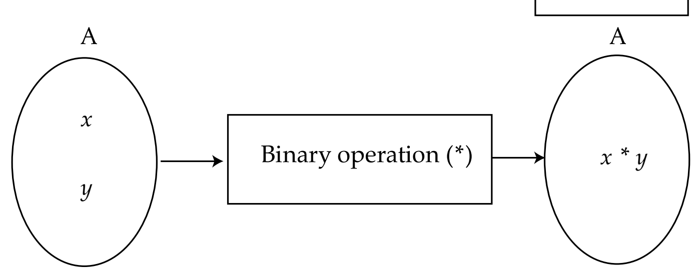
Example : $a \equiv b \pmod{m} \Leftrightarrow a - b$ is divisible by *m*. For example 26 ≡ 11(mod 5) ⇔ 26 – 11 is divisible by 5.

8. Binary Operation :

For more details, scan the code

Let S be a non-empty set. A function *f* from $\text{S} \times \text{S}$ to S is called a binary operation on S *i.e.* $f : \text{S} \times \text{S} \rightarrow \text{S}$ is a binary operation on set S.

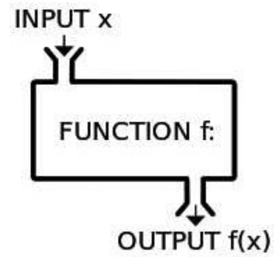
Example : Consider the binary operation ^ on the set {1, 2, 3, 4, 5} defined by *a* ^ *b* = min {*a*, *b*}.



b \ a	1	2	3	4	5
1	1	1	1	1	1
2	1	2	2	2	2
3	1	2	3	3	3
4	1	2	3	4	4
5	1	2	3	4	5

9. Function :

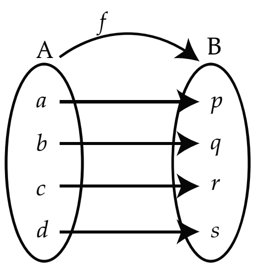
A function is like a machine which gives unique output for each input that is fed into it. But every machine is designed for certain defined inputs for e.g. a washing machine is designed for washing cloths and not the wood. Similarly the functions are defined for certain inputs which are called as its ***domain*** and corresponding outputs are called ***Range.***



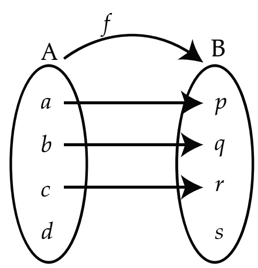
Let A and B be two sets and let there exist a rule or manner or correspondence 'f' which associates to each element of A to a unique element in B, then f is called a ***Function*** or ***Mapping*** from A to B. It is denoted by symbol

$$f:(\mathrm{A},\mathrm{B}) \text{ or } f:\mathrm{A}\to\mathrm{B} \text{ or } \mathrm{A}\xrightarrow{f}\mathrm{B}$$

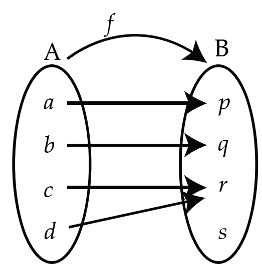
Which reads 'f is a function from A to B' or' f maps A to B.



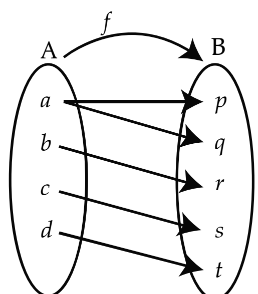
Function



Not a function



Function



Not a function

Note : Every function is a relation but every relation is not necessarily a function.

10. Domain and Range of function :

For more details, scan the code

For a relation from set A to set B i.e. a*Rb, **all the elements of set A are called as the domain of the relation R while all the elements of set B are called as the co-domain of the relation R.***

Range is the set of all second elements from the ordered pairs (a, b) in the relation $a\mathbf{R}b$.

Domain of $f = \{a \mid a \in \mathrm{A}, (a, f(a)) \in f\}$

Range of $f = \{f(a) \mid a \in \mathrm{A}, f(a) \in \mathrm{B}, (a, f(a)) \in f\}$

For the relation $a\mathrm{R}b$, domain is considered as the input to relation R while the co-domain is the possible outputs and range is the actual output.

Example : Domain (D) and Range (R) of $f(x)=\sin^{-1}(x)$ is $[-1, 1]$ and $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$ respectively.

11. Types of function :

(a) Polynomial Function : A function f is defined by

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + ... + ax + a_0$$

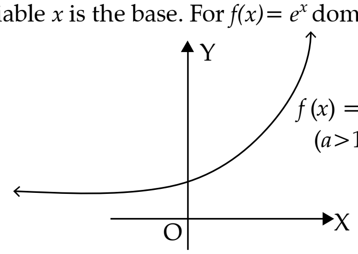
where n is a non negative integer and $a_n, a_{n-1}, \ldots, a, a_0$ are real number and $a_n \neq 0$, then f is called a polynomial function of degree n. A polynomial function is always continuous.

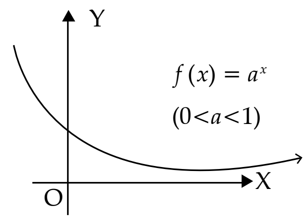
(b) Rational Function :

A function of type, $f(x) = \dfrac{g(x)}{h(x)}$, where $g(x)$ *and* $h(x)$ are polynomials functions and $h(x) \neq 0$, is called rational function.

(c) Exponential Function :

A function $f(x) = a^x$ where $(a > 0, a \neq 1, x \in R)$ is called an exponential function. $f(x) = a^x$ is called an exponential function because the variable x is the exponent. It should not be confused with power function $g(x) = x^2$ in which variable x is the base. For $f(x) = e^x$ domain is R and range is R^+.

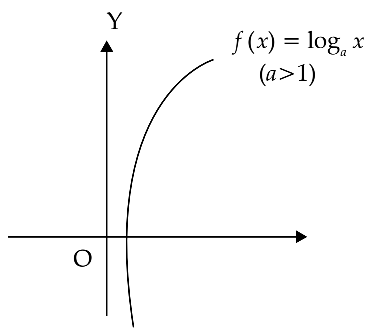


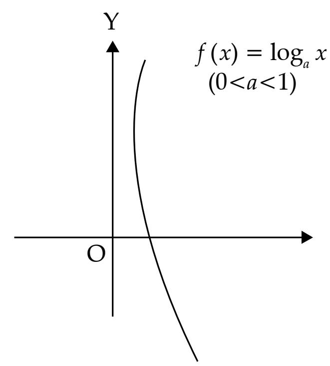


If $a > 1$ then f is increasing function.
If $0 < a < 1$ then f is strictly decreasing function.
If $a \neq 0$ then f is strictly monotonic function

(d) Logarithmic Function :

If a > 0 and a ≠ 1 then a function of the form $f(x) = \log_a x$, $x > 0$ is called general logarithmic function $D_f = (0, \infty)$.



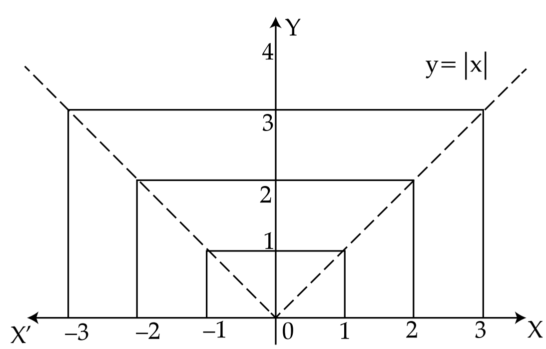


(e) Absolute Value Function (or Modulus Function) :

A function $y = f(x) = |x|$ is called the absolute value function or modulus function. It is defined as :

$$y = |x| = \begin{cases} x & if\ x \geq 0 \\ -x & if\ x < 0 \end{cases}$$

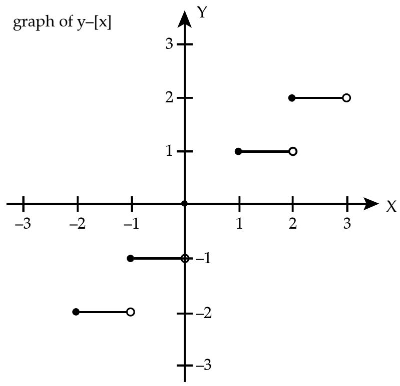
For $f(x) = |x|$, domain is R and Range is $\mathbb{R}^+ \cup \{0\}$. See below for its figure.



(f) Greatest Integer Or Step up Function :

The function $y = f(x) = [x]$ is called the greatest integer function where $[x]$ denotes the greatest integer less than or equal to x. Note that for :

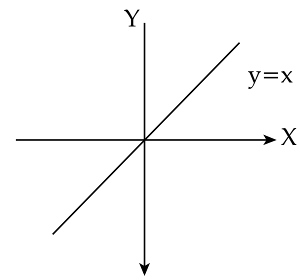
For more details, scan the code

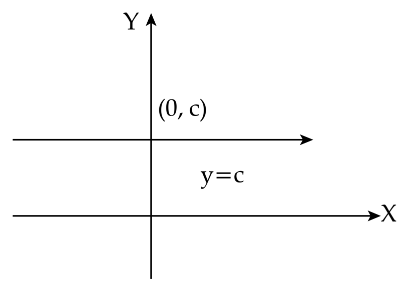


$-1 \le x < 0;\ [x] = -1$
$0 \le x < 1;\ [x] = 0$
$1 \le x < 2;\ [x] = 1$

(g) Identity Function :

The function $f : A \to B$ defined by $f(x) = x$, $\forall\ x \in A$ is called the identity of A and is denoted by I_A. The domain and range of identity function is entire real range *i.e.* R $f(x) = x$





(h) Constant Function :

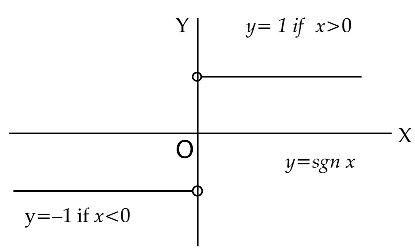
The function $f : A \to B$ is said to be a constant function if every element of A has the same f image in B. Thus $f : A \to B$; $f(x) = c$, $\forall\ x \in A$, $c \in B$ is a constant function. Note that the range of a constant function is a singleton and a constant function may be one–one or many – one, onto or into.

(i) Signum Function :

For more details, scan the code

A function $y = f(x) = sgn\ (x)$ is defined as follows $f(x) = \begin{cases} \dfrac{|x|}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$. It is also written as

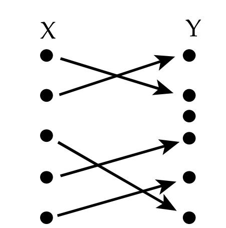
$$y = f(x) = \begin{cases} 1 \text{ if } & x > 0 \\ 0 \text{ if } & x = 0 \\ -1 \text{ if } & x < 0 \end{cases}$$



(j) One to One (or injection) Function :

A function f from X to Y is called one to one

if different elements of X have different images in Y.

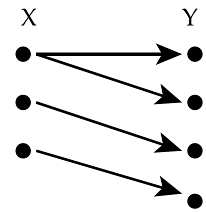


One to One function (injective)

For more details, scan the code

(k) Many to One Function :

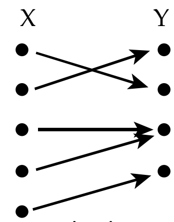
A function f from X to Y is called many to one *if* two or more elements of X have same image in Y. A function is many to one if it is not a one-one function.



For more details, scan the code

(l) Onto (or surjection) Function :

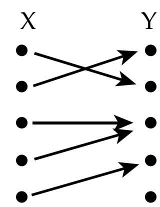
A function f from X to Y is called onto (or surjection) *if* each element of Y is the image of at least one element of X *i.e.*, co-domain of f = range of f.



surjective
(not injective)

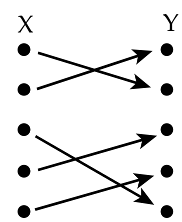
(m) Into Function :

A function f from X to Y is called into *if* there is at least one element in Y which is not the image of X *i.e., if* range of f is a proper subset of co-domain of f.
Also, a function f is said to be into *iff* it is not onto.



(n) One to One Correspondence (or bijection) :

A function f from X to Y is called one-one correspondence (or bijection) *if* f is both one-one and onto).



Bijective
(injective, surjective)

For more details, scan the code

12. Composition of Functions :

If $f : A \rightarrow B$ and $g : B \rightarrow C$ are two real valued functions then their composite or resultant of f and g *i.e.* gof is defined as, $gof(x) = g(f(x))$ for all x belonging to A.

For more details, scan the code

Given

$$f(x) = x^2 + 6$$

$$g(x) = 2x - 1$$

$$gof(x) = g(x^2 + 6)$$

$$= 2(x^2 + 6) - 1$$

$$= 2x^2 + 11$$

$$fog(x) = f(2x - 1)$$

$$= (2x - 1)^2 + 6$$

$$= 4x^2 - 4x + 1 + 6$$

$$= 4x^2 - 4x + 7$$

13. Sum, difference, product and quotients of functions :

- The sum of f & g, $f + g$ is defined as $(f + g)(x) = f(x) + g(x)$

  For example $f(x) = x^2 + 6$ and $g(x) = 2x - 1$

  then $f(x) + g(x) = x^2 + 6 + 2x - 1 = x^2 + 2x + 5$

- The difference of f & g, $f - g$ is defined as $(f - g)(x) = f(x) - g(x)$

  For example : $f(x) = x^2 + 6$ and $g(x) = 2x - 1$

  then $f(x) - g(x) = x^2 + 6 - (2x - 1) = x^2 + 6 - 2x + 1 = x^2 - 2x + 7$

- The product of f & g, $f * g$ is defined as $(f g)(x) = f(x) * g(x)$

  For example $f(x) = x^2 + 6$ and $g(x) = 2x - 1$

  then $f(x) \times g(x) = (x^2 + 6) \times (2x - 1) = 2x^3 - x^2 + 12x - 6$

- The quotient of f & g, f / g is defined as $(f / g)(x) = f(x) / g(x)$, where $g(x) \neq 0$

  For example $f(x) = x^2 + 6$ and $g(x) = 2x - 1$

  then $f(x) \div g(x) = \dfrac{(x^2 + 6)}{(2x - 1)}$

II. Sets of Concepts Clarified

Important point related to Cartesian product of Sets.

For three sets A, B and C

1. $A \times B = \phi$, if either A or B is an empty set.
2. $A \times (B \cup C) = (A \times B) \cup (A \times C)$
3. $A \times (B \cap C) = (A \times B) \cap (A \times C)$
4. $A \times (B - C) = (A \times B) - (A \times C)$
5. $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$
6. If $A \subseteq B$ and $C \subseteq D$, then $A \times C \subseteq B \times D$
7. If $A \subseteq B$, then $A \times A \subseteq (A \times B) \cap (B \times A)$
8. $A \times B = B \times A \Leftrightarrow A = B$
9. If either A or B is an infinite set, then $A \times B$ is an infinite set.
10. $A \times (B' \cup C')' = (A \times B) \cap (A \times C)$
11. $A \times (B' \cap C')' = (A \times B) \cup (A \times C)$
12. If A and B be any two non-empty sets having n elements in common, then $A \times B$ and $B \times A$ have n^2 elements in common.
13. If $A \neq B$, then $A \times B \neq B \times A$
14. If $A = B$, then $A \times B = B \times A$
15. If $A \subseteq B$, then $A \times C \subseteq B \times C$ for any set C.

Important Facts

1. If R and S are two equivalence relations on set A, then $R \cap S$ is also on equivalence relation on A.
2. The union of two equivalence relations on a set is not necessarily an equivalence relation on the set.
3. If R is an equivalence relation on a set A, then R^{-1} is also an equivalence relation on A.
4. Let A and B be two non-empty finite sets consisting of m and n elements, respectively. Then, $A \times B$ consists of mn ordered pairs. So, the total number of relations from A to B is 2^{mn}.

5. Let A be non-empty finite sets consisting of n elements, respectively. Then, $A \times A$ consists of n^2 ordered pairs. So, the total number of reflexive relations from A to A is 2^{n^2-n}.

6. Let A be non-empty finite sets consisting of n elements, respectively. Then, A × A consists of n^2 ordered pairs. So, the total number of total order relations from A to A is $n!$.

7. Let S be a finite set containing n elements

   (a) The total number of binary operations on S will be n^{n^2}.

   (b) Then the total number of commutative binary operation on S will be $n^{\frac{n(n+1)}{2}}$.

8. All the polynomials are algebraic but converse is not true. Functions which are not algebraic, are known as **Transcendental Function**.

9. If f *is* one-one and onto then f^{-1} exists with $D_f^{-1} = R_f$ and $R_f^{-1} = D_f$. Also, $f^{-1}of\ (x) = x\ \forall\ x \in D_f$ and $fof^{-1}(x) = x\ \forall\ x \in D_f^{-1} = R_f$. Further, f^{-1} is one one, so its inverse exists and $(f^{-1})^{-1} = f$.

●●

CHAPTER : 3 TRIGONOMETRY - PART-I

Trigonometric Functions

Important Trigonometric Ratios values

- $\sin 15°$ or $\sin\frac{\pi}{12} = \frac{\sqrt{3}-1}{2\sqrt{2}} = \cos 75°$ or $\cos\frac{5\pi}{12}$
- $\cos 15°$ or $\cos\frac{\pi}{12} = \frac{\sqrt{3}+1}{2\sqrt{2}} = \sin 75°$ or $\sin\frac{5\pi}{12}$
- $\tan 15° = \frac{\sqrt{3}-1}{\sqrt{3}+1} = 2-\sqrt{3} = \cot 75°$
- $\tan 75° = \frac{\sqrt{3}+1}{\sqrt{3}-1} = 2+\sqrt{3} = \cot 15°$
- $\sin\frac{\pi}{10}$ or $\sin 18° = \frac{\sqrt{5}-1}{4}$
- $\cos\frac{\pi}{5}$ or $\cos 36° = \frac{\sqrt{5}+1}{4}$

Standard General Solutions of Trigonometrical ratios

General solution: The solution consisting of all possible solutions of a trigonometric equation is called its General Solution.

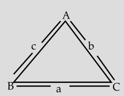
- $\sin\theta = 0 \Leftrightarrow \theta = n\pi$, $\cos\theta = 0 \Leftrightarrow \theta = (2n+1)\frac{\pi}{2}$
- $\tan\theta = 0 \Leftrightarrow \theta = n\pi$, $\sin\theta = \sin\alpha \Leftrightarrow \theta = n\pi + (-1)^n\alpha$, where $\alpha \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$
- $\cos\theta = \cos\alpha \Leftrightarrow \theta = 2n\pi \pm \alpha$, where $\alpha \in [0, \pi]$
- $\tan\theta = \tan\alpha \Leftrightarrow \theta = n\pi + \alpha$, where $\alpha \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$
- $\sin^2\theta = \sin^2\alpha \Leftrightarrow \theta = n\pi \pm \alpha$, $\cos^2\theta = \cos^2\alpha \Leftrightarrow n\pi \pm \alpha$
- $\tan^2\theta = \tan^2\alpha \Leftrightarrow \theta = n\pi \pm \alpha$, $\sin\theta = 1 \Leftrightarrow \theta = (4n+1)\frac{\pi}{2}$
- $\cos\theta = 1 \Leftrightarrow \theta = 2n\pi$, $\cos = -1 \Leftrightarrow \theta = (2n+1)\pi$
- $\sin\theta = \sin\alpha \Leftrightarrow \theta = n\pi + (-1)^n\alpha$

Solutions of Triangle

In, ΔABC,

(i) Sines Law

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



(ii) Projection formula

(a) $a\cos C + c\cos A = b$

(b) $b\cos A + a\cos B = c$

(c) $c\cos B + b\cos C = a$

(iii) Cosines Law

(a) $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

(b) $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

(c) $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

Area of Triangle

Area of triangle ABC

$\Delta = \frac{1}{2} bc \sin A = \frac{1}{2} ca \sin B = \frac{1}{2} ab \sin C$

$= \sqrt{s(s-a)(s-b)(s-c)}$; here, $s = \frac{a+b+c}{2}$

Trigonometrical Equations

The equation involving trigonometric functions of unknown angles are known as Trigonometric equations. e.g $\cos\theta = 0$, $\cos^2\theta - 4\cos\theta = 1$

A solution of trigonometric equation is the value of the unknown angle that satisfies the equation.

e.g. $\sin\theta = \frac{1}{\sqrt{2}}$

$\Rightarrow \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}, \ldots$

Factorisation of the Sum or Difference of sine and cosine with two variables

- $\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$
- $\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$
- $\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$
- $\cos C - \cos D = -2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

Transformation of Product into Sum or Difference of Sines or Cosines

- $2\sin A\cos B = \sin(A+B) + \sin(A-B)$
- $2\cos A\sin B = \sin(A+B) - \sin(A-B)$
- $2\cos A\cos B = \cos(A+B) + \cos(A-B)$
- $2\sin A\sin B = \cos(A-B) - \cos(A+B)$

Trigonometrical Function sum and Difference of Two Angles

- $\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$
- $\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$
- $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$
- $\cot(A \pm B) = \frac{\cot A\cot B \mp 1}{\cot A \pm \cot B}$
- $\sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A = \sin(A+B)\sin(A-B)$
- $\cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A = \cos(A+B)\cos(A-B)$

Degree & Radian Measures

If in a circle of radius r, an arc of length l subtends an angle of θ radians, then $l = r\theta$.

- Radian Measure $= \frac{180}{\pi} \times$ Degree Measure
- Degree Measure $= \frac{180}{\pi} \times$ Radian Measure

Trigonometric Identities

- $\sin^2\theta + \cos^2\theta = 1, -1 \le \sin\theta \le 1, -1 \le \cos\theta \le 1\ \forall\theta \in R$
- $\sec^2\theta - \tan^2\theta = 1, |\sec\theta| \ge 1, \forall\theta \in R$
- $\operatorname{cosec}^2\theta - \cot^2\theta = 1, |\operatorname{cosec}\theta| \ge 1, \theta \in R$

Multiple and Half Angles

- $\sin 2A = 2\sin A\cos A$
- $\cos 2A = \cos^2 A - \sin^2 A$
  $= 2\cos^2 A - 1$
  $= 1 - 2\sin^2 A$
- $\tan 2A = \frac{2\tan A}{1-\tan^2 A}$
- $\sin 2A = \frac{2\tan A}{1+\tan^2 A}$
- $\cos 2A = \frac{1-\tan^2 A}{1+\tan^2 A}$
- $\sin 3A = 3\sin A - 4\sin^3 A$
- $\cos 3A = 4\cos^3 A - 3\cos A$
- $\tan 3A = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$
- $\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$
- $\cos\theta = 2\cos^2\frac{\theta}{2} - 1 = 1 - 2\sin^2\frac{\theta}{2}$
- $\tan\theta = \left(2\tan\frac{\theta}{2}\right)/\left(1-\tan^2\frac{\theta}{2}\right)$

Trigonometrical Ratio of allied Angles

- $\sin(-\theta) = -\sin\theta$
- $\cos(-\theta) = \cos\theta$
- $\tan(-\theta) = -\tan\theta$
- $\cot(-\theta) = -\cot\theta$
- $\sec(-\theta) = \sec\theta$
- $\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$

- $\sin\theta\left(\frac{\pi}{2} \mp \theta\right) = \cos\theta$
- $\cos\left(\frac{\pi}{2} \mp \theta\right) = \pm\sin\theta$
- $\tan\left(\frac{\pi}{2} \mp \theta\right) = \pm\cot\theta$
- $\cot\left(\frac{\pi}{2} \mp \theta\right) = \pm\tan\theta$
- $\sec\theta\left(\frac{\pi}{2} \mp \theta\right) = \pm\operatorname{cosec}\theta$
- $\operatorname{cosec}\left(\frac{\pi}{2} \mp \theta\right) = \sec\theta$

- $\cos((\pi \mp \theta)) = -\cos\theta$
- $\sin(\pi \mp \theta) = \pm\sin\theta$
- $\tan(\pi \mp \theta) = \mp\tan\theta$
- $\cot(\pi \mp \theta) = \mp\cot\theta$
- $\sec(\pi \mp \theta) = -\sec\theta$
- $\operatorname{cosec}(\pi \mp \theta) = \pm\operatorname{cosec}\theta$

- $\sin(2\pi \mp \theta) = \mp\sin\theta$
- $\cos(2\pi \mp \theta) = \cos\theta$
- $\tan(2\pi \mp \theta) = \mp\tan\theta$
- $\cot(2\pi \mp \theta) = \mp\cot\theta$
- $\sec(2\pi \mp \theta) = \sec\theta$
- $\operatorname{cosec}(2\pi \mp \theta) = \mp\operatorname{cosec}\theta$

- $\sin\left(\frac{3\pi}{2} \mp \theta\right) = -\cos\theta$
- $\cos\left(\frac{3\pi}{2} \mp \theta\right) = \mp\sin\theta$
- $\tan\left(\frac{3\pi}{2} \mp \theta\right) = \pm\cot\theta$
- $\cot\left(\frac{3\pi}{2} \mp \theta\right) = \pm\tan\theta$
- $\sec\left(\frac{3\pi}{2} \mp \theta\right) = \mp\operatorname{cosec}\theta$
- $\operatorname{cosec}\left(\frac{3\pi}{2} \mp \theta\right) = \sec\theta$

CHAPTER - 3 TRIGONOMETRY - PART-II

Inverse Trigonometric Functions

Domain and range of inverse trigonometric functions

(i) $y = \sin^{-1}x$. Domain = [–1,1], Range = $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(ii) $y = \cos^{-1}x$. Domain = [–1,1] Range = $[0, \pi]$

(iii) $y = \text{cosec}^{-1}x$. Domain = R–(–1,1), Range = $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

(iv) $y = \sec^{-1}x$. Domain = R–(–1,1), Range = $[0, \pi] - \left\{\frac{\pi}{2}\right\}$

(v) $y = \tan^{-1}x$. Domain = R, Range = $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(vi) $y = \cot^{-1}x$. Domain = R, Range = $(0, \pi)$.

Trigonometric functions

(i) $\sin : R \to [-1,1]$

(ii) $\cos : R \to [-1,1]$

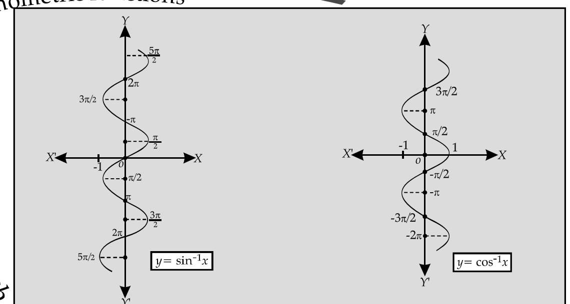
(iii) $\tan : R - \left\{x : x = (2n+1)\frac{\pi}{2}, n \in Z\right\} \to R$

(iv) $\cot : R - \{x : x = n\pi, n \in Z\} \to R$

(v) $\sec : R - \left\{x : x = (2n+1)\frac{\pi}{2}, n \in Z\right\} \to R - (-1,1)$

(vi) $\text{cosec} : R - \{x : x = n\pi, n \in Z\} \to R - (-1,1)$

Graphs of trigonometric functions and inverse trigonometric functions



Some important relations

(i) $y = \sin^{-1}x \Rightarrow x = \sin y$

(ii) $x = \sin y \Rightarrow y = \sin^{-1}x$

(iii) $\sin(\sin^{-1}x) = x$

(iv) $\sin^{-1}(\sin x) = x$

(v) $\sin^{-1}\frac{1}{x} = \text{cosec}^{-1}x$

(vi) $\cos^{-1}(-x) = \pi - \cos^{-1}x$

(vii) $\cos^{-1}\frac{1}{x} = \sec^{-1}x$

(viii) $\cot^{-1}(-x) = \pi - \cot^{-1}x$

(ix) $\tan^{-1}\frac{1}{x} = \cot^{-1}x$

(x) $\sec^{-1}(-x) = \pi - \sec^{-1}x$

(xi) $\sin^{-1}(-x) = -\sin^{-1}x$

(xii) $\tan^{-1}(-x) = -\tan^{-1}x$

(xiii) $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$

(xiv) $\text{cosec}^{-1}x + \sec^{-1}x = \frac{\pi}{2}, |x| \geq 1$

(xv) $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}, xy < 1$

(xvi) $2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}, -1 < x < 1$

(xvii) $\tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy}, xy > 1$

(xviii) $2\tan^{-1}x = \sin^{-1}\frac{2x}{1+x^2} = \cos^{-1}\frac{1-x^2}{1+x^2}$

$\sin^{-1}x \neq (\sin x)^{-1} = \frac{1}{\sin x}$ and same for other trigonometric functions.

Principal value branch and principal value

The range of an inverse trigonometric function is the principal value branch and those values which lies in the principal value branch is called the principal value of that inverse trigonometric functions.

$0 \leq \sin^{-1}x \leq \frac{\pi}{2}$	$-\frac{\pi}{2} \leq \sin^{-1}x < 0$
$0 \leq \cos^{-1}x \leq \frac{\pi}{2}$	$\frac{\pi}{2} < \cos^{-1}x \leq \pi$
$0 \leq \tan^{-1}x \leq \frac{\pi}{2}$	$-\frac{\pi}{2} \leq \tan^{-1}x < 0$
$0 \leq \cot^{-1}x \leq \frac{\pi}{2}$	$\frac{\pi}{2} < \cot^{-1}x < \pi$
$0 \leq \sec^{-1}x < \pi/2$	$\frac{\pi}{2} < \sec^{-1}x \leq \pi$
$0 < \text{cosec}^{-1}x \leq \frac{\pi}{2}$	$-\frac{\pi}{2} \leq \text{cosec}^{-1}x < 0$

CHAPTER

3 TRIGONOMETRY

Chapter Objectives

Trigonometric Functions

Positive and negative angles

Measuring angles in radians and in degrees and conversion from one measure to another

Definition of trigonometric functions with the help of unit circle

Truth of the identity $\sin^2 x + \cos^2 x = 1$, for all x

Signs of trigonometric functions

Domain and range of trigonometric functions and their graphs

Expressing $\sin(x \pm y)$ and $\cos(x \pm y)$ in terms of $\sin x$, $\sin y$, $\cos x$ & $\cos y$ and their simple applications

Deducing identities like the following :

- $\tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x.\tan y}$, $\cot(x \pm y) = \dfrac{\cot x.\cot y \mp 1}{\cot y \pm \cot x}$
- $\sin\alpha \pm \sin\beta = 2\sin\frac{1}{2}(\alpha \pm \beta).\cos\frac{1}{2}(\alpha \mp \beta)$
- $\cos\alpha + \cos\beta = 2\cos\frac{1}{2}(\alpha + \beta).\cos\frac{1}{2}(\alpha - \beta)$
- $\cos\alpha - \cos\beta = -2\sin\frac{1}{2}(\alpha + \beta).\sin\frac{1}{2}(\alpha - \beta)$
- Double angle, triple angle, half angle and one third angle formula as special cases.
- Identities related to $\sin 2x$, $\cos 2x$, $\tan 2x$, $\sin 3x$, $\cos 3x$ and $\tan 3x$
- Angle and Arc lengths
- Graphs of simple trigonometric functions for all trigonometric ratios
- Addition and subtraction formula : $\sin(A \pm B)$; $\cos(A \pm B)$; $\tan(A \pm B)$; $\tan(A + B + C)$ etc
- Sum and differences as products
- Product to sum or difference

Trigonometric equations

- General solution of trigonometric equations of the type $\sin y = \sin a$, $\cos y = \cos a$ and $\tan y = \tan a$

Properties of triangle

- Sine formula : $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$
- Cosine formula : $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$, *etc.*
- Area of triangle $= \frac{1}{2} bc \sin A$ *etc.*

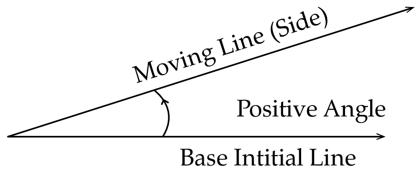
Inverse Trigonometric Functions.

- Definition, range, domain, principal value branch. Graphs of inverse trigonometric functions. Elementary properties of inverse trigonometric functions

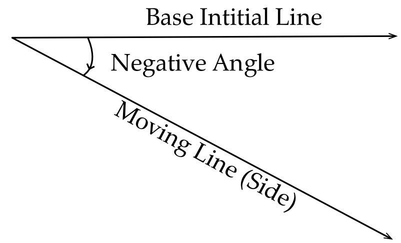
STUDY MATERIAL

I Concept Clarified :

1. Positive and negative angles :



Here, moving line moves in anti clockwise direction *w.r.t.* base line



Here, moving line moves in clockwise direction *w.r.t.* base line.

2. Measuring angles : An angle can be measured by three systems. Angles are measured in three systems

(i) English System or Sexagesimal system

In this system angle is measured in degrees, minutes and seconds.

One complete revolution = 360°

One fourth revolution = 1 right angle = 90°

1 degree = 60 minutes and 1 minute = 60 seconds or 1° = 60′ = 3600″

(ii) Circular System (Radian system)

In this system, angle is measured in radians. Radian is the angle subtended at the centre of a circle by an arc.

$$\text{Angle }(\theta) = \frac{\text{length of arc(l)}}{\text{radius of circle(r)}}$$

$$1\,\text{radian} = \frac{180}{\pi}\text{degree} \quad \left(\pi = \frac{22}{7}\right)$$

(iii) French System or Centesimal System :

In this system angle is measured in grades, minutes and seconds.

1 right angle = 100 grades = 100^g

$1^g = 100'$ and $1' = 100''$

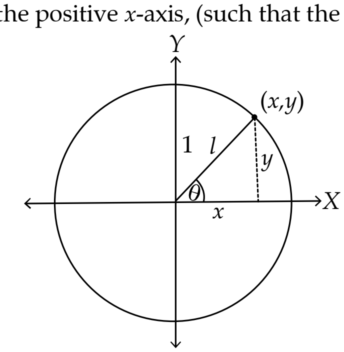
Note : The value of minutes and seconds in grade system is different from degree system.

Relation among Grade (G), Radian (R) and degree (D)

$$\frac{G}{200} = \frac{R}{\pi} = \frac{D}{180}$$

3. Definitions of Trigonometric Functions with the help of Unit Circle

In the unit circle, one can define the trigonometric functions cosine and sine as follows. If (x, y) is a point on the unit circle, and if the ray from the origin (0,0) to that point (x, y) makes an angle θ with the positive x-axis, (such that the counterclockwise direction is considered positive), then,



$$\cos\theta = \frac{x}{1} = x \text{ and } \sin\theta = \frac{y}{1} = y$$

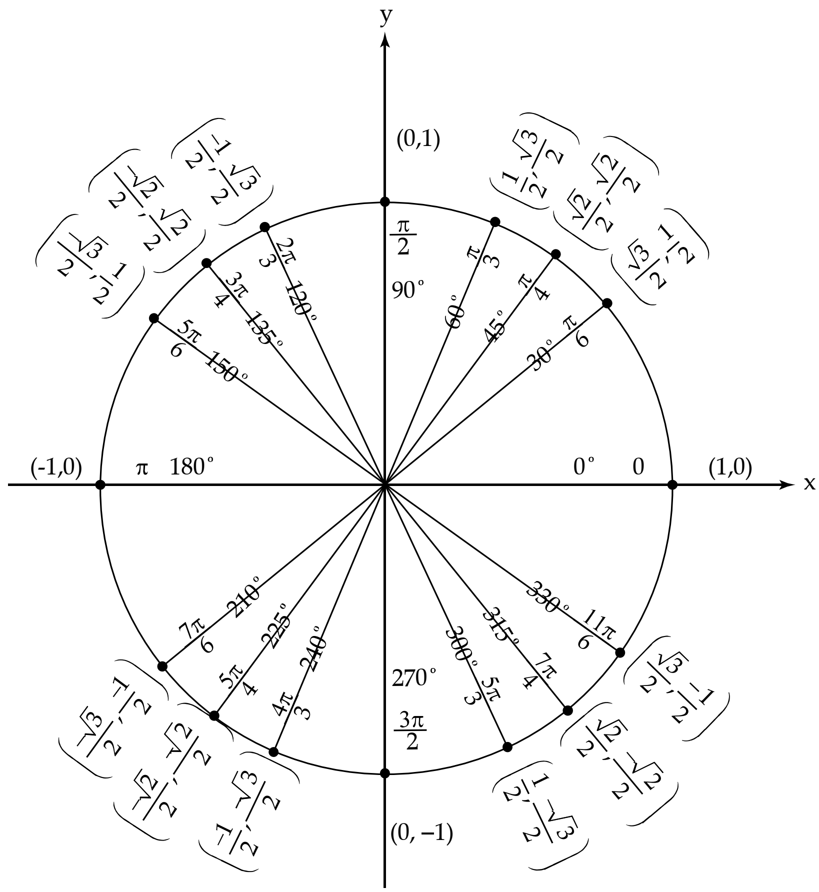
Then, each point (x, y) on the unit circle can be written as (cos θ, sin θ). Combined with the equation $x^2 + y^2 = 1$, the definitions above give the relationship $\sin^2\theta + \cos^2\theta = 1$. In addition, other trigonometric functions can be defined in terms of x and y :

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{y}{x} \qquad \sec\theta = \frac{1}{\cos\theta} = \frac{1}{x}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{x}{y} \qquad \text{cosec}\,\theta = \frac{1}{\sin\theta} = \frac{1}{y}$$

Below diagram shows a unit circle in the coordinate plane, together with some useful values of angle θ, and the points $(x, y) = (\cos\theta, \sin\theta)$. These values,

For more details, scan the code



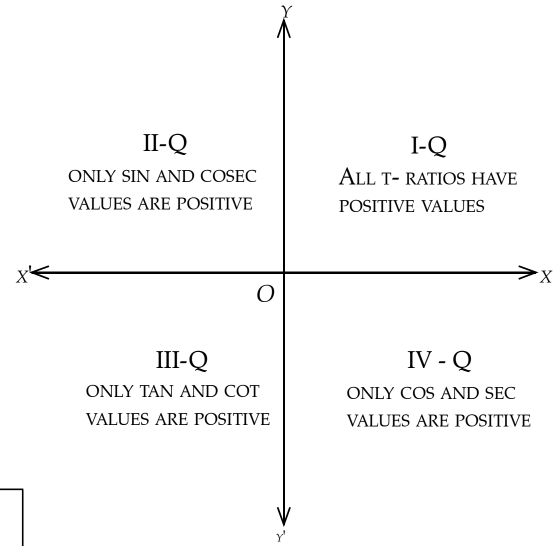
Frequently used angles and their points on the unit circle

The described values in unit circle can be tabulated as below.

Angles/ t-ratios	0°	$30° = \frac{\pi}{6}$	$45° = \frac{\pi}{4}$	$60° = \frac{\pi}{3}$	$90° = \frac{\pi}{2}$	$180° = \pi$	$270° = \frac{3\pi}{2}$	$360° = 2\pi$
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	–1	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	–1	0	1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	0	∞	0
cot	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	∞	0	∞
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞	–1	∞	1
cosec	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	∞	–1	∞

Trigonometric Identities :

$$\sin^2\theta + \cos^2\theta = 1$$
$$\sec^2\theta - \tan^2\theta = 1$$
$$\text{cosec}^2\theta - \cot^2\theta = 1$$



5. Signs of trigonometric ratios:

Quadrants	I	II	III	IV
sin	+ive	+ive	−ive	−ive
cos	+ive	−ive	−ive	+ive
tan	+ive	−ive	+ive	−ive
cot	+ive	−ive	+ive	−ive
sec	+ive	−ive	−ive	+ive
cosec	+ive	+ive	−ive	−ive

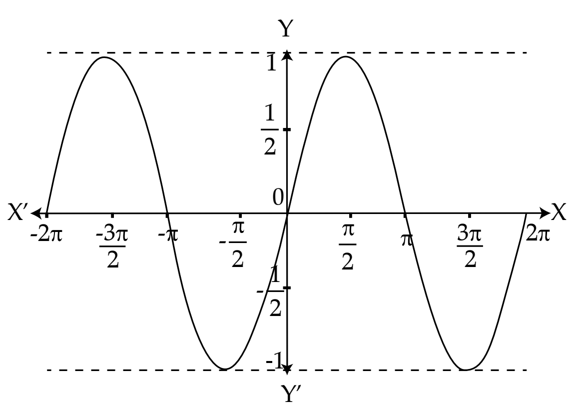
Note : In general, the angle value $(n \times 2\pi + \theta)$ lies in first quadrant, so all t-ratio's values will be positive e.g. $\sin(n \times 2\pi + \theta) = \sin\theta$. Here, n is the number of cycle in the rotation.

For complementary and supplementary angles.

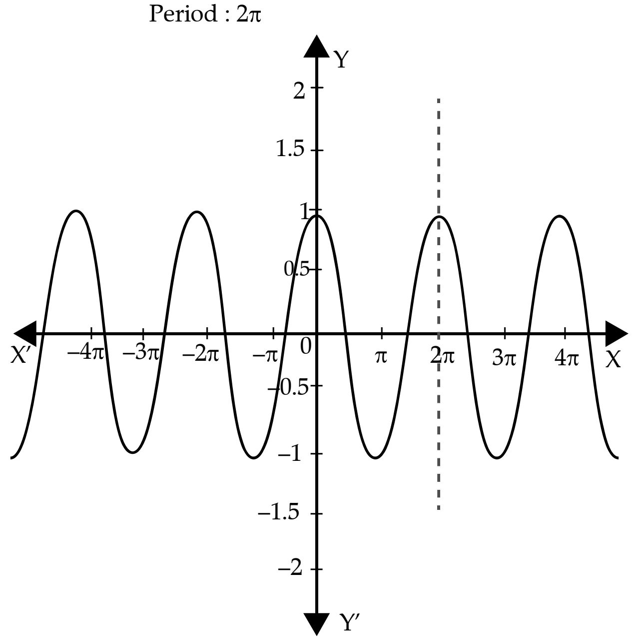
$\sin\left(\frac{\pi}{2} \pm \theta\right) = +\cos\theta$	$\sin(\pi \pm \theta) = \mp \sin\theta$
$\cos\left(\frac{\pi}{2} \pm \theta\right) = \mp\sin\theta$	$\cos(\pi \pm \theta) = -\cos\theta$
$\tan\left(\frac{\pi}{2} \pm \theta\right) = \mp\cot\theta$	$\tan(\pi \pm \theta) = \pm \tan\theta$
$\cot\left(\frac{\pi}{2} \pm \theta\right) = \mp\tan\theta$	$\cot(\pi \pm \theta) = \pm \cot\theta$
$\sec\left(\frac{\pi}{2} \pm \theta\right) = \mp\text{cosec}\,\theta$	$\sec(\pi \pm \theta) = -\sec\theta$
$\text{cosec}\left(\frac{\pi}{2} \pm \theta\right) = +\sec\theta$	$\text{cosec}(\pi \pm \theta) = \mp \text{cosec}\,\theta$

6. Domain and range of trigonometric functions and their graphs:

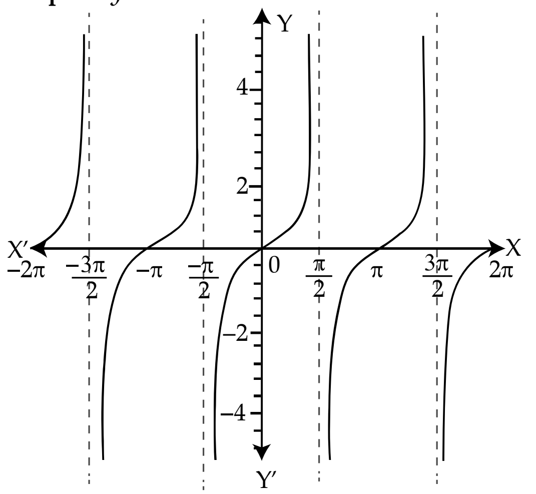
Graph of $y = \sin x$:
Domain : $-\infty \le x \le +\infty$ & Range : $-1 \le y \le 1$
Period : 2π



Graph of $y = \cos x$:
Domain : $-\infty \le x \le +\infty$ & Range: $-1 \le y \le 1$
Period : 2π



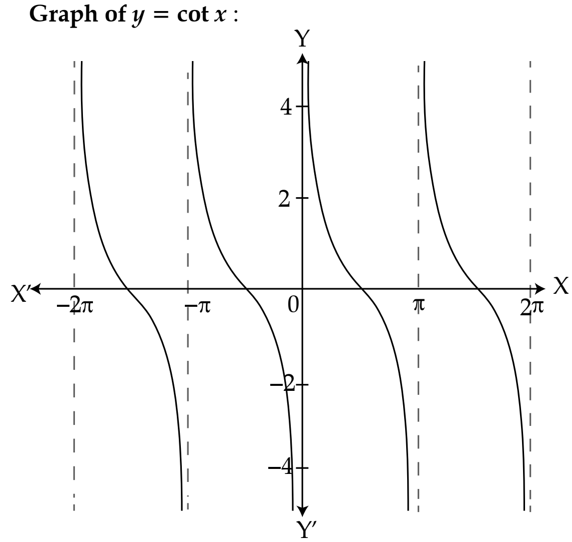
Graph of $y = \tan x$:



Domain: $\bigcup_{K\in Z}\left(\frac{(2K+1)}{2}\pi, \frac{(2K+3)\pi}{2}\right)$ &

Range: All Real Numbers, Period π

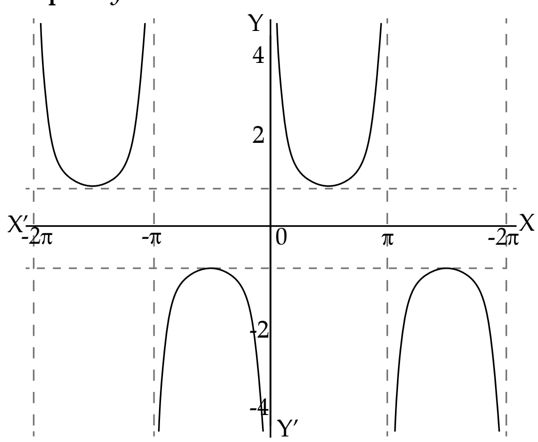
Graph of $y = \cot x$:



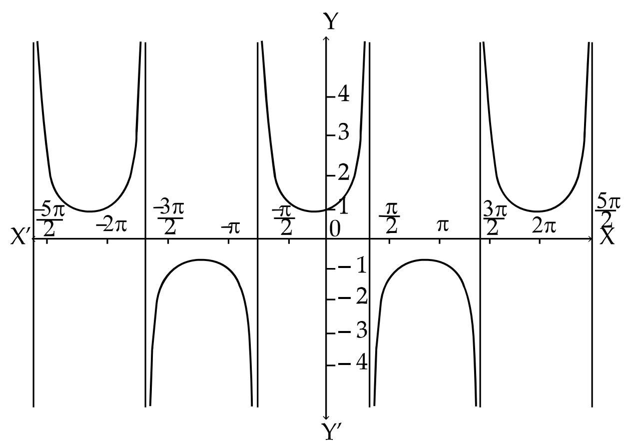
Domain: $\bigcup_{K\in Z}(K\pi, (K+1\pi)$ &

Range: All Real Numbers, Period π

Graph of $y = \operatorname{cosec} x$:



Graph of $y = \sec x$:



Domain: $\{x \mid x \neq \ldots -2\pi, -\pi, 0, \pi, 2\pi, \ldots\}$

& Range : $\{ y \mid y \leq -1 \text{ or } y \geq 1\}$, Period 2π

Domain: $\{ x \mid x \neq \ldots \frac{-3\pi}{2}, \frac{-\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2} \ldots\}$

& Range: $\{ y \mid y \leq -1 \text{ or } y \geq 1\}$, Period 2π

7. Expressing sin $(x \pm y)$ and cos $(x \pm y)$ in terms of sin x, sin y, cos x & cos y and Reduction of other Identities.

(a)

$$\sin(x \pm y) = \sin x.\cos y \pm \cos x.\sin y \quad .....(i)$$

$$\cos(x \pm y) = \cos x.\cos y \mp \sin x.\sin y \quad ......(ii)$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x.\tan y};\ \cot(x \pm y) = \frac{\cot x.\cot y \mp 1}{\cot y \pm \cot x}$$

$$\left[\tan(A+B+C) = \frac{\tan A + \tan B + \tan C - \tan A.\tan B.\tan C}{1 - \tan A.\tan B - \tan B.\tan C - \tan C.\tan A}\right]$$

(b) From equations (i) and (ii) in (a) part

$$2\sin x.\cos y = \sin(x+y) + \sin(x-y)$$
$$2\cos x.\sin y = \sin(x+y) - \sin(x-y)$$
$$2\cos x.\cos y = \cos(x+y) + \cos(x-y)$$
$$2\sin x.\sin y = \cos(x-y) - \cos(x+y)$$

(c) Let $x + y = \alpha$ and $x - y = \beta$ then from above result (b)

$$\sin\alpha \pm \sin\beta = 2\sin\frac{1}{2}(\alpha \pm \beta).\cos\frac{1}{2}(\alpha \mp \beta)$$

$$\cos\alpha + \cos\beta = 2\cos\frac{1}{2}(\alpha + \beta).\cos\frac{1}{2}(\alpha - \beta)$$

$\cos\alpha - \cos\beta = -2\sin\frac{1}{2}(\alpha+\beta).\sin\frac{1}{2}(\alpha-\beta)$

8. Identities related to sin 2x, cos 2x, tan 2x, sin 3x, cos 3x and tan 3x:

$\sin 2x = 2\sin x\cos x = \frac{2\tan x}{1+\tan^2 x}$

$\cos 2x = \cos^2 x - \sin^2 x, = 1 - 2\sin^2 x, = 2\cos^2 x - 1, = \frac{1-\tan^2 x}{1+\tan^2 x}$

$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$

$\sin 3x = 3\sin x - 4\sin^3 x$

$\cos 3x = 4\cos^3 x - 3\cos x$

$\tan 3x = \frac{3\tan x - \tan^3 x}{1-3\tan^2 x}$

Identities for half angle

- $\sin x = 2\sin\frac{x}{2}.\cos\frac{x}{2} = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$
- $\cos x = \cos^2\frac{x}{2} - \sin^2\frac{x}{2}, = 2\cos^2\frac{x}{2} - 1 = 1 - 2\sin^2\frac{x}{2} = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$
- $\tan x = \frac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}}$

Identities for one third angle

- $\sin x = 3\sin\frac{x}{3} - 4\sin^3\frac{x}{3}$
- $\cos x = 4\cos^3\frac{x}{3} - 3\cos\frac{x}{3}$
- $\tan x = \frac{3\tan\frac{x}{3} - \tan^3\frac{x}{3}}{1-3\tan^2\frac{x}{3}}$

Trigonometric Ratios for special angles

Angle/t-ratio	$7\frac{1}{2}^\circ$	15°	18°	$22\frac{1}{2}^\circ$	36°
sin θ	$\frac{\sqrt{2-\sqrt{2+\sqrt{3}}}}{2}$	$\frac{\sqrt{3}-1}{2\sqrt{2}}$	$\frac{\sqrt{5}-1}{4}$	$\frac{1}{2}\sqrt{2-\sqrt{2}}$	$\frac{1}{4}\sqrt{10-2\sqrt{5}}$
cos θ	$\frac{\sqrt{2+\sqrt{2+\sqrt{3}}}}{2}$	$\frac{\sqrt{3}+1}{2\sqrt{2}}$	$\frac{1}{4}\sqrt{10+2\sqrt{5}}$	$\frac{1}{2}\sqrt{2+\sqrt{2}}$	$\frac{\sqrt{5}+1}{4}$
tan θ	$(\sqrt{3}-\sqrt{2})(\sqrt{2}-1)$	$2-\sqrt{3}$	$\frac{\sqrt{5}-1}{\sqrt{10+2\sqrt{5}}}$	$\sqrt{2}-1$	$\frac{\sqrt{10-2\sqrt{5}}}{\sqrt{5}+1}$

9. General solution of trigonometric equations of the type sin y = sin a, cos y = cos a and tan y = tan a :

$\sin y = \sin a \Rightarrow y = n\pi + (-1)^n a$, where $n \in z$

$\cos y = \cos a \Rightarrow y = 2n\pi \pm a$, where $n \in z$

$\tan y = \tan a \Rightarrow y = n\pi + a$, where $n \in z$

$\sin y = 0 \Rightarrow y = n\pi$, where $n \in z$

$a\cos y + b\sin y = c \Rightarrow y = 2n\pi + \alpha \pm \beta$, where

$|c| \le \sqrt{a^2+b^2}$ $a, b, c \in R$ & $n \in z$

$\cos\beta = \frac{c}{\sqrt{a^2+b^2}}$; $\cos\alpha = \frac{a}{\sqrt{a^2+b^2}}$

and $\sin\alpha = \frac{b}{\sqrt{a^2+b^2}}$

$\cos y = 0 \Rightarrow y = (2n+1)\frac{\pi}{2}$, where $n \in z$

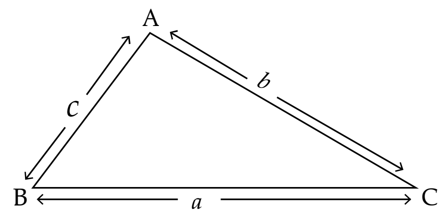
$\tan y = 0 \Rightarrow y = n\pi$, where $n \in z$

$\sin^2 y = \sin^2 a \Rightarrow y = n\pi \pm a$, where $n \in z$

$\cos^2 y = \cos^2 a \Rightarrow y = n\pi \pm a$, where $n \in z$

$\tan^2 y = \tan^2 a \Rightarrow y = n\pi \pm \alpha$, where $n \in z$, where $n \in z$

10. Properties of triangles :



(a) Laws of sines or sine formula

The sides of any triangle ABC are proportional to the sines of the angles opposite to them

i.e., $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

(b) Projection Formula: In any triangle ABC,

(i) $A = b\cos C + c\cos B$

(ii) $B = a\cos C + c\cos A$

(iii) $C = a\cos B + b\cos A$

(c) Laws of cosines or cosine formulae : In triangle ABC,

(i) $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

(ii) $\cos B = \dfrac{c^2 + a^2 - b^2}{2ac}$

(iii) $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

(d) Napier's Analogies (Laws of tangents) : In triangle ABC,

(i) $\tan\dfrac{B-C}{2} = \dfrac{b-c}{b+c}\cot\dfrac{A}{2}$

(ii) $\tan\dfrac{C-A}{2} = \dfrac{c-a}{c+a}\cot\dfrac{B}{2}$

(iii) $\tan\dfrac{A-B}{2} = \dfrac{a-b}{a+b}\cot\dfrac{C}{2}$

(e) Semi – Sum formulae or Half Angle Formulae: If s be the half perimeter of triangle ABC, *i.e.,* $2s = a + b + c$, then

(i) $\sin\dfrac{A}{2} = \sqrt{\dfrac{(s-b)(s-c)}{bc}}$, $\sin\dfrac{B}{2} = \sqrt{\dfrac{(s-a)(s-c)}{ac}}$, $\sin\dfrac{C}{2} = \sqrt{\dfrac{(s-a)(s-b)}{ab}}$

(ii) $\cos\dfrac{A}{2} = \sqrt{\dfrac{s(s-a)}{bc}}$, $\cos\dfrac{B}{2} = \sqrt{\dfrac{s(s-b)}{ac}}$, $\cos\dfrac{C}{2} = \sqrt{\dfrac{s(s-c)}{ab}}$

(iii) $\tan\dfrac{A}{2} = \sqrt{\dfrac{(s-b)(s-c)}{s(s-a)}}$, $\tan\dfrac{B}{2} = \sqrt{\dfrac{(s-a)(s-c)}{s(s-b)}}$, $\tan\dfrac{C}{2} = \sqrt{\dfrac{(s-a)(s-b)}{s(s-c)}}$, where $2s = a+b+c$

(f) Area of a Triangle : In a triangle ABC, its area of Δ is given by

$$\Delta = \frac{1}{2}bc\sin A = \frac{1}{2}ca\sin B = \frac{1}{2}ab\sin C$$

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

(g) If $A + B + C = 180°$, then

(i) $\sin 2A + \sin 2B + \sin 2C = 4\sin A\sin B\sin C$

(ii) $\cos 2A + \cos 2B + \cos 2C = -1 - 4\cos A\cos B\cos C$

(iii) $\sin A + \sin B + \sin C = 4\cos\dfrac{A}{2}\cos\dfrac{B}{2}\cos\dfrac{C}{2}$

(iv) $\cos A + \cos B + \cos C = 1 + 4\sin\dfrac{A}{2}\sin\dfrac{B}{2}\sin\dfrac{C}{2}$

(v) $\tan A + \tan B + \tan C = \tan A\tan B\tan C$

(vi) $\tan\dfrac{A}{2}\tan\dfrac{B}{2} + \tan\dfrac{B}{2}\tan\dfrac{C}{2} + \tan\dfrac{C}{2}\tan\dfrac{A}{2} = 1$

11. Periods of trigonometric functions :

A function is said to be periodic if there exists a constant real quantity p such that

$$f(x + p) = f(x)\ \forall\ x \in R$$

Each real number in the set $\{-2\pi, 2\pi, -4\pi, 4\pi, \ldots\}$ will be periodic for both $\sin x$ and $\cos x$ functions.

The functions $\sec x$ and $\operatorname{cosec} x$ are periodic and the period of either is 2π.

The functions $\tan x$ and $\cot x$ are periodic and the period of either is π.

For more details, scan the code

12. Inverse Trigonometric Functions :

A one-one onto function is called invertible function.

For the one-one onto function, $f : X \rightarrow Y$, f^{-1} is defined as

$f^{-1}: Y \rightarrow X: f^{-1}(y) = x \Leftrightarrow f(x) = y$, for each $y \in Y \exists x \in X$.

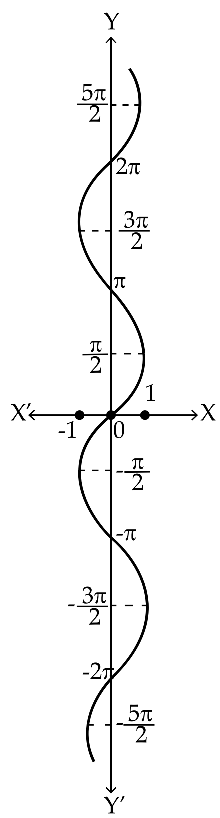
It is obvious,

Domain (f^{-1}) = Range (f) and Range (f^{-1}) = Domain (f)

Function	Domain	Range
1. $y = \sin^{-1} x$	$[-1, 1]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
2. $y = \cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
3. $y = \tan^{-1} x$	R	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
4. $y = \cot^{-1} x$	R	$(0, \pi)$
5. $y = \text{cosec}^{-1} x$	R – (–1, 1)	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$
6. $y = \sec^{-1} x$	R – (–1, 1)	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$

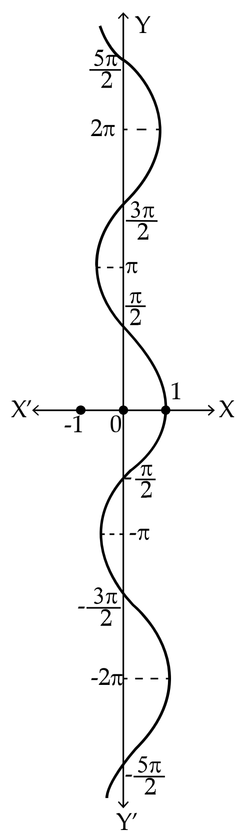
13. Graphs of inverse trigonometric functions :

Graph of $y = \sin^{-1}x$



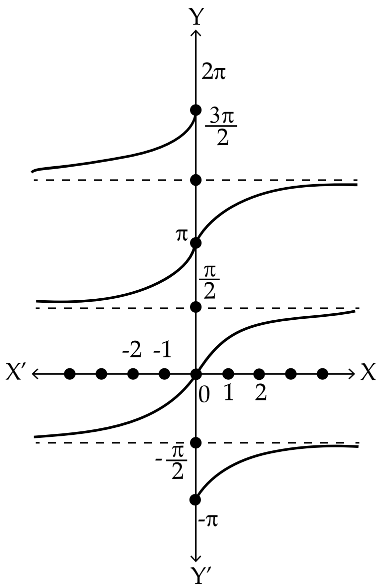
Domain : $\{x | -1 \le x \le 1\}$ & Range : $\left\{y \,|\, \frac{-\pi}{2} < y < \frac{\pi}{2}\right\}$

Graph of $y = \cos^{-1} x$



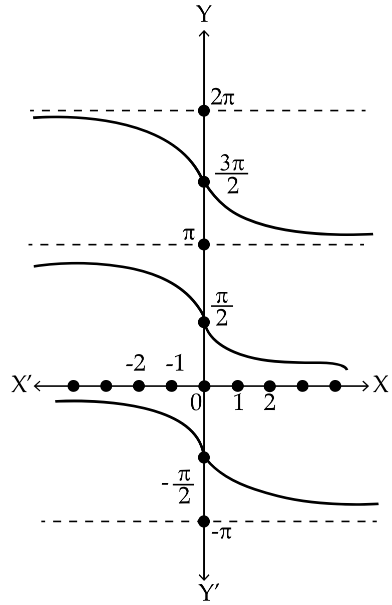
Domain : $\{x | -1 \le x \le 1\}$ & Range : $\{y | 0 \le y \le \pi\}$

Graph of $y = \tan^{-1} x$



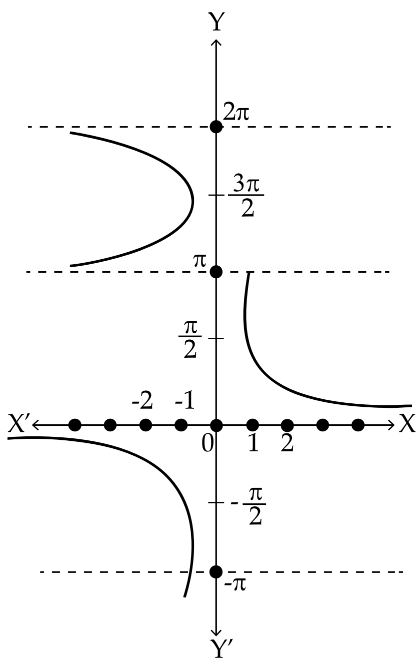
Domain : All real numbers $(-\infty \le x \le \infty)$ & Range : $\left\{y \mid \frac{-\pi}{2} < y < \frac{\pi}{2}\right\}$

Graph of $y = \cot^{-1} x$



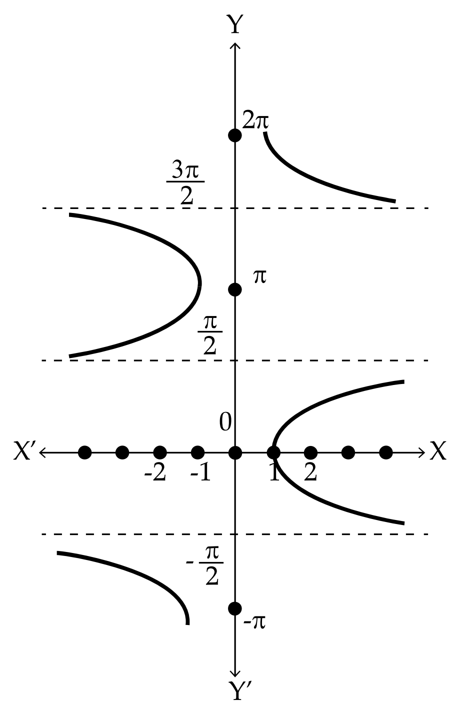
Domain : All real numbers & Range : $\{y \mid 0 < y < \pi\}$

Graph of $y = \operatorname{cosec}^{-1} x$



Domain : $\{x \mid x \le -1 \text{ or } x \ge 1\}$ & Range : $\left\{y \mid -\frac{\pi}{2} \le y \le \frac{\pi}{2}, y \ne 0\right\}$

Graph of $y = \sec^{-1} x$



Domain : $\{x \mid x \le -1 \text{ or } x \ge 1\}$ & Range : $\left\{y \mid 0 \le y \le \pi, y \ne \frac{\pi}{2}\right\}$

Principle values for inverse Trigonometric Functions

Principle values for $x \geq 0$	Principle values for $x < 0$
$0 \leq \sin^{-1}x \leq \frac{\pi}{2}$	$-\frac{\pi}{2} \leq \sin^{-1}x < 0$
$0 \leq \cos^{-1}x \leq \frac{\pi}{2}$	$\frac{\pi}{2} < \cos^{-1}x \leq \pi$
$0 \leq \tan^{-1}x < \frac{\pi}{2}$	$-\frac{\pi}{2} < \tan^{-1}x < 0$
$0 < \cot^{-1}x \leq \frac{\pi}{2}$	$\frac{\pi}{2} < \cot^{-1}x < \pi$
$0 \leq \sec^{-1}x < \frac{\pi}{2}$	$\frac{\pi}{2} < \sec^{-1}x \leq \pi$
$0 < \text{cosec}^{-1}x \leq \frac{\pi}{2}$	$-\frac{\pi}{2} \leq \text{cosec}^{-1}x < 0$

Relations Among Inverse trigonometric Functions.

(a) $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$ for all $x \in [-1, 1]$

$\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$ for all $x \in R$

$\sec^{-1}x + \text{cosec}^{-1}x = \frac{\pi}{2}$ for all $x \in (-\infty\ -1] \cup [1, \infty)$

(b) $\text{cosec}^{-1}x = \sin^{-1}\left(\frac{1}{x}\right)$ *for all* $x \in (-\infty, -1] \cup [1, \infty)$

$\sec^{-1}x = \cos^{-1}\left(\frac{1}{x}\right)$ for all $x \in (-\infty, -1] \cup [1, \infty)$

$\cot^{-1}x = \tan^{-1}\left(\frac{1}{x}\right)$ for $x > 0$,

$\tan^{-1}\left(\frac{1}{x}\right) = -\pi + \cot^{-1}x,$ *for* $x < 0$

(c) $\sin^{-1}(-x) = -\sin^{-1}x$ for all $x \in [-1, 1]$

$\cos^{-1}(-x) = \pi - \cos^{-1}x$ for all $x \in [-1, 1]$

$\tan^{-1}(-x) = -\tan^{-1}x$ for all $x \in R$ $\text{cosec}^{-1}(-x) = -\text{cosec}^{-1}x$

$\cot^{-1}(-x) = \pi - \cot^{-1}x$ $\sec^{-1}(-x) = \pi - \sec^{-1}x$

(d) $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right);\ xy < 1$

$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right);\ xy > -1$

$\tan^{-1}x + \tan^{-1}y = \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right);\ xy > 1,\ x, y > 0$

$\tan^{-1}x + \tan^{-1}\left(\frac{2x}{1-x^2}\right) = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right);\ |x| < \frac{1}{\sqrt{3}}$

(e) $2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right),\ |x| \leq 1$

$= \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right),\ x \geq 0$

$= \tan^{-1}\left(\frac{2x}{1-x^2}\right),\ -1 < x < 1$

(f) $\sin^{-1}(2x\sqrt{1-x^2}) = 2\sin^{-1}x,\quad -\frac{1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}}$

$\cos^{-1}(2x^2-1) = 2\cos^{-1}x,\quad$ if $0 \leq x \leq 1$

(g)
$\sin(\sin^{-1}x) = x,\quad$ if $x \in [-1,1]$

$\cos(\cos^{-1}x) = x,\quad$ if $x \in [-1,1]$

$\tan(\tan^{-1}x) = x,\quad$ if $x \in R$

$\cot(\cot^{-1}x) = x,\quad$ if $x \in R$

$\sec(\sec^{-1}x) = x,\quad x \in [-\infty,-1| \cup |1,\infty]$

$\text{cosec}(\text{cosec}^{-1}x) = x,\quad$ if $x \in [-\infty,-1| \cup |1,\infty]$

(h)
$\sin^{-1}(\sin\theta) = \theta,\quad$ if $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\cos^{-1}(\cos\theta) = \theta,\quad$ if $\theta \in [0,\pi]$

$\tan^{-1}(\tan\theta) = \theta,\quad$ if $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\cot^{-1}(\cot\theta) = \theta,\quad$ if $\theta \in (0,\pi)$

$\sec^{-1}(\sec\theta) = \theta,\quad$ if $\theta \in [0,\pi], \theta \neq \frac{\pi}{2}$

$\text{cosec}^{-1}(\text{cosec}\theta) = \theta,\quad$ if $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right], \theta \neq 0$

(i) $\sin^{-1}x + \sin^{-1}y = \sin^{-1}\left\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\right\}$,

if $-1 \leq x, y \leq 1$ and $x^2 + y^2 \leq 1$

or, if $xy < 0$ and $x^2 + y^2 > 1$

For more details, scan the code

$\sin^{-1}x - \sin^{-1}y = \sin^{-1}\left\{x\sqrt{1-y^2} - y\sqrt{1-x^2}\right\}$,

if $-1 \le x, y \le 1$ and $x^2 + y^2 \le 1$

or, $xy > 0$ and $x^2 + y^2 > 1$

$\cos^{-1}x + \cos^{-1}y = \cos^{-1}\left\{xy - \sqrt{1-x^2}\sqrt{1-y^2}\right\}$,

if $-1 \le x, y \le 1$ and $x + y \ge 0$

$\cos^{-1}x - \cos^{-1}y = \cos^{-1}\left\{xy + \sqrt{1-x^2}\sqrt{1-y^2}\right\}$,

if $-1 \le x, y \le 1$ and $x \le y$.

II. Important Formulae

- Relation among G, R and D

  $\frac{G}{200} = \frac{R}{\pi} = \frac{D}{180}$

- Angle = $\frac{\text{length of arc}}{\text{radius of the circle}}$, *i.e.*, $\theta = \frac{l}{r}$
- Domain & Range of $y = \sin x$: $-\infty \le x \le +\infty$ & $-1 \le y \le 1$
- Domain & Range of $y = \cos x$: $-\infty \le x \le +\infty$ & $-1 \le y \le 1$
- Domain & Range of $y = \tan x$: $\underset{K\in Z}{U}\left(\frac{(2K+1)}{2}\pi, \frac{(2K+3)}{2}\pi\right)$ & R
- Domain & Range of $y = \cot x$: $\underset{K\in Z}{U}\left(K\pi, (K+1)\pi\right)$ & R
- Domain & Range of cosec $\{x \mid x \ne \ldots -2\pi, -\pi, 0, \pi, 2\pi, \ldots\}$ $\{y \mid y \le -1 \text{ or } y \ge 1\}$
- Domain & Range of $y = \sec x$: $\{x \mid x \ne \ldots \frac{-3\pi}{2}, \frac{-\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$ $< y \mid y \le -1\ y \ge 2\}$

- **Sum and Difference as products**

- $\sin\alpha \pm \sin\beta = 2\sin\frac{1}{2}(\alpha \pm \beta).\cos\frac{1}{2}(\alpha \mp \beta)$
- $\cos\alpha + \cos\beta = 2\cos\frac{1}{2}(\alpha + \beta).\cos\frac{1}{2}(\alpha - \beta)$
- $\cos\alpha - \cos\beta = -2\sin\frac{1}{2}(\alpha + \beta).\sin\frac{1}{2}(\alpha - \beta)$

- **Product to the sum or difference**

- $2\sin x.\cos y = \sin(x + y) + \sin(x - y)$
- $2\cos x.\sin y = \sin(x + y) - \sin(x - y)$
- $2\cos x.\cos y = \cos(x + y) + \cos(x - y)$
- $2\sin x.\sin y = \cos(x - y) - \cos(x + y)$

t – Ratios for Compound Angles

- sin (A+B+C) = cos A cos B sin C + cos A sin B cos C + sin A cos B cos C – sin A sin B. sin C
- cos (A+B+C) = cos A cos B cos C – sin A sin B cos C – cos A sin B sin C – sin A cos B sin C
- $\tan(A+B+C) = \frac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan C \tan A}$

For general form

- $\sin(A_1 + A_2 + A_3 + \ldots\ldots A_n) = (\cos A_1 \cos A_2 \cos A_3 \ldots\ldots \cos A_n) \times (S_1 - S_3 + S_5 - S_7 + \ldots\ldots)$
- $\cos(A_1 + A_2 + A_3 + \ldots\ldots A_n) = (\cos A_1 \cos A_2 \cos A_3 \ldots\ldots \cos A_n) \times (1 - S_2 + S_4 - S_6 + \ldots\ldots)$
- $\tan(A_1 + A_2 + A_3 + \ldots\ldots A_n) = \frac{(S_1 - S_3 + S_5 - S_7 + \ldots\ldots)}{(1 - S_2 + S_4 - S_6 + \ldots\ldots)}$

Here, $S_1 = \tan A_1 + \tan A_2 + \tan A_3 + ... \tan A_n$

$S_2 = \tan A_1 \tan A_2 + \tan A_2 \tan A_3 + \tan A_3 \tan A_4 + ... \tan A_{n-1} \tan A_n$

$S_3 = \tan A_1 \tan A_2 \tan A_3 + \tan A_2 \tan A_3 \tan A_4 + ...$

If All angles are equal to A, then

$S_1 = n \tan A,\ S_2 = {}^nC_2 \tan^2 A,\ S_3 = {}^nC_3 \tan^3 A\ ......\ S_n = {}^nC_n \tan^n A$

Product of t – ratios

➢ $\cos A \cos 2A \cos 4A ...... \cos 2^{n-1} A = \dfrac{1}{2^n \sin A} \sin(2^n A)$

Sum of t – ratios

➢ $\sin A + \sin(A+B) + \sin(A+2B) + ... \sin(A+\overline{n-1}B) = \dfrac{\sin\left[A + (n-1)\dfrac{B}{2}\right] \sin\dfrac{nB}{2}}{\sin\dfrac{B}{2}}$

➢ $\cos A + \cos(A+B) + \cos(A+2B) + ... \cos(A+\overline{n-1}B) = \dfrac{\sin\dfrac{nB}{2}}{\sin\dfrac{B}{2}} \times \cos\left[A + \dfrac{n-1}{2}B\right]$

Trigonometric Equations

➢ $\sin\left(\dfrac{n\pi}{2} + \theta\right) = (-1)^{\frac{n}{2}} \cos\theta$, if n is odd

$= (-1)^{\frac{n}{2}} \sin\theta$, if n is even

➢ $\cos\left(\dfrac{n\pi}{2} + \theta\right) = (-1)^{\frac{n-1}{2}} \sin\theta$, if n is odd

$= (-1)^{\frac{n}{2}} \cos\theta$, if n is even

Inverse Trigonometric functions

➢ $\tan^{-1} A + \tan^{-1} B + \tan^{-1} C = \tan^{-1}\left(\dfrac{A + B + C - ABC}{1 - AB - BC - CA}\right)$

if A, B, C > 0 and (AB + BC + CA) < 1

➢ If $\tan^{-1} A + \tan^{-1} B + \tan^{-1} C = \dfrac{\pi}{2}$, then

(a) $AB + BC + CA = 1$
(b) $A + B + C = ABC$
(c) $A^2 + B^2 + C^2 + 2ABC = 1$

➢ If $\sin^{-1} A + \sin^{-1} B + \sin^{-1} C = \pi$, then $A\sqrt{1-A^2} + B\sqrt{1-B^2} + C\sqrt{1-C^2} = 2\,ABC$

➢ If $\sin^{-1} A + \sin^{-1} B + \sin^{-1} C = \dfrac{3\pi}{2}$, then $AB + BC + CA = 3$

➢ If $\cos^{-1} A + \cos^{-1} B + \cos^{-1} C = \pi$, then $A^2 + B^2 + C^2 + 2ABC = 1$

➢ If $\cos^{-1} A + \cos^{-1} B + \cos^{-1} C = 3\pi$, then $AB + BC + CA = 3$

➢ If $\cos^{-1}\dfrac{x}{a} + \cos^{-1}\dfrac{y}{b} = \theta$, then $\dfrac{x^2}{a^2} - \dfrac{2xy}{ab}\cos\theta + \dfrac{y^2}{b^2} = \sin^2\theta$

➢ In a triangle ABC, its area Δ is given by

$\Delta = \dfrac{1}{2} bc \sin A = \dfrac{1}{2} ca \sin B = \dfrac{1}{2} ab \sin C = \sqrt{s(s-a)(s-b)(s-c)}$

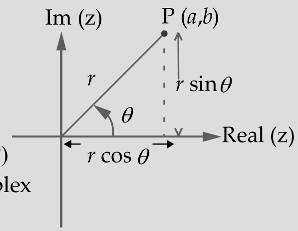
CHAPTER : 4 & 5 COMPLEX NUMBERS & QUADRATIC EQUATIONS

Complex Numbers & Quadratic Equations

Polar Representation

Let $a = r\cos\theta$
$b = r\sin\theta$
where,
$r = |z|$
and $\theta = \arg(z)$
$\therefore z = a + ib = r(\cos\theta + i\sin\theta)$
The argument 'θ' of complex number $z = a+ib$ is called principal argument of z if

$-\pi < \theta \leq \pi$.

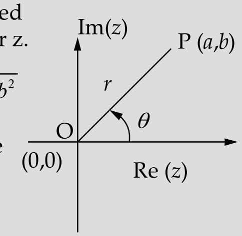


Modulus & Argument of Complex Number

If z= a+ib is a complex number
(i) Distance of z from origin is called as modulus of complex number z.

It is denoted by $r = |z| = \sqrt{a^2 + b^2}$

(ii) Angle θ made by OP with +ve direction of X-axis is called argument of z.



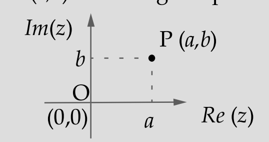
Powers of 'i'

$i = \sqrt{-1}, i^2 = -1$

In general, $i^{4k+r} = \begin{cases} 1; r = 0 \\ i; r = 1 \\ -1; r = 2 \\ -i; r = 3 \end{cases}$

Argand Plane

A complex number $z=a+ib$ can be represented by a unique point $P(a,b)$ in the argand plane



$z=a+ib$ is represented by a point P (a,b)

Square root of Complex Number

Let $z=x+iy=\sqrt{a+ib}$, squaring both sides, we get $(x+iy)^2= a+ib$ i.e. $x^2-y^2=a$, $2xy=b$ solving these equations, we get square root of z.

Multiplicative Inverse of complex Number

For a non-zero complex number $z=a+ib$ $(a \neq 0, b \neq 0)$,

there exists a complex number $\frac{a}{a^2+b^2} + \frac{-b}{a^2+b^2}i$

denoted by $\frac{1}{z}$ or z^{-1}, called multiplicative inverse of z

Such that: $(a+ib)\left(\frac{a}{a^2+b^2} + i\frac{-b}{a^2+b^2}\right) = 1 + 0i = 1$

Algebra of Complex Numbers

Let: $z_1 = a+ib$ and $z_1 = c+id$ be two complex numbers, where $a,b,c,d \in R$ and $i = \sqrt{-1}$

1. Addition: $z_1 + z_2 = (a+ib) + (c+id) = (a+c) + i(b+d)$

2. Subtraction: $z_1 - z_2 = (a+ib) - (c+id) = (a-c) + (b-d)i$

3. Multiplication: $z_1 . z_2 = (a+ib)\,(c+id)$
$= a(c+id) + ib(c+id)$
$= (ac-bd) + (ad+bc)i$
$(\because i^2 = -1)$

4. Division: $\frac{z_1}{z_2} = \frac{a+ib}{c+id} = \frac{a+ib}{c+id} . \frac{c-id}{c-id}$
$= \left(\frac{ac+bd}{c^2+d^2}\right) + \left(\frac{bc-ad}{c^2+d^2}\right)i$

Note: If $a+ib = c+id$
$\Leftrightarrow a = c \ \& \ b = d$

Solution of Quadratic Equation

- General form of quadratic equation in x is $ax^2+bx+c=0$,
  Where $a, b, c \in R$ & $a \neq 0$
  The solutions of given quadratic equation are given by $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$
  (i) If $b^2 - 4ac > 0$, roots are real and unequal
  (ii) If $b^2 - 4ac = 0$, roots are real and equal
  (iii) If $b^2 - 4ac < 0$, roots are imaginary

Note: • A monomial equation has one root.
• A polynomial equation of degree n has n roots.

Definition of Complex Numbers

A number of the form $a+ib$, where $a,b \in R$ and $i = \sqrt{-1}$ is called a complex number and denoted by 'z'.

$z = a+ib$ → a: Real part, ib: Imaginary part

Conjugate of a complex number: For a given complex number $z=a+ib$, its conjugate is defined as $\bar{z} = a - ib$

CHAPTER

4 COMPLEX NUMBERS

Chapter Objectives

Need for complex numbers, especially $\sqrt{-1}$, to be motivated by inability to solve some of the quadratic equations, algebraic properties of complex numbers, conjugate complex numbers, Argand plane and polar representation of complex numbers, modulus and argument, triangle inequality, loci on the Argand Diagram, product and quotient for complex numbers in modulus-argument form, De Moivre's theorem, square root of complex number, cube root of unity, Euler's Formula.

STUDY MATERIAL

I. Concept Clarified

1. Introduction

Integers (positive or negative) solve equations of the form $(x+1)(x-3)=0$.

Rational numbers (ratios of integers) solve equations of the form $(3x+5)(2x-1)=0$.

Irrational numbers solve equations of the form $x^2-2x-6=0$.

So far we have used numbers which may all be called 'real'; but no real number satisfies an equation such as $x^2+6x+11=0$, equivalent to $(x+3)^2=-2$.

To continue with this equation

$$x+3=\pm\sqrt{-2} \Leftrightarrow x=-3\pm\sqrt{-2}$$

If we now write $\sqrt{(-1)}=i$, we have obtained 'solutions' to the equation in the form $-3\pm i\sqrt{2}$. Such numbers of the form $a+ib$, where a and b are real, are called complex numbers; a is known as the real part and b as the imaginary part of $a+ib$. The quantity $\sqrt{-1}$ is an imaginary number and is denoted by letter 'i' called **iota.**

➢ **Important Points :**

(i) The real negative numbers expressed in terms of square root are called imaginary numbers.

Examples : $\sqrt{-2},\sqrt{-3},\sqrt{-5}$ (In general $\sqrt{-n}$ where $n=1,2,3,\ldots\ldots$)

(ii) $\sqrt{ab}=\sqrt{a}\sqrt{b}$ when $a>0, b>0$

(iii) $\sqrt{-ab}=\sqrt{-1}\sqrt{a}\sqrt{b}$

(iv) $\sqrt{-ab}\neq-\sqrt{ab}$ $(\because \sqrt{-1}=i)$

2. Integral power of iota

$$i^0=1,\ i=\sqrt{-1}\ and\ i^2=-1$$
$$i^3=i(i^2)=i(-1)=-i$$
$$i^4=(i^2)^2=(-1)^2=1$$